

Using the Properties of Wavelet Coefficients of Time Series for Image Analysis and Processing

Vyacheslav V. Lyashenko¹, Rami Matarneh^{2,*}, Zhanna V. Deineko³

¹Department of Informatics, Kharkov National University of Radio Electronics, Kharkov, Ukraine

²Department of Computer Science Prince Sattam Bin Abdulaziz University Saudi Arabia, Al-Kharj

³Department of Media Systems and Technology, Kharkov National University of Radio Electronics, Kharkov, Ukraine

*Corresponding author: ramimatarneh@gmail.com

Abstract Image processing is used in many fields of knowledge; because it allows to automate processes to get more information about the object being studied. Image processing techniques are many and varied. Wavelet analysis is one of such techniques. Among various methods and approaches of wavelet processing we distinguish the ideology of multiresolution wavelet analysis. The essence of this ideology is to perform wavelet decomposition on test data and the subsequent analysis of the relevant factors of this decomposition (the wavelet coefficients). An important aspect is the consideration of the properties of the wavelet coefficients. Based on this, we have examined the feasibility of using the properties of detailing wavelet coefficients to study and compare different images. We have introduced additional characteristics of images on the basis of sets of detailing wavelet coefficients decomposition. These characteristics reflect the dynamics of change in mean and variance for the detailing of the coefficients of the wavelet decomposition. We have shown that the dynamic changes in mean and variance of detailing coefficients of wavelet decomposition can be used to analyze and compare different images.

Keywords: *image, wavelet transform, image processing, wavelet coefficients, detailing coefficients, approximating coefficients, levels of wavelet decomposition*

Cite This Article: Vyacheslav V. Lyashenko, Rami Matarneh, and Zhanna V. Deineko, "Using the Properties of Wavelet Coefficients of Time Series for Image Analysis and Processing." *Journal of Computer Sciences and Applications*, vol. 4, no. 2 (2016): 27-34. doi: 10.12691/jcsa-4-2-1.

1. Introduction

Image analysis is a powerful tool in various fields of knowledge; because of significant part of information that can be received. At the same time, various methods of processing and interpretation of images are form the basis of some of artificial intelligent systems. If we take into consideration the possibility of applying various methods of image analysis and processing in a combined form with an interconnected methodology, this inevitably will play an important role to solve practical problems.

On the other hand, if we take into consideration the possibility of applying various methods of image analysis and processing in a separate form, this will allow us to: (1) choose the most comprehensive methods of image analysis and processing, (2) to optimize the structure of artificial intelligent systems which are based on the analysis of images and (3) to increase the productivity and the overall performance of data analysis. Usually, images of the real world are presented in a two dimensional form, which allows the use of separate methods of image analysis and processing. These methods can be classified into three category: (1) Methods of preliminary image processing (noise suppression, contrast increase, localization of separate sites of the image) [1,2], (2) Methods of preliminary analysis (segmentation, contour allocation) [3,4], (3) Cognitive recognition methods [5,6,7].

The mentioned methods use different mathematical apparatus: group theory, linear algebra, graph theory. Currently, the image processing technique is widely use wavelet analysis [8,9,10]. This is due to the fact that the use of wavelet analysis procedure yields: the possibility of processing for various aspects of the studied images from the point of view of their method of presentation and the ability to use different approaches to highlight any aspects presentation of visual images during subsequent processing. These reasons determines the basis of our research idea.

2. Materials and Methods

2.1. Wavelet Analysis as a Tool for Image Processing

The main idea of wavelet transform is a time-and-frequency signal notation [11]. Wavelet transform is a decomposition with the use of functions, each of which is a shifted and scaled copy of one function – mother wavelet [11,12]. Wavelet in this case is a function rapidly decreasing to infinity with average value equals to zero. If the signal is discontinuous, only those wavelets will have high amplitudes, where the maximum value will appear near the discontinuity point. At the same time, discontinuity point is a sharp intermittent transition during some process. Quantitatively, it can be estimated by the

value of the first derivative of such process, taking into consideration that the first derivative of intermittent transitions is very high [10,12]. The real processes in reality cannot have perfect discontinuity points. In fact, the measured fractal transitions are characterized by the finite value of the derivative. The sharper the transition,

the higher the derivative value is. Smooth transitions will have small derivative values. This allows us to determine the presence of special characteristics of the analyzed image, as well as the point where these characteristics may arise [10,12]. At the same time we can present two-dimensional image as a vector (Figure 1).

