

# Forecasting Financial Assets Volatility Using Integrated GARCH-Type Models: International Evidence

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**Abstract** In this article we compare the forecasting ability of two symmetric integrated GARCH models (FIGARCH & HYGARCH) with an asymmetric model (FIAPARCH) based on a skewed Student distribution. Each model is used for forecasting the daily conditional variance of 10 financial assets, for a sample period of about 18 years. This exercise is done for seven stock indexes (Dow Jones, NASDAQ, S&P500, DAX30, FTSE100, CAC40 and Nikkei 225) and three exchange rates vis-a-vis the US dollar (the GBP- USD, YEN-USD and Euro-USD). Results indicate that the skewed Student AR (1) FIAPARCH (1.d.1) relatively outperforms the other models in out-of-sample forecasts for one, five and fifteen day forecast horizons. Results indicate also, no difference for the AR (1) FIGARCH (1.d.1) and AR (1) HYGARCH (1.d.1) models since they have the same forecasting ability.

**Keywords:** forecasting volatility, skewed Student distribution, Long-range memory

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## 1. Introduction

ARCH-type models have been the most commonly used models for time-series analysis. The ARCH model developed by Engle [20] is a model that allows the conditional variance to be time-varying, while the unconditional variance is constant. Bollerslev [9] focuses on extending the ARCH model to allow for a more flexible lag structure and he develops the generalized ARCH (GARCH) model. To account for some financial time series stylized facts, many variants of GARCH class models were proposed such as EGARCH, GJR-GARCH, APARCH, FIGARCH, HYGARCH...etc. The models and their later extensions were quickly found to be relevant for the conditional volatility of financial returns. More precisely, those models are usually used for both volatility modeling and forecasts of financial time series. see Bollerslev, Chou and Kroner [11], Bollerslev, Engle and Nelson [10], Bera and Higgins [8] and Diebold and Lopez [17]. Ghysels, Harvey and Renault [26], Allen et al. (2005), Angelidis et al. [3], Assaf (2009), Chiu et al. (2006), Cheong (2008), Shieh and Wu (2007), Lee and Saltoglu (2002), Awartani Corradi (2005), Gençay Selçukc (2004), Fan et al. (2004), Tse (1991) Giot and Laurent (2004), Bollerslev [12], Wright, (2008), Bali, (2000), Engle (1990), Hamilton and Susmel (1999), Gallo and Pacini (1998). In general, models that allow for volatility asymmetry come out well in the forecasting contest because of the strong negative relationship between volatility and shock. Cao and Tsay [13], Heynen and Kat [28], Lee [35] and Pagan and Schwert [45] favour the EGARCH model for volatility of stock indices and exchange rates, whereas Brailsford and Faff (1996) and

Taylor, J. [52] find GJR-GARCH outperforms GARCH in stock indices. During the last decades long-memory processes (the presence of statistically significant correlations between observations that are a large distance apart.) have evolved into a vital and important part of the time series analysis. A long memory series has autocorrelation coefficients that decline slowly at a hyperbolic rate. These features change dramatically the statistical behavior of estimates and predictions. As a consequence, many of the theoretical results and methodologies used for analyzing short-memory time series, for instance, ARMA and standard GARCH processes, are no longer appropriate for long-memory models (Plama, 2007). An important property of fractionally integrated GARCH models is their ability to capture both volatility clustering and long memory in financial time series. During recent years, several researches have been concerned with the long-range memory on both price variations and price volatilities. More precisely, the empirical literature is focused on volatility modelling when studied time series are governed by a long memory process. See Tang and Shieh [50], Yu So (2010), Assaf (2009), Chiu et al. (2006), Cheong (2008), Shieh and Wu (2007), Lee and Saltoglu (2002), Kang and Yoon (2007), Mabrouk and Aloui (2010), Mabrouk and Saadi (2012)...etc. These studies showed that stock market and exchange market volatility are governed by a long memory process. They concluded that the long memory GARCH class models outperform the other models. The long memory characteristic of financial market volatility has important implications for volatility forecasting and option pricing. Comparing forecasting performance of studied models is crucial for any forecasting exercise. In contrast to the efforts made in the

