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Calculation Method for Assessing Contact Parameters in the Hip Prosthesis Made of Thermo-diffusion Nitrided Grade 2 / ultra-high Molecular Weight Polyethylene

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Abstract In this paper, the author proposes a new calculation method for calculating contact parameters (i.e., maximum contact pressures, angle and diameter of contact) in the hip prosthesis made of thermo-diffusion nitrided (TDN) Grade 2 and ultra-high molecular weight polyethylene (UHMWPE). The paper investigates the impact of hip joint load, prosthesis head diameter and radial clearance on the above contact parameters. Relationships between maximum contact pressures and the above-mentioned contact parameters are determined. Both increasing radial clearance and endoprosthesis loading cause a linear increase in contact pressure. However, when the head diameter increases, there is a non-linear reduction of contact pressure. The contact diameter increases linearly as the head diameter increases. According to the given method, the endoprosthesis with non-spherical surfaces of its elements was also tested (Alpharabola geometry). An analysis of the effect of head deviation from sphericity in the form of oval on contact pressure, contact angle and contact diameter was performed. Beneficial effects of this geometry were determined.

Keywords: *hip prosthesis, calculation method, Alpharabola geometry, maximum contact pressures*

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1. Introduction

The hip prosthesis is the most frequent joint replacement in arthroplasty. Over 1000000 hip replacement surgeries are performed every year. For this reason, when selecting hip replacement implants for particular groups of patients it is vital to ensure the highest contact strength of the prosthesis components at the initial stage of use. The primary objective is to minimize the wear of prosthesis components, the rate of which also depends on contact pressures because the accumulation of excessive wear products can have various negative effects, ultimately meaning that the prosthesis will only last 10-12 years and will have to be replaced.

However, the issue of reliable assessment of contact pressure in endoprostheses is still not fully resolved. This is due to the lack of relatively simple and effective calculation methods ensuring the possibility of their estimation depending on the compressive force, connection size, radial clearance and the materials used. In the literature [1] the method based on the use of the Airy function in displacements for the shferical ball / metal - backed plastic lauer system has been presented. It was used for finite element method (FEM) verification. Later [2] presents the simple elasticity method after modifying the method [1]. The UHMWPE cup and the metal-backed UHMWPE cup were tested using this method and FEM. However, by [3] a method based on work [4] with the use of equivalent bearing radius in the ball-on-plane scheme was approved, approving the ball-in-socked hip endoprosthesis system. The numerical solution is quite complicated, which limited the use of this method to verify numerical FEM analysis. Comparison of the efficiency or accuracy of these calculation methods does not seem expedient because they were used only to verify the results of obtained by FEM.

The problem of contact strength as a function of contact pressures must be considered when designing hip prostheses, particularly for the metal - on - plastic (MoP) combination. An even more important criterion is the resistance to wear caused by the friction force, with the wear rate being a result of action of contact pressures, according to the Amontons-Coulomb friction law. The literature review shows lack of methods for solving axisymmetric contact problems for balls with similar diameters in order to calculate contact pressures. Many researchers used FEM modelling for estimating contact pressures in the hip prosthesis made of different material combinations (MoM, MoP) [3,5-15]. The first stage of these studies usually involved determining contact pressures which were then used to model wear. Depending on the approach to FEM, the quantitative results of maximum contact pressures and their distributions in the hip prosthesis significantly differ.

The works [16,17,18] provide a contact problem solution method for calculating internal contact parameters in circular-section bodies with similar diameters. The above method was effectively used for investigating contact parameters in fixed cylindrical joints, as well as for determining initial contact pressures in slide bearings and cylindrical guides [19,20]]. The above method is employed in this study to numerically analyse the hip joint prosthesis made of TDN Grade 2 and UHMWPE.

The aim of the paper is estimation of maximum contact pressures and contact parameters, as well as their quantitative and qualitative change from the main factors of influence. In particular, from the load on the endoprosthesis, the diameter of its head, the radial cleance in the joint and the small ovality of the head.

2. Formulation of a Contact Problem

The author's method was taken as a basis to study the flat contact problem of the theory of elasticity. The parameters of hip joint endoprosthesis were analyzed as a piece of the skeleton system. That ended up to the following transformations. The hip endoprosthesis as a ball joint with acetabulum and head (3D system) (Figure 1) is replaced by a model cylindrical joint (3D system) with an simulated (efektive) radius of the endoprosthesis. This system is loaded with compressive force N occurring in both parts - in the real human hip joint and the endoprosthesis.