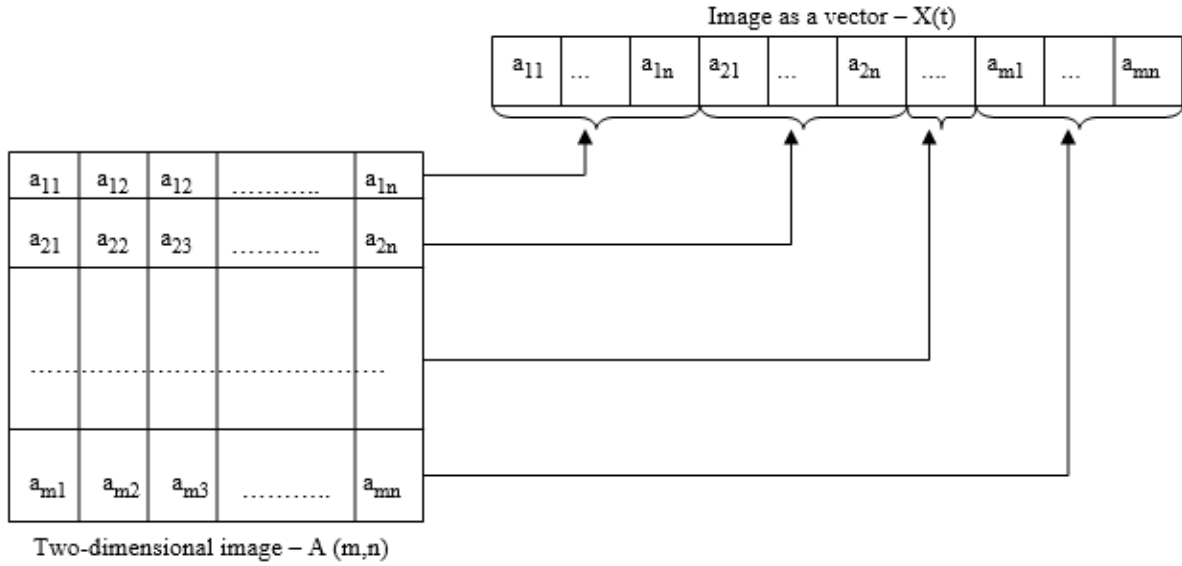


Figure 1. Two-dimensional image as a vector

Then, for the image analysis, we can use the so-called multiresolution analysis method, on the basis of the theory of wavelet transformations [13,14,15]. A multiresolution wavelet-analysis transforms time series to hierarchical structure by means of the wavelet transformations which results to the set of wavelet coefficients. On each new level of wavelet-expansion there is a division of an approximating signal of the previous level of detail (presented by some time series) on its high-frequency component and on more smoothed approximating signal [13, 15]. According to discrete wavelet-transformation time series $X(t)$, ($t = t_1, t_2, \dots, X(t_1) = a_{11}, X(t_2) = a_{12}, X(t_3) = a_{13}, \dots, X(t_{mn}) = a_{mn}$) consists of a set of coefficients – detailing and approximating [16]:

$$X(t) = \sum_{k=1}^{N_a} apr(N, k) \phi_{N,k}(t) + \sum_{j=1}^N \sum_{k=1}^{N_j} det(j, k) \psi_{j,k}(t) \quad (1)$$

- Where $apr(N, k)$ – Approximating wavelet-coefficients of level N ;
- $det(j, k)$ – Detailing wavelet-coefficients of level j ;
- N – Chosen maximum level of expansion;
- N_j – Quantity of detailing coefficients at j level of expansion;
- N_a – Quantity of approximating coefficients at level N
- $\psi(t)$ – Mother wavelet-function
- $\phi(t)$ – Corresponding scaling-function.

At set the mother wavelet and $\psi(t)$ corresponding scaling-function approximating $\phi(t)$ coefficients and $a_X(j, k)$ detailing coefficients of $d_X(j, k)$ DWT (discrete wavelet transformation) for the process $X(t)$ can be defined as follows [16]:

$$a_X(j, k) = \int_{-\infty}^{+\infty} X(t) \phi_{j,k}(t) dt \quad (2)$$

$$d_X(j, k) = \int_{-\infty}^{+\infty} X(t) \psi_{j,k}(t) dt \quad (3)$$

In particular, on each level of discrete wavelet transformation detailing coefficients represent features, detail of the investigated signal, arising at transition from one scale to another and are equal [16]:

$$d_X(j, k) = \langle X, \psi_{j,k} \rangle, \quad (4)$$

Where

$d_X(j, k)$ – Detailing wavelet-coefficients $k = \overline{1, N_j}$ on level j ,

$\langle X, \psi_{j,k} \rangle$ – Scalar product of investigated sequence of data in the form of time series $X(t)$ and a mother wavelet ψ on corresponding level of expansion j .

Thus, the main tool for the research of studied processes is processing of the wavelet-coefficients which have been received on different scales. First of all it concerns detailing wavelet-coefficients, which emphasize the characteristics of the time series.

2.2. Basic Properties of Wavelet Coefficients of Self-similar Time Series

1. Detailing coefficients of DWS on each level of expansion j have normal distribution with a zero average $N(0, \sigma)$.

- On each level of expansion j detailing wavelet-coefficients $d_X(j, k)$, $k = \overline{1, N_j}$ are self-similar [16]:

$$\begin{aligned} & (d_X(j, 0), d_X(j, 1), \dots, d_X(j, N_j - 1)) \\ & \cong 2^{\frac{j(H+\frac{1}{2})}{2}} (d_X(0, 0), d_X(0, 1), \dots, d_X(0, N_j - 1)) \end{aligned} \quad (5)$$

- The wavelet-coefficients received as a result of expansion of process with stationary increments are stationary on each scale 2^j .
- If there are moments of p order then for the wavelet-coefficients which were received as a result of expansion of process $X(t)$, the following equality is carried out [16]:

$$M \left[|d_X(j, k)|^p \right] = M \left[|d_X(0, k)|^p \right] 2^{jp(H+\frac{1}{2})} \quad (6)$$

Where $M[\dots]$ – expectation value of the process, which is studied.