construction of volatility models and forecasts, little attention has been paid to forecast evaluation in the volatility forecasting literature. Figlewski [24] finds GARCH superiority confined to the stock market and for forecasting volatility over a short horizon only. Vilasuso (2002) tested FIGARCH against GARCH and IGARCH for volatility prediction for five major currencies. Vilasuso (2002) finds FIGARCH produces significantly better 1- and 10-day-ahead volatility forecasts for five major exchange rates than GARCH and IGARCH. Zumbach [56] produces only one-day-ahead forecasts and find no difference among model performance. Li [36], Martens and Zein [41] and Pong, Shackleton, Taylor and Xu [52], compared long memory volatility model forecasts with option implied volatility. In most applications, the excess kurtosis implied by the GARCH class model under a normal density is not enough to mimic what we observe on real data. Other distributions are possible. Bollerslev [9] proposed to use the Student-t distribution, since it implies conditional leptokurtosis and, therefore, stronger unconditional leptokurtosis. To account for excess kurtosis, the generalized error (GE) distribution was proposed by Nelson [42]. As reported by Pagan [44], the use of symmetric heavy-tailed distributions (such as Student-t distribution and the generalized error distribution) is common in the finance literature. In particular, Bollerslev [9], Hsieh (1989), Baillie and Bollerslev [6] and Palm and Vlaar (1997) among others show that these distributions perform better in order to capture the excess kurtosis. However, many financial times series returns are fat tailed and skewed. To account for both asymmetric and fat tail in the empirical density, Fernandez and Steel [23] proposed skewed-Student density which has been extended by Lambert and Laurent [34]. The aim of this paper is to contribute to the finance literature on volatility forecasting of financial time series. Thus, our goal is to compare the volatility forecasting ability of same non-linear models. Since all studied financial time series returns are skewed, fat tailed, exhibit ARCH effect and long memory, we adopt GARCH-type models with asymmetric innovation distributions (a skewed Student distribution) to forecast financial time series volatility for three horizons. More specifically, we are concerned with three GARCH-type models: the FIGARCH, FIAPARCH and HYGARCH. To the best of our knowledge, this paper represents one of the first studies focused on volatility forecasting of financial assets using the FIGARCH, FIAPARCH and HYGARCH processes under a skewed Student-t distribution. In addition, we compare the forecast accuracy under same alternative approaches. In our empirical application, we search for models that capture the features of the analyzed data and that provide accurate out-of-sample forecasts. Thus, our analysis has greater emphasis on in-sample fit, while our forecasting exercise will necessarily concentrate on out-of-sample outcomes. . The remainder of this paper proceeds as follows: Section 2 provides a description of GARCH-class models, density model and forecasting evaluation model employed in this paper. Section 3 presents the data and the empirical results of out-of-sample forecasting. Section 4 summarizes this paper.

## 2. Long-memory Models

### 2.1. The Fractional Integrated GARCH Model

Bailey, Bollerslev and Mikkelsen (1996) applied the concept of fractional integration to the conditional variance of a time series, proposing the fractionally integrated GARCH model (FIGARCH). Unlike the GARCH model is I(0) and IGARCH model that is I(1), the integrated fractional process I(d), distinguishes between short memory and long memory in time series.

According to Bailey et al. (1996), the FIGARCH (pdq) model is defined by:

$$y_t = \mu + \varepsilon_t, \varepsilon_t = z_t \sigma_t, z_t \sim N(0,1) \quad (1)$$

$$\varnothing(L)(1-L)^d \varepsilon_t^2 = \omega + [(1-\beta(L))v_t]$$

with,  $v_t = \varepsilon_t^2 - \sigma_t^2$ ; all the roots of  $\varnothing(L)$  and  $[(1-\beta(L))]$  is outside the unit circle

(1) can be rewritten as follows:

$$\sigma_t^2 = \omega + \left(1 - [1-\beta(L)]^{-1} \varnothing(L)(1-L)^d\right) \varepsilon_t^2 \quad (2)$$

$$\sigma_t^2 = \omega + \lambda(L) \sigma_t^2 \quad (3)$$

with,  $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + \dots$  and  $0 \leq d \leq 1$ .

For the FIGARCH process (pdq) is well defined and that the conditional variance is positive for all, all the coefficients of ARCH representation shall be positive.

$\lambda_j \geq 0$  for  $j = 1, 2, \dots$ . Consider the condition proposed by Bollerslev and Mikkelsen (1996), which is necessary and sufficient to ensure the non negativity  $\lambda_j$ :

$$\beta_1 - d \leq \varnothing_1 \leq \frac{2-d}{3} \quad (4)$$

$$\text{and } d \left[ \varnothing_1 - \frac{1-d}{2} \right] \leq \beta_1 (\varnothing_1 - \beta_1 + d)$$

For FIGARCH model (p.d.q), the persistence of the conditional variance or the long memory degree is measured by the parameter  $d$ . Thus, the model FIGARCH (p.d.q) will be attractive for  $0 < d < 1$ , which is an intermediate situation (finite long memory)