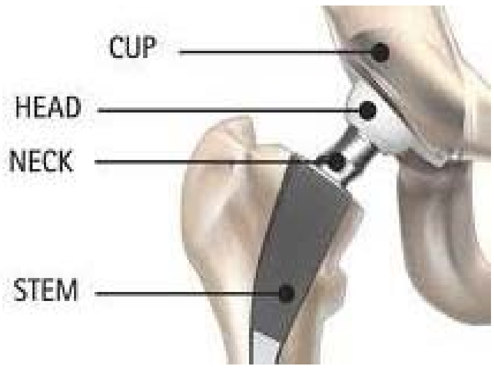

Figure 1. Hip joint endoprosthesis

Then, this system of cylindrical bodies with diameters $D_1 = 2R_1$ (acetabular cup 1), $D_2 = 2R_2$ (femoral head 2) and possible small ovality $\delta_k(\alpha) \langle\langle R_k$ of their outlines were brought to a flat system loaded with force $N' = N / D_2$ (Figure 2a, b).

The diameters are similar, i.e. $R_1 \approx R_2$ in endoprosthesis. But there is always an $\varepsilon = R_1 - R_2 \geq 0 \langle\langle R$ radial clearance. The compressive load N' generated contact pressures $p_{\alpha\delta}$ in the contact area described by the angle $2\alpha_{0\delta}$.

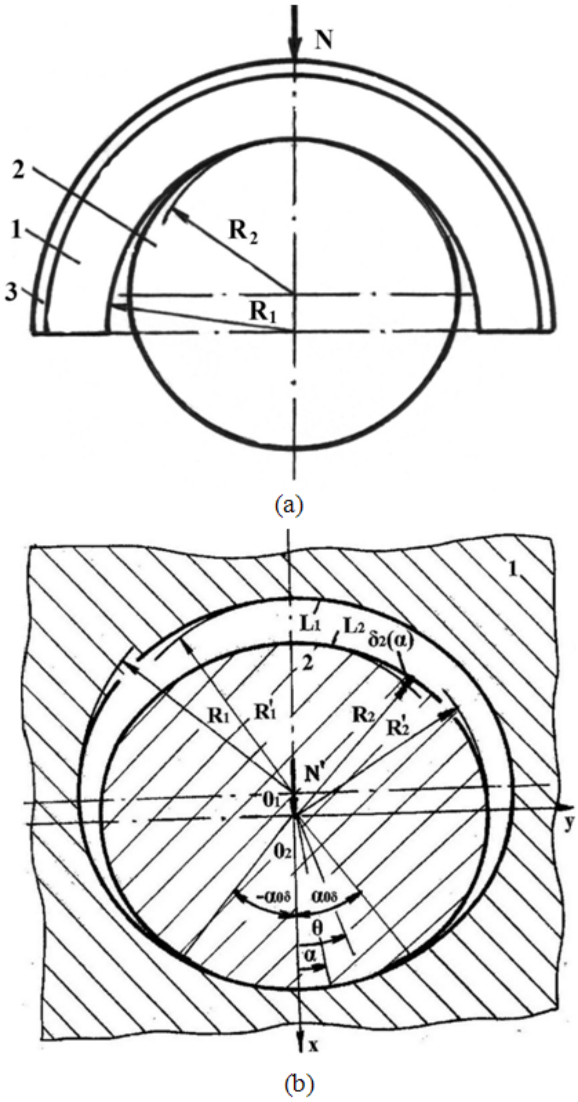

Figure 2. Schemes computing joint replacement: a) endoprostheses, b) the model cylindrical joints

3. Solution Methods

The equation describes a touch $p_{\alpha\delta}$ using the strict author's method [16,19]. The analysed case includes the following details - the symmetric contact as a function of load for the components with small circularity deviations of the profile. Below is the equation:

$$c_1 \int_{-\alpha_{0\delta}}^{\alpha_{0\delta}} \cot\frac{\alpha-\theta}{2} p'_\theta d\theta = c_2 p_{\alpha\delta} + c_3 \int_{-\alpha_{0\delta}}^{\alpha_{0\delta}} p_{\alpha\delta} d\alpha + c_4 \cos\alpha \int_{-\alpha_{0\delta}}^{\alpha_{0\delta}} p_{\alpha\delta} \cos\alpha d\alpha + \frac{\varepsilon}{R^2}\left[1 - \frac{\delta_1}{2\varepsilon} D_{1\alpha} - \frac{\delta_2}{2\varepsilon} D_{2\alpha}\right], \quad (1)$$

where $p'_\theta = dp / d\theta$; $0 \leq \alpha \leq \theta, 0 \leq \theta \leq \alpha_{0\delta}$; α - polar angle; $\varepsilon = R_1 - R_2 \rangle 0$; $1 - \frac{\delta_1}{2\varepsilon} D_{1\alpha} - \frac{\delta_2}{2\varepsilon} D_{2\alpha} = \Sigma_\delta$;

$$c_1 = \frac{1}{8\pi}\left(\frac{1+\kappa_1}{G_1R_1} + \frac{1+\kappa_2}{G_2R_2}\right); c_2 = \frac{1}{4}\left(\frac{1-\kappa_1}{G_1R_1} - \frac{1-\kappa_2}{G_2R_2}\right);$$

$$c_3 = \frac{1+\kappa_1}{8\pi G_1R_1}; c_4 = \frac{1}{2\pi}\left(\frac{\kappa_1}{G_1R_1} + \frac{1}{G_2R_2}\right);$$