The above listed properties of wavelet-expansion can be used for the analysis of various images in practice. For this we look at the various images and the analysis detailing wavelet-coefficients.

2.3. Data for Analysis

For the analysis, we use a variety of images (these images are in public internet access): Figure 2: Cytological preparation. Mammary gland puncture (image size 1328x996 pixels) and Figure 3: Image for eye fundus (image size 933x937 pixels).

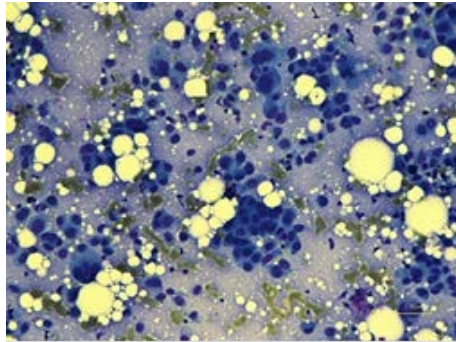


Figure 2. Cytological preparation. Mammary gland puncture



Figure 3. Image for eye fundus

The image size determines the length of the time series and the number of levels of wavelet decomposition.

3. Results and Discussion

First of all, we transform the two-dimensional original image (Figure 2 and Figure 3) in the one-dimensional series in accordance with Figure 1. The converted original image (Figure 2 and Figure 3) are shown in Figure 4 and Figure 5, respectively.

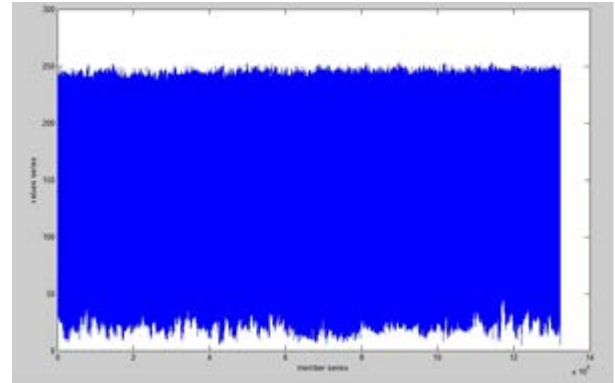


Figure 4. One-dimensional series of image in Figure 2

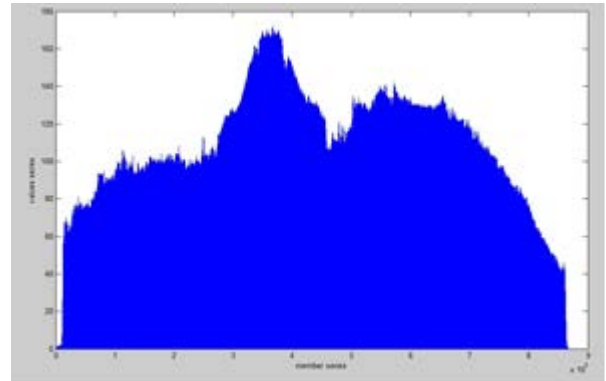


Figure 5. One-dimensional series of image in Figure 3

It is noticeable that the transformed series (Figure 4 and Figure 5) and the original image (Figure 2 and Figure 3) are different, which may allow us to make general conclusions about the possibility of using the wavelet expansion coefficients for image analysis.

In accordance with the size of the original images (Figure 2 and Figure 3) we will make the expansion of one-dimensional series (Figure 4 and Figure 5) into 10 levels with the help of different wavelets (db1, db2 and db4). We will also calculate basic statistical parameters for detailing the coefficients of the wavelet decomposition (expectation value, dispersion, intervals for the parameter that estimates the standard deviation). Thus, for each of source image, we will obtain a specific set of statistical characteristics at each level of decomposition. Table 1 presents the basic statistical characteristics for detailing wavelet coefficients of image in Figure 2, which is represented as a one-dimensional series in Figure 4. Table 2 presents the basic statistical characteristics for detailing wavelet coefficients of image in Figure 3, which is represented as a one-dimensional series in Figure 5. Analysis of Table 1 and Table 2, as well as the detailing wavelet coefficients show that detailing wavelet

coefficients of the series, which represent the images at Figure 2 and Figure 3 is the first property of the Wavelet coefficients decomposition.

Table 1. Basic statistical characteristics for detailing wavelet coefficients of image in Figure 2

Decomposition Level	Expected Value	Dispersion	Intervals For the Parameter Estimates the Standard Deviation (significance level 0.05)	
wavelet db1				
1	0,01	41,70	6,45	6,47
2	0,03	278,12	16,64	16,72
3	0,06	1436,82	37,78	38,04
4	0,63	5391,36	73,07	73,78
5	-0,01	16779,40	128,66	130,43
6	0,03	39701,97	197,36	201,20
7	-0,07	75568,27	271,21	278,71
8	0,57	106487,89	320,18	332,77
9	0,48	120980,58	338,66	357,64
10	-0,97	127273,47	339,87	366,33
wavelet db2				
1	0,01	12,53	3,53	3,55
2	-0,02	157,48	12,52	12,58
3	-0,03	1111,62	33,23	33,46
4	0,16	4772,90	68,76	69,42
5	0,00	16291,09	126,77	128,51
6	-0,02	40859,19	200,21	204,11
7	0,11	82835,04	283,95	291,80
8	0,39	117056,77	335,70	348,90
9	0,59	132558,98	354,50	374,36
10	-0,82	135207,52	354,20	382,60
wavelet db4				
1	0,01	8,16	2,85	2,86
2	0,00	105,97	10,27	10,32
3	0,04	1008,34	31,65	31,86
4	-0,30	4459,41	66,46	67,10
5	0,01	16153,79	126,24	127,97
6	-0,02	41077,21	200,75	204,65
7	0,00	86204,75	289,67	297,68
8	-0,37	119885,34	339,73	353,08
9	0,52	134238,39	356,74	376,72
10	0,31	150581,51	373,82	403,74