### 2.2. The Hyperbolic GARCH Model

Davidson has developed in 2004 a model called Hyperbolic GARCH which represents an extension of FIGARCH model. In fact, this model is based on the fact to test how non-stationary model of FIGARCH. Extending from the model HYGARCH, the FIGARCH resides in the addition of weight. The conditional variance of HYGARCH model can be formulated as follows:

$$\sigma_t^2 = \omega [1-\beta(L)]^{-1} + \left\{ \begin{array}{l} 1 - [1-\beta(L)]^{-1} \rho(L) \\ \left. \right) [1 + \alpha \{(1-L)^d\}] \end{array} \right\} \varepsilon_t^2 \quad (5)$$

The HYGARCH model becomes a simple GARCH when  $\alpha = 0$  and a model FIGARCH in case  $\alpha = 1$ . Therefore, GARCH and FIGARCH models are only special cases of HYGARCH model.

### 2.3. The FIAPARCH Model The Fractional Integrated Asymmetric Power ARCH Model

The FIAPARCH model can be considered an extension of the FIGARCH model with the APARCH model of Ding, Granger and Engle [18]. This model can capture both long memory and asymmetry in the conditional variance. The FIAPARCH model (p.d.q) can be specified as follows:

$$\sigma^\delta = \omega + \left[ 1 - (1 - \beta(L))^{-1} (1 - \varnothing(L))(1 - L)^d \right] \times (|\varepsilon_t| - \gamma \varepsilon_t)^\delta \tag{6}$$

$\delta > 0$ ,  $-1 < \gamma < 1$  and  $0 < d < 1$ . when,  $\gamma > 0$ , a negative shock increases volatility than a positive shock and vice versa. :

The FIAPARCH model becomes a FIGARCH model when  $\delta = 2$  and  $\gamma = 0$ . That 's why we can say that the FIAPARCH model is a generalization of FIGARCH model.

### 2.4. Density Model

In order to overcome the shortcomings of the symmetric Student-t distribution and to take into account both the skewness and excess kurtosis, we consider the skewed Student-t distribution proposed by Lambert and Laurent [34]. The latter distribution captures both asymmetry and thick tail (fat tail).

If  $z \sim SKST(0,1,k,\nu)$ , the log probability distribution function skewed Student-t is formulated as follows

$$L_{skst} = T \left\{ \begin{array}{l} \ln \Gamma \left( \frac{\nu+1}{2} \right) - \ln \Gamma \left( \frac{\nu}{2} \right) \\ - \frac{1}{2} \ln \left[ \pi (\nu - 2) \right] \\ + \ln \left( \frac{2}{k + \frac{1}{k}} \right) + \ln(s) \end{array} \right\} \tag{7}$$

$$- \frac{1}{2} \sum_{t=1}^T \left[ \ln(\sigma_t^2) + (1 + \nu) \ln \left[ 1 + \left( 1 + \frac{(sz_t + m)^2}{(\nu - 2)} \right) k^{-2I_t} \right] \right]$$

with,  $I_t = 1$  si  $z_t \geq m/s$  or  $I_t = -1$  si  $z_t < m/s$ ,  $k$  is a parameter of asymmetry, the constant  $m = m(k, \nu)$  and  $s = \sqrt{s^2(k, \nu)}$ , are the mean and standard deviation of the distribution skewed Student-t, respectively :

$$m(k, \nu) = \frac{\Gamma(\frac{\nu-1}{2})\sqrt{\nu-2}}{\sqrt{\pi}\Gamma(\frac{\nu}{2})} \left( k - \frac{1}{k} \right) \tag{8}$$

$$s^2(k, \nu) = \left( k^2 + \frac{1}{k^2} - 1 \right) - m^2 \tag{9}$$

The value of  $\ln(k)$  denotes the degree of asymmetry in the distribution of the residual term. Where,  $\ln(k) > 0$ ,

the density is asymmetric to right. If  $\ln(k) < 0$ , the density is asymmetric left. When  $\ln(k) = 0$ , that's means  $k = 1$ , the skewed Student-t distribution reduces to a general distribution of Student-t.

### 3. Forecast Evaluation Criteria

Since volatility itself is unobservable, the comparison of volatility forecasts relies on an observable proxy for the latent volatility process. Several criteria for measuring the predictive ability of the models were developed namely MSE (Mean Squared Error), MAE (Mean Absolute Error), TIC (Theil Inequality Coefficient) and the Mincer-Zarnowitz (MZ) regression, [40], which involves regressing the realization of a variable on a constant and its forecast. In our study we will use these four evaluation criteria to measure the predictive capabilities of the following long memory models: FIGARCH, HYGARCH and FIAPARCH.