G_1, G_2 - the elasticity moduli of the component materials; ν_1, ν_2 - the Poisson ratios of the component materials; $\kappa = 3 - 4\nu$ - two-dimensional strain; $D_{1\alpha}$, $D_{2\alpha}$ [17,18] - the characteristics of circularity deviation of the component profiles (ovality, eliptisity); δ_1, δ_2 - deviations of the component profiles from circularity, $\delta_k(\alpha)\langle\langle R_k$, $\delta_k \le 0.5\varepsilon$; k - the number of components; $\delta_1 = R_1 - R_1'$, $\delta_2 = R_2' - R_2$; $R_1 = a_1$ - the bigger semi-axis of the cup hole with ovality, $R_1' = b_1$ - the smaller semi-axis of the cup hole, $R_2' = a_2$ - the bigger semi-axis of the disk (ball), $R_2 = b_2$ - the smaller semi-axis of the disk (ball) (Figure 2b).

By solving the problem, it is possible to determine the maximum contact pressures, the angle of contact, as well as the distribution of pressures in the contact area. An effective way of solving Eq. (1) describing contact pressures $p_{\alpha\delta}$ is to use the collocation method, well-known in the elasticity theory.

According to [19], the function of contact pressures $p_{\alpha\delta}$ has the form:

$$p_{\alpha\delta} \approx E_\delta \varepsilon_\delta \sqrt{\tan^2\frac{\alpha_{0\delta}}{2} - \tan^2\frac{\alpha}{2}}, \quad (2)$$

where

$$\varepsilon_\delta = \varepsilon\Sigma_\delta, E_\delta R \left(/ ^* \right)\cos^2(\alpha_{0\delta}/4), ,$$

$$e = 4E_1E_2/Z, E = 2G/(1+\nu)$$

$$Z = (1+\kappa_1)(1+\nu_1)E_2 + (1+\kappa_2)(1+\nu_2)E_1,$$

E is the Young modulus of the material, R^* - effective radius of the prosthesis.

The radii of endoprosthesis $R_1 \approx R_2 = R$ elements were replaced in the model cylindrical joint of bodies with similar diameters by its effective radius R^*. He defines himself by relationship

$$R^* = \sqrt{R_1^* R_2^*} = 0.5\sqrt{(R_2+\varepsilon)R_2}. \quad (3)$$

The maximum contact pressures $p_{0\delta}$ occur when $\alpha = 0$

$$p_{0\delta} \approx E_\delta \varepsilon_\delta tg\frac{\alpha_{0\delta}}{2}. \quad (4)$$

The half contact angle $\alpha_{0\delta}$ is calculated from the condition of equilibrium of forces acting on disc 2

$$N' = R^* \int_{-\alpha_{0\delta}}^{\alpha_{0\delta}} p_{\alpha\delta}\cos\alpha\, d\alpha = 4\pi R^* E_\delta \varepsilon_\delta \sin^2\frac{\alpha_{0\delta}}{4}. \quad (5)$$

Loads acting on the hip joint are variable. According to the literature data [21,22], the load N acting on the hip prosthesis head is a geometric sum of two forces: the body weight, K, and the muscular force, M. During movement (physiological gait) it varies in the range of $1.45K \le N \ge (3.0...4.4)K$. The maximum, average and minimum value of the compressive force, respectively, are calculated at K = 700 N.

4. Numerical Calculations, Results

Contact parameters of the hip prosthesis were calculated using the following data: N_{max} = 2900 N, N_s = 1900 N, N_{min} = 1000 N; D_2 = 28, 38, 48, 58 mm; $\varepsilon = 0.05$ ... 0.2 mm. In a model system, the compressive force $N' = N / D_2$. Two cases were investigated:

A) Radial clearance $\varepsilon = 0.05$, 0.1, 0.2 mm; $\delta_1, \delta_2 = 0$;

B) Radial clearance $\varepsilon =$ 0.1, 0.2 mm; $\delta_1 = 0$, $\delta_2 > 0$.