In Table 1 and Table 2, we see that the expectation value in most cases is zero. Consequently, for detailing the coefficients of wavelet decomposition we have a normal distribution with zero average $N(0, \sigma)$ at each level of decomposition. This allows us to use other properties of detailing the coefficients of wavelet decomposition. Given the previously mentioned properties 2 through 4, it is possible to say that each image is characterized by its own set of detailing coefficients at each level of decomposition. Then as individual characteristics of the image can be considered indicators as follows:

Given the properties of 2-4 can say that each image is characterized by its own set of detailing coefficients at each level of decomposition. Then, as individual characteristics of the image the following indicators can be considered:

$$pM0 = \frac{|M[d_X(j,k)]|}{|M[d_X(0,k)]|} \quad (7)$$

$$pM1 = \frac{|M[d_X(j+1,k)]|}{|M[d_X(j,k)]|} \quad (8)$$

$$pD0 = \frac{D[d_X(j,k)]}{D[d_X(0,k)]} \quad (9)$$

$$pD1 = \frac{D[d_X(j+1,k)]}{D[d_X(j,k)]} \quad (10)$$

Where

$M[d_X(0,k)]$ – Expected value of detailing wavelet coefficients at the first level,

$M[d_X(j,k)]$ – Expected value of detailing wavelet coefficients at j level ($j = \overline{1, N-1}$),

$D[d_X(0,k)]$ – Dispersion of detailing wavelet coefficients at the first level,

$D[d_X(j,k)]$ – Dispersion of detailing wavelet coefficients at j level ($j = \overline{1, N-1}$),

$j, j+1$ – Previous and subsequent levels of wavelet decomposition.

Table 2. Basic statistical characteristics for detailing wavelet coefficients of image in Figure 3

Decomposition level	expected value	dispersion	intervals for the parameter estimates the standard deviation (significance level 0.05)	
wavelet db1				
1	0,00	1,81	1,34	1,35
2	0,00	8,25	2,86	2,88
3	0,00	33,46	5,76	5,81
4	0,01	134,47	11,53	11,67
5	0,02	545,09	23,15	23,55
6	-0,01	2153,37	45,86	46,96
7	-0,01	9629,12	96,52	99,81
8	0,05	46604,30	210,91	221,16
9	-0,10	194742,78	427,10	456,75
10	-0,26	267206,82	493,80	542,99
wavelet db2				
1	0,00	0,99	0,99	1,00
2	0,00	5,55	2,35	2,36
3	0,00	23,06	4,78	4,82
4	-0,01	95,11	9,70	9,81
5	0,02	381,27	19,36	19,69
6	0,02	1338,37	36,16	37,02
7	-0,01	5445,80	72,58	75,06
8	-0,01	32360,72	175,75	184,29
9	-0,12	241884,95	476,01	509,03
10	-0,04	291812,82	516,07	567,41
wavelet db4				
1	0,00	0,75	0,86	0,86
2	0,00	4,97	2,22	2,24
3	0,00	20,90	4,55	4,59
4	0,01	86,01	9,22	9,33
5	-0,02	362,09	18,87	19,19
6	0,00	1218,40	34,50	35,33
7	0,02	4428,70	65,46	67,69
8	0,00	22108,85	145,27	152,32
9	0,01	277289,20	509,68	544,99
10	-0,10	271643,19	497,96	547,38

In Figure 6 and Figure 7 presented dynamic indicators $pM0$ and $pM1$ for image in Figure 2 taking into consideration that the decomposition is in different wavelet functions (allocated colors).