The MZ regression is based on regression of realized volatility on a constant and expected volatility. Formally, the regression of MZ regression may be presented as follows

$$\sigma_{\text{realized}(t+1)} = \alpha + \beta \times \sigma_{\text{forecast}(t+1)} + \varepsilon_t \tag{10}$$

The MZ regression allows to evaluate two different aspects of the volatility forecast. First, the MZ regression allows to test the presence of systematic over or under predictions, that is, whether the forecast is biased, by testing the joint hypothesis  $H_{(0)} \alpha = 0 \cup \beta = 1$ . Second, being the  $R^2$  of Equation 10, an indicator of the correlation between the realization and the forecast, it can be used as an evaluation criterion of the accuracy of the forecast. Indeed, the model with the  $R^2$  closer to 1 indicates a great predictive power compared to those with  $R^2$  near 0. The  $R^2$  of the MZ regression has frequently been used as a criterion for ordering over a set of volatility forecasts [1,2].

To assess the forecasting power of the GARCH- type models under skewed Student-t distribution, we used different loss functions rather than make an individual choice. To compare the predicative power of studied models, many researchers used, Mean Square Error (MSE), Mean Absolute Error (MAE), Theil Inequality Coefficient (TIC) among other. see, Hansen and Lunde [30], Patton [46], Wang and Wu (2012) and Byun and Cho (2013). These three criteria may be presented respectively as follows:

$$MAE = \frac{1}{N} \sum_{t=1}^N |\varepsilon_t| = \frac{1}{N} \sum_{t=1}^N \left| \hat{\sigma}_t - \sigma_t \right| \tag{11}$$

$$MSE = \frac{1}{N} \sum_{t=1}^N \varepsilon_t^2 = \frac{1}{N} \sum_{t=1}^N \left( \hat{\sigma}_t - \sigma_t \right)^2 \tag{12}$$

$$TIC = \frac{\sum_{t=1}^N \left( \hat{\sigma}_t - \sigma_t \right)^2}{\sum_{t=1}^N \left( \hat{\varepsilon}_t^{BM} - \sigma_t \right)^2} \tag{13}$$

The forecasting exercise will focus on three horizons are; 1-day, 5-day and 15-day ahead.

## 4. Empirical Analysis

### 4.1. Data Description

To investigate the volatility forecasting power of GARCH- class models , the data employed in this study consists of daily closing prices of seven stock indexes (Dow Jones, Nasdaq, S&P500, DAX30, FTSE100, CAC40 and Nikkei 225) and three exchange rates vis-a-vis the US dollar (the GBP- USD, YEN-USD and Euro-USD). The raw data sets are downloaded from the web

<http://finance.yahoo.com>. For each financial asset's time series, the sample period and the number of observations are displayed in Table 1. The continuously compounded daily returns are computed as follows:

$$r_t = 100 \ln \left( \frac{P_t}{P_{t-1}} \right) \quad (14)$$

where  $r_t$  and  $P_t$  are the return in percent and the closing price on day (t), respectively. We should mention that for each asset the data set is subdivided into two subsets. The last 1000 day returns are reserved for the out-of-sample analysis.

**Table 1. Data sets**

<i>asset</i>	<i>noun</i>	<i>Sample period</i>	<i>Observations</i>
<i>indices</i>	<i>DAX</i>	01/02/1990 – 10/10/2008	4498
	<i>DOW JONES</i>	01/02/1990 – 10/10/2008	4734
	<i>NASDAQ</i>	01/02/1990 – 10/10/2008	4720
	<i>NIKKEI</i>	01/02/1990 – 10/10/2008	4607
	<i>CAC40</i>	01/02/1990 – 10/10/2008	4678
	<i>FTSE100</i>	01/02/1990 – 10/10/2008	4727
	<i>S&amp;P500</i>	01/02/1990 – 10/10/2008	4720
<i>exchange rates</i>	<i>EURO/USD</i>	12/31/1998 – 10/10/2008	2498
	<i>GBP/USD</i>	01/02/1990 – 10/10/2008	4777
	<i>YEN/USD</i>	01/02/1990 – 10/10/2008	4777

Descriptive statistics and unit root and stationarity tests for all the daily returns are reported in Table 2.