The hip prosthesis is made of the following materials: acetabular component (cup) 1 - UHMWPE with E_1 = 0.625 GPa (37°C) and ν_1 = 0.46; head (ball) 2 - TDN Grade 2 [23,24] with E_2 = 112 GPa and ν_2 = 0.32. In the analysed combination of materials, the ratio between the Young moduli is E_2 / E_1 = 112 / 0.625 = 179.2. Therefore, it can be claimed that the head made of TDN Grade 2 is totally rigid when compared to the acetabular cup made of UHMWPE and does not deform under pressure.

4.1. Case A (Spherical Elements)

According to the data from hip prosthesis manufacturers, the initial radial clearance $\varepsilon = 0.05...0.25$ mm [8]. Results are given in Figure 3, Figure 4. Figure 3 shows the effect of the radial clearance ε, the prosthesis head diameter D_2 and the compressive force N on the maximum contact pressures p_0.

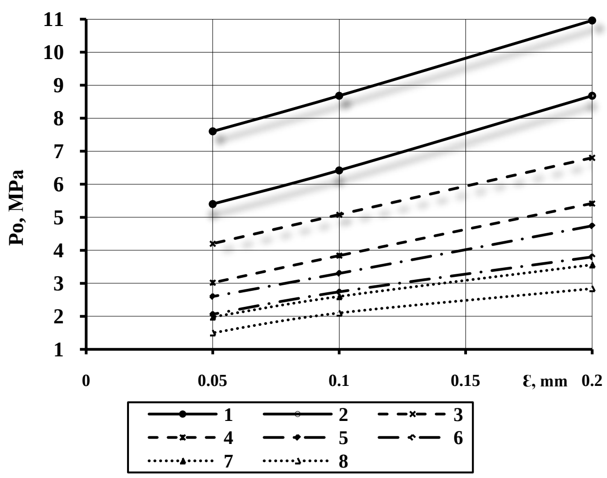

Figure 3. Effect of radial clearance and prosthesis head diameter on maximum contact pressures: 1 - N_{max} = 2900 N, D_2 = 28 mm; 2 - N_s = 1900 N, D_2 = 28 mm; 3 - N_{max} = 2900 N, D_2 = 38 mm; 4 - N_s = 1900 N, D_2 = 38 mm; 5 - N_{max} = 2900 N, D_2 = 48 mm; 6 - N_s = 1900 N, D_2 = 48 mm; 7 - N_{max} = 2900 N , D_2 = 58 mm; 8 - N_s = 1900 N, D_2 = 58 mm

In the range of $0.05 \le \varepsilon \le 0.2$ mm one can observe a nearly linear dependence between p_0 and ε when D_2 = const. The maximum contact pressures p_0 increase considerably in the tested range of the radial clearance ε when the head diameter D_2 is decreased. The head diameter D_2 is of crucial importance for prosthesis selection. Figure 4 shows the maximum contact pressures for the maximum and mean compressive forces.

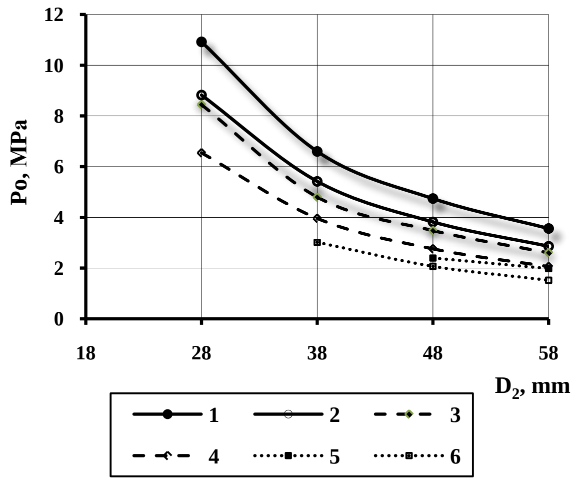

Figure 4. Prosthesis head diameter versus maximum contact pressures: 1 - N_{max} = 2900 N, $\varepsilon = 0.2$ mm; 2 - N_s = 1900 N, $\varepsilon = 0.2$ mm; 3 - N_{max} = 2900 N, $\varepsilon = 0.1$ mm; 4 - N_s = 1900 N, $\varepsilon = 0.1$ mm; 5 - N_{max} = 2900 N, $\varepsilon = 0.05$ mm; 6 - N_s = 1900 N, $\varepsilon = 0.05$ mm

The impact of the load on the head of the endoprosthesis during physiological gait is shown in Figure 5. A virtually linear increase in maximum pressure with an increase in load is observed.