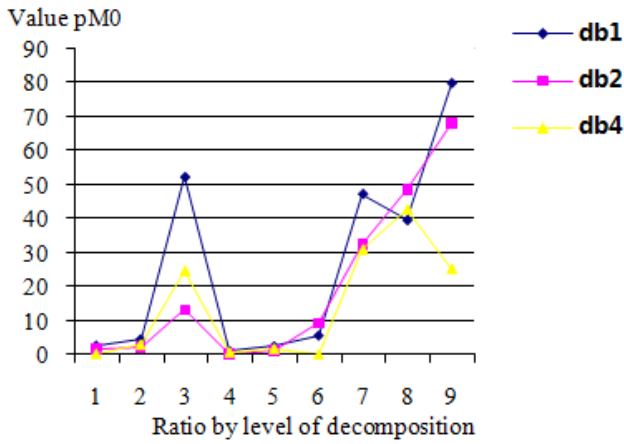


Figure 6. Dynamic indicator $pM0$ for Figure 2

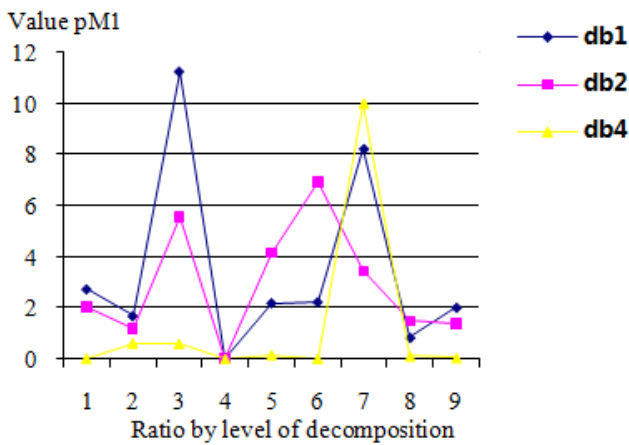


Figure 7. Dynamic indicator $pM1$ for Figure 2

In Figure 8 and Figure 9 presented dynamic indicators $pM0$ and $pM1$ for image in Figure 3 taking into consideration that the decomposition is in different wavelet functions (allocated colors).

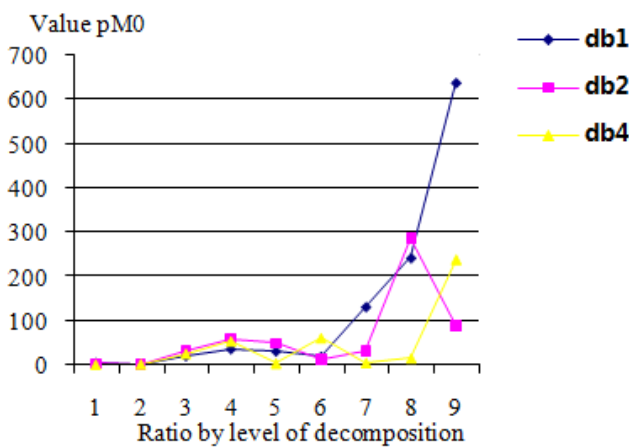


Figure 8. Dynamic indicator $pM0$ for Figure 3

Analysis of Figure 6, Figure 7, Figure 8 and Figure 9 shows that changes in dynamic indicators $pM0$ and

$pM1$ for wavelet decomposition at different wavelet functions (db1, db2 and db4) is approximately the same for the same image, and different for different images. At the same time we can say that the indicators $pM0$ and $pM1$ reflect the various manifestations and features of detailing wavelet coefficients at decomposition at different levels depending on the wavelet function (db1, db2 and db4), which was used to decompose (see Figure 10 through Figure 13).