### 4.2. Preliminary Analysis

**Table 2. Descriptive statistics, unit root and stationarity tests**

<i>Asset</i>	<i>Mean</i>	<i>Median</i>	<i>SD</i>	<i>Skewness</i>	<i>Kurtosis</i>
<i>DAX</i>	0.0262	0.0741	1.3733	-0.394	7.6728
<i>DOW JONES</i>	0.0239	0.0476	1.009	-0.387	8.139
<i>NASDAQ</i>	0.0266	0.1014	1.5784	-0.291	10.1645
<i>NIKKEI</i>	-0.0317	-0.0232	1.3951	0.158	6.3244
<i>CAC40</i>	0.0112	0.0591	1.464	-0.362	9.9244
<i>FTSE100</i>	0.0120	0.0262	1.1053	-0.046	8.1293
<i>S&amp;P500</i>	0.0198	0.0435	1.042	-0.432	8.630
<i>EURO</i>	0.0077	-0.00004	0.5905	0.013	3.8602
<i>GBP</i>	0.0021	0.003	0.586	-0.405	8.811
<i>YEN</i>	0.0061	0.0077	0.672	-0.460	7.056

SD is the standard deviation. For all the time series, the descriptive statistics for cash daily returns are expressed in percentage.

As it's shown on the table above, all the daily return series have a positive mean unless the Nikkei stock index return which have a negative one. Furthermore, those time series data are not normally distributed in fact that the 3<sup>rd</sup> and the 4<sup>th</sup> moment respectively are different from zero and three. More precisely, the return series are skewed and fat tailed. Furthermore, for all time series the null hypothesis of presence of unit root is absolutely rejected by the Augmented Dickey-Fuller [19] (ADF), Phillips-Perron [47] (PP)<sup>1</sup> unit root tests. The Kwiatkowski,

Phillips, Schmidt and Shin [33] (KPSS) stationarity test<sup>2</sup> indicates that all time series returns are stationary at a 1% significant level.

### 4.3. Graphical Data Analysis

In Figure 1, we present graphs of the daily returns.

In Figure 1, we present graphs of the daily returns. The graph of the return series clearly shows that there are periods of low volatility followed by periods of high volatility (some tranquil periods as well as turbulent ones)

<sup>1</sup> The lag length or the ADF test regressions is set using the Schwarz information criteria (SIC) and the bandwidth for the PP test regressions is set using a Bartlett Kernel.

<sup>2</sup> These unit root and stationary test results could be considered with caution because these tests have been later refined by several authors including Elliot et al. (1996), Ng and Perron (2001).



which suggests volatility clustering and confirm the presence of ARCH effect.