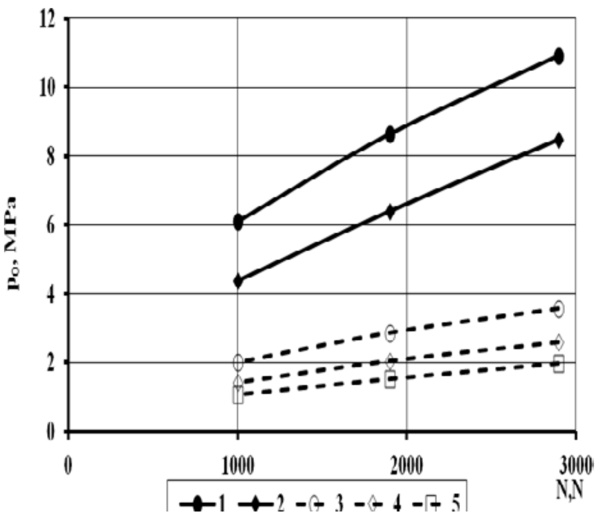

Figure 5. Impact of endoprosthesis loading on maximum contact pressure: 1 - D_2 = 28 mm, $\varepsilon = 0.2$ mm; 2 - D_2 = 28 mm, $\varepsilon = 0.1$ mm; 3 - D_2 = 48 mm, $\varepsilon = 0.2$ mm; 4 - D_2 = 28 mm, $\varepsilon = 0.2$ mm; 5 - D_2 = 28 mm, $\varepsilon = 0.05$ mm

Maximum contact pressures p_0 obtained with the proposed method are compared with the results of $p_{i\,max}$ obtained by the methods [8,25]. According to [25], p_{max} is calculated with the formula

$$p_{max} = c_0 \frac{\varepsilon E_1}{R}, \tag{6}$$

where $E_2 = \infty, E_1$ = 0.625 GPa, c_0 is the coefficient of collocation dependent on α_0.

In turn, according to [5]

$$p_{max} = a_1 + a_2 N^{a_3} + a_4 \varepsilon^{a_5} + a_6 N^{a_3} \varepsilon^{a_5}, \tag{7}$$

where $a_1, a_2, \ldots$ are the coefficients of approximation.

Table 1 gives the results of $p_{i\,max}$ calculated in compliance with the above-mentioned methods and their relative variation when compared to results obtained by the authors of this study.

Table 1. Maximum contact pressures

Formula Parameters	D_2 = 58 mm, N = 2900 N		(6)		(7)	
ε, mm p_0, MPa	0.1 2.59	0.2 3.56	0.1 2.74	0.2 3.81	0.1 3.0	0.2 4.0
$2\alpha_0^0$	106.6	74.0	106.6	74.0	—	—
p_{max} / p_0	1.0	1.0	1.06	1.07	1.16	1.12

When calculated with the method given in [25], the pressures p_{max} are slightly higher (by 1.07 times) than those obtained with the proposed method, whereas the results obtained with the method proposed in [8] are higher by 1.16 times than the results obtained with the method proposed by the authors of this study.

Also in [3], a calculation method was given, according to which the maximum contact pressure for the MoP endoprosthesis was determined (steel - E_2 = 210 GPa, ν_2 = 0.3; UHMWPE - E_1 = 1.0 GPa, ν_1 = 0.4), where no final formula was given p_{max}. For N = 2500 N, D_2 = 28 mm, ε = 0.25 mm, respectively p_{max} = 14.94 MPa was determined, and according to the author's method p_0 = 13.44 MPa and then p_{max} / p_0 = 1.11 times.

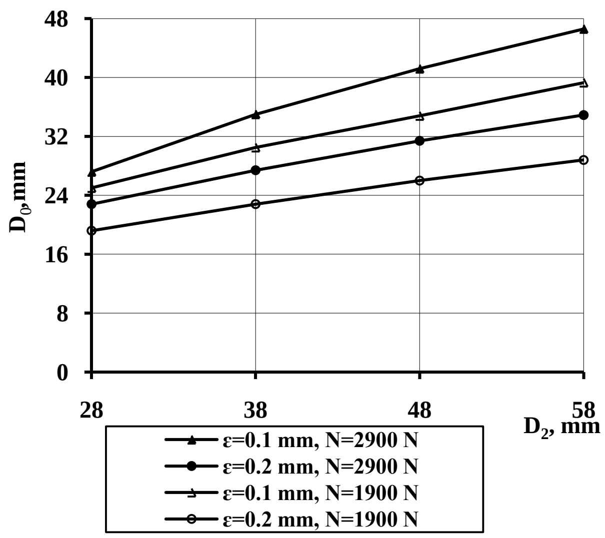

Figure 6. Variation in the contact diameter in the hip prosthesis

Figure 6 shows the variation in the contact diameter D_0 caused by changing the prosthesis head diameter D_2, radial clearance ε and compressive force N acting on the head. In an axisymmetric contact problem (hip prosthesis) the contact diameter D_0 is similar to the contact angle $2\alpha_0$ in a two-dimensional contact problem.