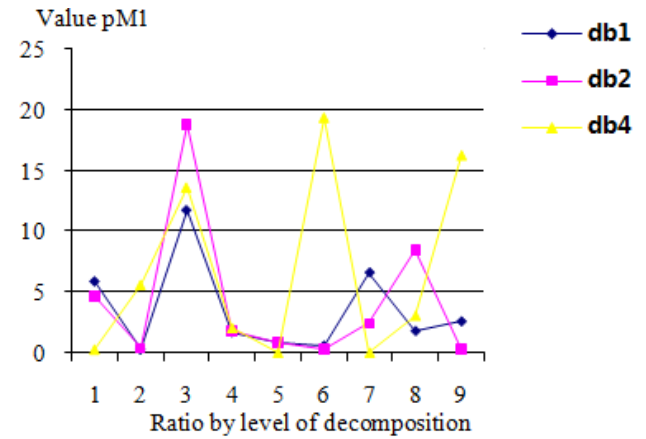


Figure 9. Dynamic indicator $pM1$ for Figure 3

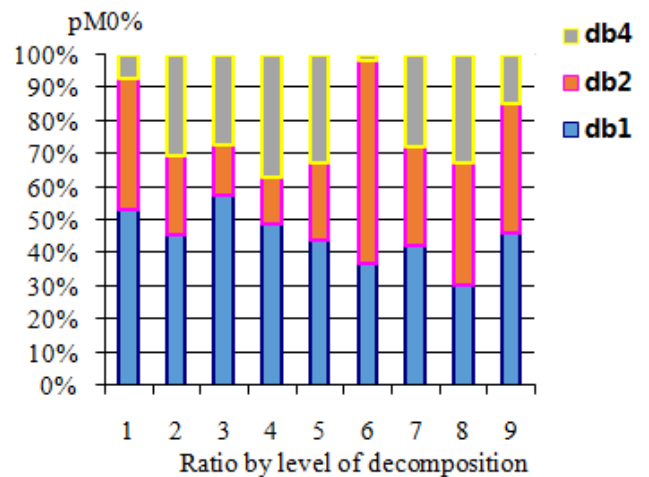


Figure 10. Dynamic percentage $pM0$ for Figure 6

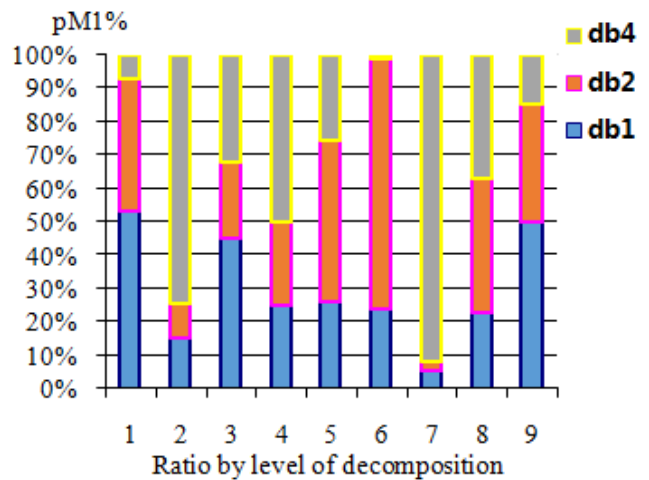


Figure 11. Dynamic percentage $pM1$ for Figure 7

In Figure 10 and Figure 11 presented dynamic indicators $pD0$ and $pD1$ for image in Figure 2 taking into consideration that the decomposition is in different wavelet functions (allocated colors).

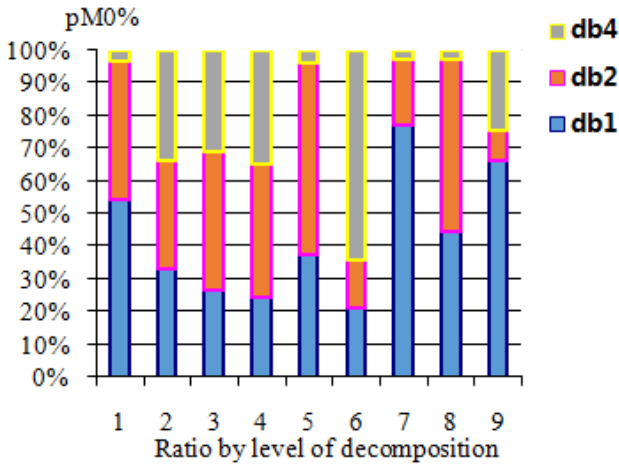


Figure 12. Dynamic percentage $pM0$ for Figure 8

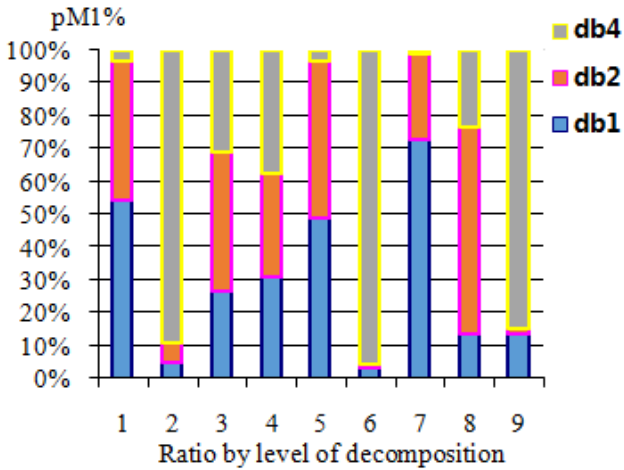


Figure 13. Dynamic percentage $pM1$ for Figure 9

In Figure 12 and Figure 13 presented dynamic indicators $pD0$ and $pD1$ for image in Figure 3 taking into consideration that the decomposition is in different wavelet functions (allocated colors).