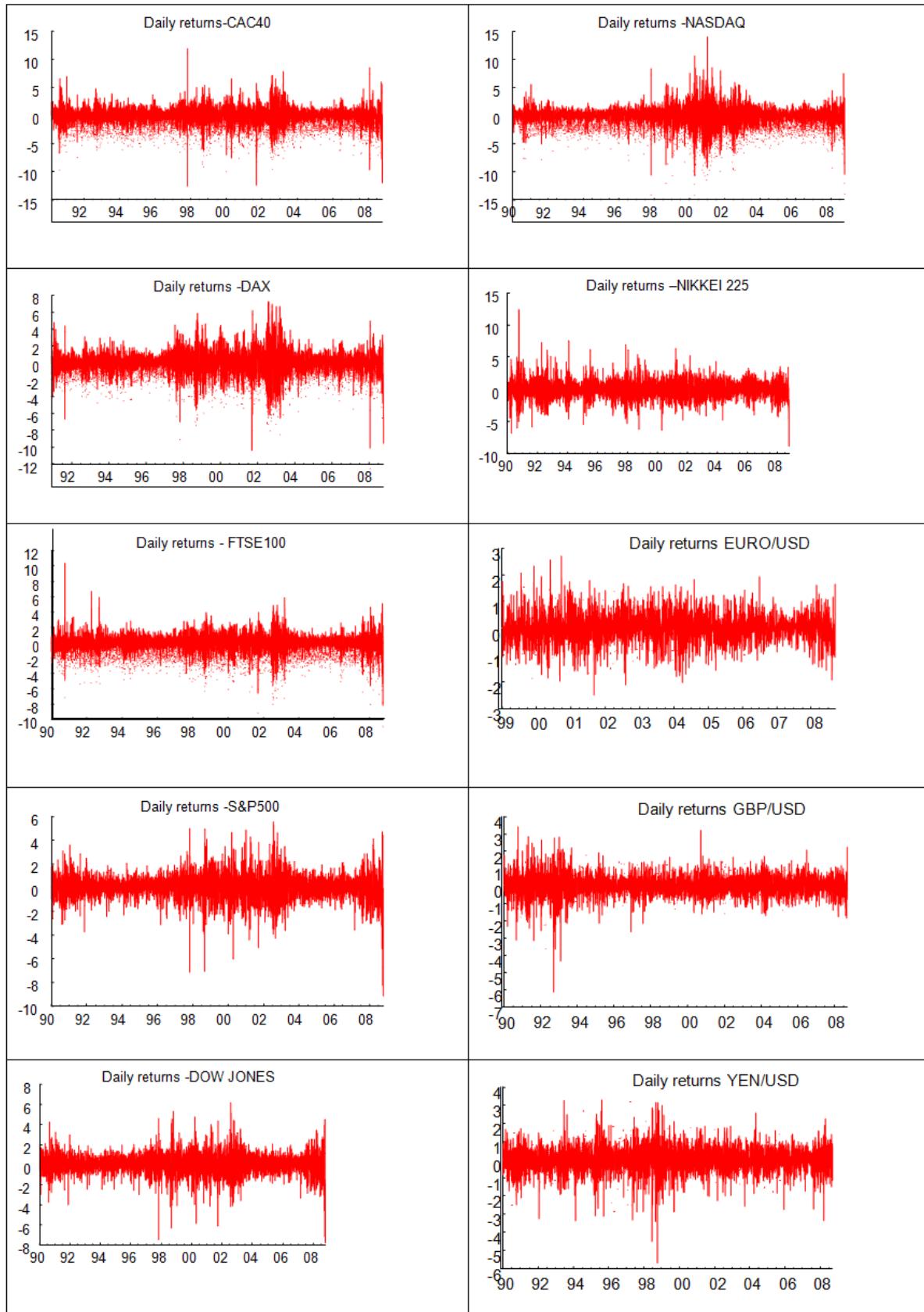


Figure 1.

#### 4.4. Long Range Memory Tests

Testing the presence of long range memory is important. Indeed, like many previous studies we use the absolute

returns and the daily squared volatility returns as two proxies of daily volatility. To test long memory we employed two long-range memory tests: Lo's [37] test and the log-periodogram regression (GPH) of Geweke and Porter-Hudak [25].

Table 3. long memory tests

Panel. a	$ r_t $				$r_t^2$			
	DAX	DOW JONES	NASDAQ	NIKKEI	DAX	DOW JONES	NASDAQ	NIKKEI
GPH Test $m = T^{0.5}$ $m = T^{0.6}$ $m = T^{0.7}$	0.62	0.48	0.49	0.25	0.59	0.35	0.45	0.26
	0.52	0.44	0.47	0.35	0.47	0.35	0.38	0.27
	0.45	0.43	0.38	0.29	0.41	0.37	0.36	0.24
Lo's RS Test	2.62746 {< 0.005}	1.89117 {< 0.025}	2.9925 {< 0.005}	0.90332 {< 0.95}	3.37581 {< 0.005}	2.56252 {< 0.005}	3.80299 {< 0.005}	1.61096 {< 0.02}
Panel. b	$ r_t $			$r_t^2$				
	CAC40	FTSE100	S&P500	CAC40	FTSE100	S&P500		
GPH Test $m = T^{0.5}$ $m = T^{0.6}$ $m = T^{0.7}$	0.50	0.45	0.38	0.33	0.36	0.20		
	0.44	0.40	0.26	0.29	0.33	0.16		
	0.39	0.42	0.24	0.28	0.32	0.17		
Lo's RS Test	1.70413 {< 0.1}	1.53375 {< 0.2}	1.52429 {< 0.2}	2.35328 {< 0.005}	2.00091 {< 0.025}	1.83822 {< 0.05}		
Panel. c	$ r_t $			$r_t^2$				
	EURO	GBP	YEN	EURO	GBP	YEN		
GPH $m = T^{0.5}$ $m = T^{0.6}$ $m = T^{0.7}$	0.34	0.45	0.41	0.28	0.30	0.31		
	0.26	0.37	0.41	0.21	0.33	0.40		
	0.16	0.26	0.32	0.10	0.25	0.29		
Lo's RS Test	0.93629 {< 0.9}	1.67277 {< 0.1}	1.01412 {< 0.9}	1.56165 {< 0.2}	2.94725 {< 0.01}	2.02818 {< 0.025}		

Notes:  $(r_t)$ ,  $(r_t^2)$ , and  $|r_t|$  are respectively log return, squared log return and absolute log return. (m) denotes the bandwidth for the Geweke and Porter-Hudakk's (1983) test.