Accordingly

$$D_0 = D_2 \sin \alpha_0. \tag{8}$$

The convergence of literature data and calculations of the authors of the contact diameter was determined. The results of the experimental data and the calculated FEM [7] endoprosthesis Steel (femoral head) - UHMWPE (cup) when D_2 = 28 mm, ε = 0.079 mm, the thickness of the skull 11.32 mm and D_2 = 32 mm, ε = 0.098 mm, the thickness of the skull 9.42 mm, are given in Table 2.

Table 2. Experimental and calculated contact diameter

Autors	***N***, N			
	1000	1500	2000	2500
[7] Eksper., D_2 = 28 mm	23.2	23.3	25.0	26.5
[7] FEM	19.9 / 14.2%	21.0 / 9.9%	23.8 / 4.8%	25.6 / 3.4%
Own rezults	19,91 / 14.2%	21.62 / 7.05%	24.02 / 3.9%	25.76 / 2.8%
[7] Eksper., D_2 = 32 mm	23.6	25.2	26.4	27.0
[7] FEM	20.7 / 12.3%	22.6 / 11.5%	23.8 / 9.85%	25.9 / 4.1%
Own rezults	20,2 / 14.4%	22.1 / 12.3%	24.12 / 8.6%	26.26 / 2.7%

Analysis of the given results indicates that the results of experimental and computational tests using FEM and the author's method are approaching the increase in load. The author's method and [7] show similar values of contact diameters.

According to [11]: a) N = 2500 N, D_2 = 32 mm, ε = 0.098 mm, femoral head CoCrMo - UMHWPE cup, D_0 = 19.2 mm, and according to the authors D_0 = 17.6 mm - (8.3%); b) N = 2500 N, D_2 = 32 mm, ε = 0.098 mm, D_0 = 20.0 mm, and according to the authors D_0 = 19.22 mm - (3.9%).

4.2. Case B (ovality elements, N = 1900 N)

Deviations from the ideal circular profile are inevitable when manufacturing prosthesis components. According to the literature data, the maximum allowable deviations of the hear diameter are $\delta_{\max} = (0.5...2)\cdot 10^{-4} D_2$ (mm). There are, however, no studies that would establish optimum values of profile circularity deviation. It is generally claimed that the smaller the deviations are, the higher the prosthesis properties relating to contact pressures and durability become. According to the general standards, in combinations of similar diameter components the maximum allowable $\delta_{\max} \le 0.5\varepsilon$.

It is assumed that the UHMWPE cup is free from ovality and has $D_{20} = 1$. The ball made of thermo-diffusion nitrided titanium Grade 2 has the profile ovality $\delta_2 = R_2' - R_2 \rangle 0$ (Figure 2b). Two variants of location of the head (ball) with ovality relative to the acetabular component (cup) are considered (Figure 7).