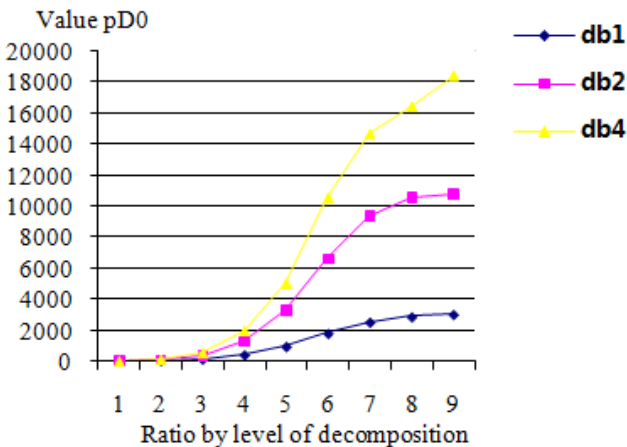


Figure 14. Dynamic indicator $pD0$ for Figure 2

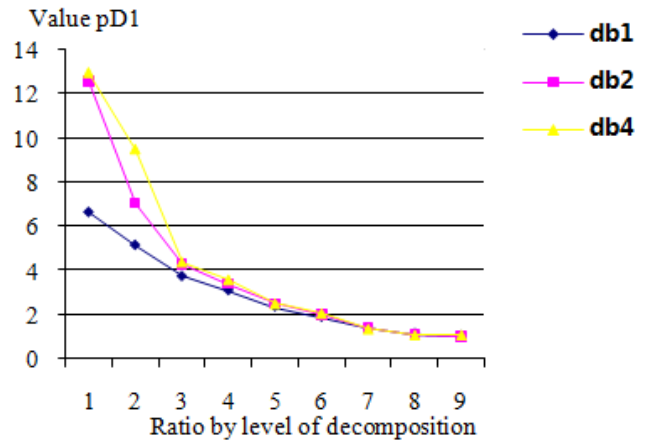


Figure 15. Dynamic indicator $pD1$ for Figure 2

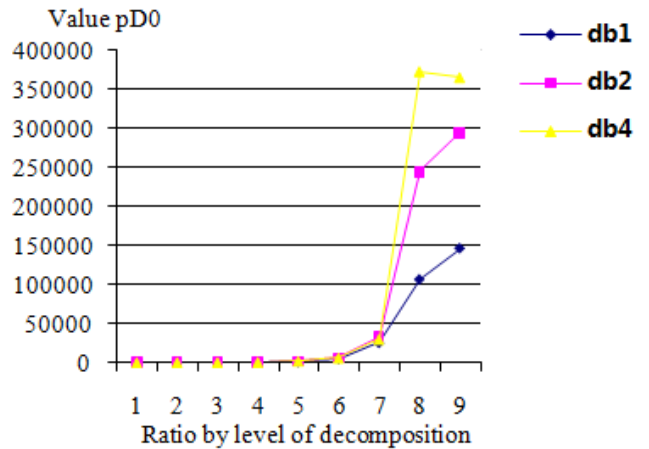


Figure 16. Dynamic indicator $pD0$ for Figure 3

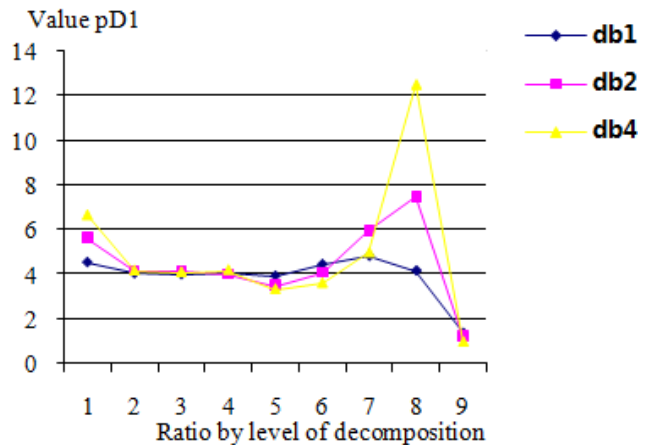


Figure 17. Dynamic indicator $pD1$ for Figure 3

Analysis of Figure 14, Figure 15, Figure 16 and Figure 17 shows that changes in dynamic indicators $pM0$ and $pM1$ for wavelet decomposition at different wavelet functions (db1, db2 and db4) is approximately the same for the same image, and different for different images. Therefore, the indicators $pM0$, $pM1$, $pD0$ and $pD1$ can be used to compare the images with each other, identifying the distinguishing features between them based on the levels of wavelet decomposition, which is used for image analysis (see Figure 18 through Figure 21).