Table 4. The predictive power of the FIGARCH, HYGARCH and FIAPARCH models under the skewed Student-t distribution ( 1-day , 5-day and 15-day horizons)

	Horizons		Indices						exchange rate			
			DAX	Dow Jones	NASDAQ	NIKKEI	CAC40	FTSE 100	S&P 500	Euro	GBP	Yen
AR(1) - FIGARCH	1-day	MSE	0.210	0.22	19.14	1.079	2.003	10.43	2.789	0.410	0.029	0.076
		MAE	0.458	0.469	4.375	1.039	0.909	3.23	1.67	0.640	0.172	0.276
		TIC	0.459	0.796	0.715	0.964	0.690	0.798	0.621	0.460	0.833	0.854
	5- day	MSE	1.365	0.644	4.391	3.391	1.429	2.174	0.701	0.156	0.163	0.069
		MAE	0.957	0.641	1.534	1.467	0.977	0.401	0.642	0.358	0.299	0.251
		TIC	0.538	0.690	0.606	0.581	0.500	0.714	0.523	0.402	0.593	0.415
	15- day	MSE	0.998	0.799	2.091	1.944	1.049	0.844	0.348	0.178	0.113	0.095
		MAE	0.848	0.692	1.058	1.185	0.887	0.560	0.465	0.370	0.265	0.289
		TIC	0.470	0.552	0.458	0.482	0.454	0.620	0.452	0.475	0.541	0.496
AR(1) - HYGARCH	1- day	MSE	0.207	0.218	18.95	1.117	1.905	10.37	2.726	0.411	0.029	0.076
		MAE	0.455	0.466	4.353	1.057	1.38	3.221	1.651	0.641	0.171	0.277
		TIC	0.458	0.795	0.709	0.964	0.479	0.794	0.610	0.461	0.832	0.854
	5- day	MSE	1.364	0.646	4.367	3.389	1.4	2.168	0.691	0.156	0.163	0.069
		MAE	0.954	0.738	1.539	1.48	0.985	0.897	0.646	0.395	0.298	0.251
		TIC	0.539	0.484	0.600	0.577	0.488	0.709	0.512	0.403	0.594	0.415
	15- day	MSE	0.996	0.798	2.08	1.948	1.042	0.848	0.355	0.178	0.113	0.095
		MAE	0.846	0.691	1.065	1.196	0.89	0.568	0.472	0.371	0.264	0.289
		TIC	0.471	0.553	0.478	0.479	0.443	0.616	0.445	0.475	0.542	0.496
AR(1) - FIAPARCH	1-day	MSE	0.266	0.422	19.19	1.132	1.76	10.54	2.501	0.150	0.031	0.082
		MAE	0.516	0.650	4.380	1.064	1.327	3.246	1.582	0.671	0.176	0.286
		TIC	0.489	0.844	0.716	0.964	0.452	0.805	0.570	0.493	0.836	0.858
	5 -day	MSE	1.381	0.538	4.425	3.363	1.356	2.187	0.667	0.163	0.164	0.071
		MAE	1.007	0.691	1.555	1.463	1.005	0.880	0.665	0.366	0.300	0.253
		TIC	0.520	0.393	0.594	0.578	0.465	0.722	0.475	0.419	0.591	0.414
	15- day	MSE	1.056	0.833	2.149	1.932	1.065	0.830	0.411	0.179	0.113	0.099
		MAE	0.889	0.780	1.109	1.181	0.902	0.540	0.529	0.370	0.265	0.296
		TIC	0.457	0.500	0.452	0.482	0.421	0.646	0.434	0.482	0.540	0.496

Table 4 displays the results of long-memory tests including two tests: Lo's R/S and GPH test for three BANDWIDTH  $m = T^{0.5}$ ,  $m = T^{0.6}$  and  $m = T^{0.7}$ . As shown, Lo's R/S that tests in null hypothesis  $H_0$  for the presence

of short memory vs. Long memory (long dependence) indicates the presence of long memory in both absolute log return and squared log return. Furthermore, the GPH test rejects the null hypothesis of short memory. Indeed, the two time series are governed by long-memory process. Thus, the long memory ARCH type models are highly recommended for the volatility modelling of the studied return series. For each return series, we fit three specific models, namely FIGARCH, HYGARCH and FIAPARCH. The models are estimated on a rolling basis, using a window of 1000 observations, and under a skewed Student-t distribution. The three models are then used to produce one, five and fifteen -day-ahead variance forecasts. The models are compared using some of the methods described in the previous section.

## 5. Forecasts Evaluation Results

In our study we consider a multiple comparison without control test of Hansen et al. (2011)), where all the forecasts are compared against each other.

Moving to the out-of-sample comparison, we start from the outcomes of the MSE, MAE and TIC loss functions. In order to evaluate model performance across different market, we consider last 1000 observations for our out-of-sample forecasting exercise.

The evaluation results of the predictive ability of the FIGARCH, FIAPARCH and HYGARCH models adjusted by the skewed Student-t distribution for different horizons are included in Table 4 and Table 5.