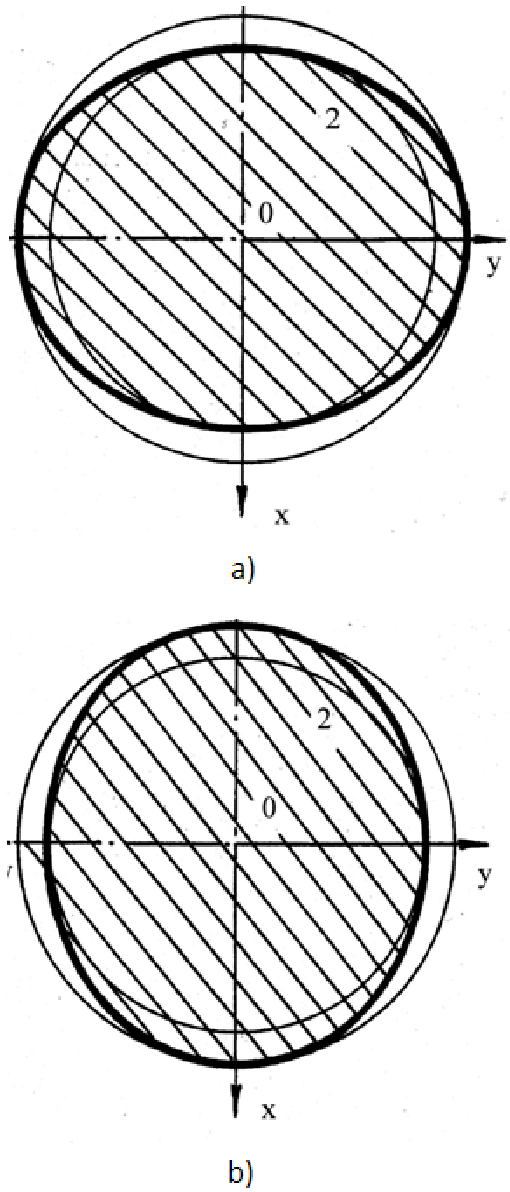

Figure 7. Tested variants of ball ovality

For the more convenient position of the ball shown in Figure 7a, $D_{2\alpha} = 1 + 3\cos 2\alpha$ and when $\alpha = 0$ then $D_{20} = 4$ [17,18]. In contrast, when the major axis is vertical (Figure 7b), then $D_{2\alpha} = 1 - 3\cos 2\alpha$, $D_{20} = -2$. Results of the maximum contact pressures $p_{0\delta}$ (MPa), contact angles $2\alpha_{0\delta}$ (degree) and the contact diameter D_0 (mm) obtained for the two investigated cases are listed in Table 3, whereas the relative variation in $p_{0\delta} / p_0$ is shown in Figure 8.

Table 3. Contact parameters

ε, mm	δ_2, mm	D_2, mm: 28	D_2, mm: 58
0.2		$D_{20}=4$	$D_{20}=4$
	0.05 0.02 0.01	6.5/124.2/24.75 7.84/97.8/21.10 8.27/91.8/20.11	2.05/85.2/39.26 2.56/66.8/31.93 2.71/62.9/30.26
		$D_{20}=1$	$D_{20}=1$
	0	8.67/86.9/19.26	2.85/59.6/28.82
		$D_{20}=-2$	$D_{20}=-2$
	0.01 0.02 0.05	8.87/84.7/18.86 9.06/82.6/18.48 9.6/77.3/17.49	2.92/58.1/28.16 2.98/56.8/27.59 3.17/53.2/25.97
0.1		$D_{20}=4$	$D_{20}=4$
	0.0164 / 0.05 0.01 / 0.02 0.005 / 0.01	5.57/158.72/27.52 5.9/143.3/26.58 6.16/133.9/26.3	1.64/111.9/48.05 1.86/95.2/42.83 1.96/90.1/41.05
		$D_{20}=1$	$D_{20}=1$
	0	6.42/126.3/24.98	2.052/85.3/39.30
		$D_{20}=-2$	$D_{20}=-2$
	0.01 0.02 0.05	6.67/119.8/24.22 6.92/114.2/23.51 7.61/110.9/23.06	2.15/81.2/37.75 2.23/77.5/36.30 2.48/69.0/32.85

The results demonstrate that the ball ovality shown in Figure 7a causes the pressures $p_{0\delta}$ to decrease (lower diagrams), whereas the ball ovality shown in Figure 7b causes the pressures $p_{0\delta}$ to increase (upper diagrams).

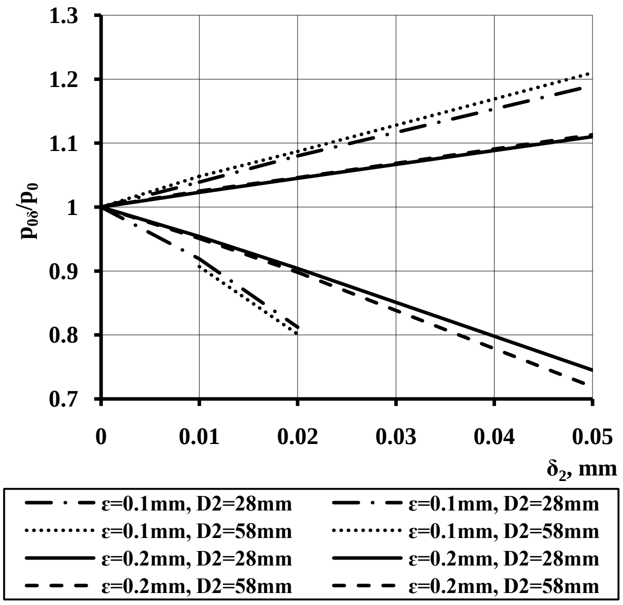

Figure 8. Relative variation in maximum contact pressures

The deviations from circularity of the ball (head) shown in Figure 7a have a positive effect on reducing the contact pressures. The higher the deviation δ_2 is, the lower the pressures $p_{0\delta}$ become.

5. Comparison Results

The new method proposed in this paper is used to determine maximum contact pressures p_{0i} in different variants of the hip prosthesis described by the parameters given in [5,6,8,9,11,12,15] to compare results (Table 4).