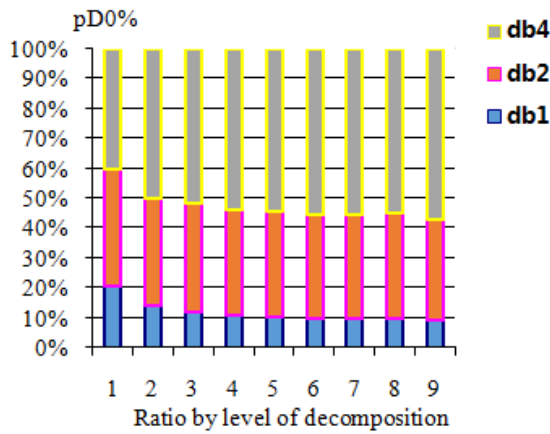


Figure 18. Dynamic percentage $pD0$ for Figure 14

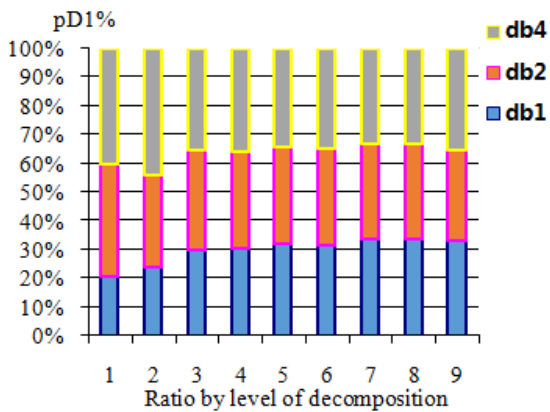


Figure 19. Dynamic percentage $pD1$ for Figure 15

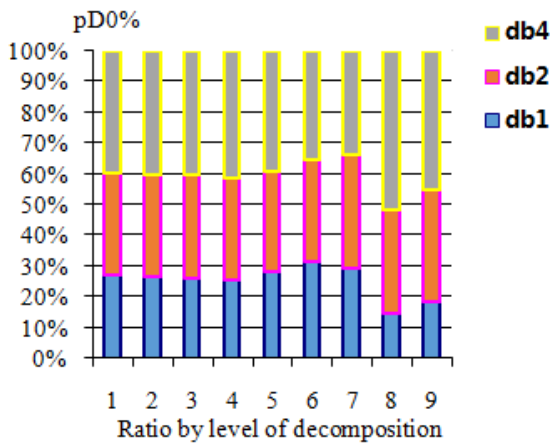


Figure 20. Dynamic percentage $pD0$ for Figure 16

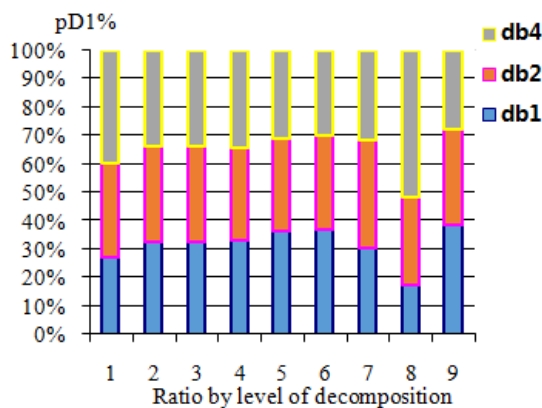


Figure 21. Dynamic percentage $pD1$ for Figure 17

4. Conclusions

We have considered the possibility of using the properties of wavelet coefficients decomposition as a comparative analysis for image analysis and processing. For this task we convert two-dimensional images into one-dimensional data sets using multiresolution wavelet analysis, then we find the detailing wavelet coefficients at each level of decomposition for each data set, after that, we build new features that reflect the dynamic change in mean and variance for the detailing wavelet coefficients decomposition at each level of decomposition.

The transformation of the image into new characteristics based on the properties of the wavelet coefficients of decomposition takes into consideration the following:

- Unique representation of the original image as a set of detailing coefficients,
- The presence of the self-similarity of the wavelet coefficients for detailing the various levels of decomposition,
- Conformity of detailing wavelet coefficients to the law of normal distribution at each level of decomposition for all data set.

We also investigated the dynamics of change in mean and variance for the detailing wavelet coefficients decomposition for different wavelet functions. We have also shown that the dynamics are identical to the same images and different for different images, which is allow to build comparison and analysis procedures of how to identify images and compare individual parts. In general, the results have shown the feasibility of using the properties of wavelet coefficients decomposition as a comparative analysis of images.

References

- [1] Ji, L., & Yi, Z., "A mixed noise image filtering method using weighted-linking PCNNs". *Neurocomputing*, 71(13), 2986-3000, 2008.
- [2] Khan, M. B., Lee, X. Y., Nisar, H., Ng, C. A., Yeap, K. H., & Malik, A. S., "Digital image processing and analysis for activated sludge wastewater treatment", In *Signal and image analysis for biomedical and life sciences*, Springer International Publishing, (pp. 227-248), 2015.
- [3] Aksoy, S., Yalniz, I. Z., & Tasdemir, K., "Automatic detection and segmentation of orchards using very high resolution imagery", *IEEE Transactions on geoscience and remote sensing*, 50(8), 3117-3131, 2012.
- [4] Pedroso, M., Taylor, J., Tisseyre, B., Charnomordic, B., & Guillaume, S., "A segmentation algorithm for the delineation of agricultural management zones", *Computers and Electronics in Agriculture*, 70(1), 199-208, 2010.
- [5] Wang, Y., Wu, G., Chen, G. S., & Chai, T., "Data mining based noise diagnosis and fuzzy filter design for image processing". *Computers & Electrical Engineering*, 40(7), 2038-2049, 2014.
- [6] Javed, U., Riaz, M. M., Ghafoor, A., & Cheema, T. A., "SAR image segmentation based on active contours with fuzzy logic", *IEEE Transactions on Aerospace and Electronic Systems*, 52(1), 181-188, 2016.
- [7] Yan, J., Schwartz, L. H., & Zhao, B., "Semiautomatic segmentation of liver metastases on volumetric CT images", *Medical physics*, 42(11), 6283-6293, 2015.
- [8] Gorgel, P., Sertbas, A., & Ucan, O. N., "A wavelet-based mammographic image denoising and enhancement with homomorphic filtering", *Journal of medical systems*, 34(6), 993-1002, 2010.
- [9] Kobylin, O., & Lyashenko, V., "Comparison of standard image edge detection techniques and of method based on wavelet

- transform", *International Journal of Advanced Research*, 2(8), 572-580, 2014.
- [10] Lyashenko, V., Matarneh, R., Kobylin, O., & Putyatin, Y., "Contour detection and allocation for cytological images using Wavelet analysis methodology", *International Journal of Advance Research in Computer Science and Management Studies*, 4(1), 85-94, 2016.
- [11] Kingsbury, N., "Image processing with complex wavelets", *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 357(1760), 2543-2560, 1999.
- [12] Lyashenko, V., Kobylin, O., & Ahmad, M. A., "General Methodology for Implementation of Image Normalization Procedure Using its Wavelet Transform", *International Journal of Science and Research*, 3(11), 2870-2877, 2014.
- [13] Delbeke, L., & Abry, P., "Stochastic integral representation and properties of the wavelet coefficients of linear fractional stable motion", *Stochastic Processes and their Applications*, 86(2), 177-182, 2000.
- [14] Mallat, S., & Hwang, W. L., "Singularity detection and processing with wavelets", *IEEE transactions on information theory*, 38(2), 617-643, 1992.
- [15] Flandrin, P., "Wavelet analysis and synthesis of fractional Brownian motion", *IEEE Transactions on information theory*, 38(2), 910-917, 1992.
- [16] Lyashenko, V., Deineko, Z., & Ahmad, A., "Properties of wavelet coefficients of self-similar time series", *International Journal of Scientific and Engineering Research*, 6(1), 1492-1499, 2015.