**Table 5. The predictive ability of the FIGARCH, HYGARCH and FIAPARCH models under the skewed Student-t distribution based on MZ regression (1969)**

		AR(1) - FIGARCH(1,d,1)			AR(1) - HYGARCH(1,d,1)			AR(1) - FIAPARCH(1,d,1)		
		$\alpha$	$\beta$	R <sup>2</sup>	$\alpha$	$\beta$	R <sup>2</sup>	$\alpha$	$\beta$	R <sup>2</sup>
Indices	DAX	0.11	-0.04	0.000533006	0.11	-0.04	0.000537348	0.08	-0.01	9.61483e-005
	DOWJONES	-0.16	0.38	0.00643073	-0.16	0.38	0.00641623	-0.14	0.306	0.00722727
	NASDAQ	-0.033	0.07	0.000642019	-0.03	0.07	0.000691542	-0.02	0.06	0.000665357
	NIKKEI	0.068	-0.002	1.57335e-006	0.06	0.000	2.12012e-007	0.04	0.01	8.50767e-005
	CAC40	0.06	-0.01	9.88671e-005	0.07	-0.01	9.80916e-005	0.033	0.026	0.00028156
	FTSE 100	0.04	0.01	9.27359e-005	0.04	0.01	6.49319e-005	0.02	0.05	0.0010579
	S&P 500	-0.11	0.28	0.00369096	-0.11	0.29	0.00383687	-0.13	0.29	0.0070623
Exchange rate	Euro	0.03	-0.08	4.03567e-005	0.04	-0.10	5.87205e-005	-0.002	0.03	1.14566e-005
	GBP	0.17	-0.64	0.00614568	0.18	-0.65	0.00619664	0.21	-0.77	0.00943436
	Yen	0.03	-0.08	8.88553e-005	0.03	-0.08	8.78054e-005	0.03	-0.08	0.000116371

Table 4 reports the full set of empirical results. Focusing on the MSE and MAE loss functions, all empirical models seem very similar for all indices and exchange rate, with the null hypothesis of zero loss function differential being rejected only in few cases. When we consider the TIC loss function, the null hypothesis is rejected more frequently, with the finding seemingly independent of the sample used for model evaluation. In this case, there are some differences across exchange rate and indices, but the outcomes suggest a preference of FIGARCH and FIAPARCH over HYGARCH. All models are equivalent as they are all included in the confidence set independently of the loss function used for their evaluation. In summary, there is not a clear preference for a specific model. Model preference depends on the loss function under consideration and on the sample period used for model evaluation.

Table 5 highlights that FIAPARCH (1,d,1) is always preferred to its FIGARCH(1,d,1) and HYGARCH (1,d,1) counterpart for all return series. The results for FIGARCH (1,d,1) and HYGARCH (1,d,1) are quite similar. Therefore, FIAPARCH is the preferred conditional volatility model. This is confirmed by a relatively higher R<sup>2</sup> of the MZ regression (1969) for FIAPARCH model than those of FIGARCH and HYGARCH models. This finding is not surprising as FIAPARCH is more flexible than FIGARCH and HYGARCH, can exhibit long memory, volatility clustering, asymmetry and leverage, and there are no restrictions on the parameters of the model.

The main conclusion is that when we give a great importance to volatility spikes, most models seem relevant, and simple specifications may perform as well as their

more flexible counterparts. If we consider the evolution over time of the conditional volatility, then more flexible models are to be preferred. Our findings are consistent with those of Chortareas et al (2011), Balaban (2004) Bollerslev, Poon and Granger (2003) and Xekalaki and Degiannakis (2010) who they show that the predictive ability of the AR (1) -APARCH (1,1) adjusted by the skewed Student-t distribution outperforms those of the AR (1) -GJR (1,1), and AR (1) -IGARCH (1,1) models. Bollerslev et al. (1994), Diebold and Lopez (1996) and Lopez (2001), confirm that none of the GARCH models can out-perform all of the others under the criteria of different loss functions.

## 6. Conclusion

In this paper, we focused on the predictive capacity of three conditional variance models with long memory. More precisely, our study involved ten sets of financial assets. The skewed Student-t distribution was used to adjust the conditional volatility models to take account of the asymmetry criterion of the daily return series. We focused on forecast evaluation and comparison where the forecast accuracy is measured by a statistical criterion. We tried to study the predictive ability of FIGARCH, HYGARCH and FIAPARCH models adjusted by the skewed Student-t distribution for three horizons (one, five and fifteen days). The results of the forecasting exercise show that the three models have the same predictive power. However, result based on the MZ regression (1969) confirms that the FIAPARCH model has relatively better predictive ability compared to FIGARCH and HYGARCH models.

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