Table 4. Maximum contact pressures

Authors	p_{0i} / p_{max}, MPa	p_{0i} / p_{max}, times	Prosthesis parameters
[5]	21.61 / 22.0	0.982	N = 3200 N; D_2 = 58.6 mm; ε = 0.05 mm; CoCr - CoCr
[6]	22.16 / 20.0 19.46 / 18.0	1.109 1.082	N = 2722 N; D_2 = 54.6 mm; ε = 0.05 mm; CoCr - CoCr N = 2100 N; D_2 = 54.6 mm; ε = 0.05 mm; CoCr - CoCr
[8]	2.8 / 3.24 3.88 / 4.31 11.81 / 13.75 8.77 / 10.65	0.864 0.9 0.856 0.823	N = 2650 N; D_2 = 58 mm; ε = 0.1 mm; Ti - UMHWPE N = 2650 N; D_2 = 58 mm; ε = 0.2 mm; Ti - UMHWPE N = 2650 N; D_2 = 28 mm; ε = 0.1 mm; Ti - UMHWPE N = 2650 N; D_2 = 28 mm; ε = 0.2 mm; Ti - UMHWPE
[9]	15.24 / 17.1 15.24 / 17.16 15.24 / 18.14	0.89 0.888 0.84	N = 2500 N; D_2 = 22,225 mm; ε = 0.1825 mm; Stainless steel - UMHWPE $\beta = 45^{\circ}$ - inclination angle $\beta = 55^{\circ}$ $\beta = 65^{\circ}$
[12]	9.99/10.75 9.99/12.98	0.929 0.77	N = 2500 N; D_2 = 36 mm; ε = 0.3 mm; Ti - UMHWPE $\beta = 35^{\circ}$ $\beta = 45^{\circ}$
[11]	8.84 / 8.5 9.07 / 8.9 10.2 / 10.2 11.04 / 11.25	1.04 1.019 1.0 0.981	D_2 = 32 mm; ε = 0.098 mm; CoCrMo - UMHWPE N = 1900 N N = 2000 N N = 2500 N N = 2900 N
[15]	16.7 / 21.12	0.788	N = 3000 N; D_2 = 22 mm; ε = 0.1 mm; $\beta = 45^{\circ}$ CoCrMo - UMHWPE

So the author's results are different from the results according to the numerical methods proposed in [5,6,8,11] where for the inclination angle $\beta = 0$ the ratio is 0.823 times $\geq p_0 / p_{max} \geq 1.109$ times, depending on the prosthesis parameters. When $\beta = 35^{\circ} ... 65^{\circ}$ [9,12,15], the ratio is 0.77 times $\geq p_0 / p_{max} \geq 1.106$ times. The differences between the maximum contact pressures obtained with the method developed by the authors of this study and those calculated by FEM probably result from different approaches to this numerical method. Given its strictly analytical nature, the proposed method for calculating maximum contact pressures helps prevent such discrepancies or the potential unreliability of results.

6. Conclusion

1. The aim of the study of the endoprosthesis MoP hip joint completed totally. Quantitative values of contact parameters, as well as regularities of their qualitative change from the investigated factors of influence, are established.

2. The maximum contact pressures p_0 depend on the compressive force N acting on the prosthesis head, head diameter D_2 and radial clearance ε in the system (Figure 3, Figure 4, Figure 5). With increasing D_2 by 2.071 times the pressures p_0 decrease by 2.96 up to 4.24 times, depending on the values of N and ε.

3. The contact pressures show a practically linear increase with increasing the radial clearance in the range of $0.05 \leq \varepsilon \leq 0.2$ mm (Figure 3).

4. The investigation of the effect of the low-ovality head on the maximum contact pressures p_0 has demonstrated that the pressures significantly decrease if the head with ovality is located conveniently relative to the acetabular component, when compared to the pressures obtained for the prosthesis components with the ideal circular profile (Figure 8).

5. The maximum contact pressures obtained with the proposed method show good agreement with the results obtained with the FEM by other authors and other analytical methods.

6. The purpose of further studies is to analyze the parameters of contact in the endoprostheses of MoM, CoC, CoP.

7. The presented analitical method of estimating contact characteristics is an integral part of the author's methodology for investigating the wear kinetics of tribological sliding friction systems. The author further envisages the study of wear and duration of safe use of various types of endoprostheses.

Competing Interests

The author has no competing interests.

List of Abbreviations

TDN - Thermo-diffusion nitrided
UHMWPE - Ultra-high molecular weight polyethylene
FEM - Finite element method
MoP - Metal - on - plastic
MoM - Metal - on - metal
CoP - Ceramic - on - plastic

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