Journal of Biomedical Engineering and Technology, 2017, Vol. 5, No. 1, 6-11
Available online at http://pubs.sciepub.com/jbet/5/1/2

DOI:10.12691/jbet-5-1-2

Does Temperature Effects the Growth of Microcracks in Tibia due to Volleyball?

M. Tsili*, D. Zacharopoulos

School of Engineering, Democritus University of Thrace, Xanthi, Greece
*Corresponding author: martsili@otenet.gr

Abstract In present paper we considered if temperature plays role to the growth of microcracks in a tibia due to volleyball. We dealed with three particular points of bone and we based upon theories: of adaptive elasticity and upon energy density. We showed that both: neglecting or accounting temperature after a long time tibia at points of our interest will be strengthened (the mean length of their microcracks will be decreased). The result coincides with that of corresponding problem at macroscopic area. We concluded that temperature does not effects the growth of microcracks.

Keywords: *theory of adaptive elasticity, accounting and neglecting temperature, SED theory, tibia locally at some points, volleyball, microcracks strengthened*

Cite This Article: M. Tsili, and D. Zacharopoulos, "Does Temperature Effects the Growth of Microcracks in Tibia due to Volleyball?" *Journal of Biomedical Engineering and Technology*, vol. 5, no. 1 (2017): 6-11. doi: 10.12691/jbet-5-1-2.

1. Introduction

It is well known that bone fracture due to many factors: as age [1-14], microstructure [14-24], bone density [2,3,8,9,10,14,17,24,25] and loading mode [14,23-28].

From the other hand very few known studies investigated the effect of temperature in bone disease [23,25].

The purpose of this paper is to study if temperature plays role to the growth of microcracks in a tibia, due to volleyball. For that reason we will use the theories: of adaptive elasticity [29,30] both neglecting and accounting temperature and energy density [31,32,33].

Macroscopically the bone has a volume V and a surface S. The volume V microscopically consists of microvolumes $\underline{\Delta V}$ which generally are not homogenous [32,33]. Expanding the theory of adaptive elasticity [[32], p. 322] at microscopic area we assume:

i) Every microvolume $\underline{\Delta V}$ of the bone consists of a elastic microvolume ΔV (micromatrix bone) and of microcracks (pores) that is:

$$\underline{\Delta V} = \Delta V + \Delta p \tag{1}$$

where Δp is the volume of microcracks.

Sih [[33], p.179] showed that every microvolume ΔV contains an homogenous microvolume. We suppose that the elastic microvolume ΔV given by (1) is an homogenous microvolume.

ii) The mechanical properties of microvolume of bone $\underline{\Delta V}$ coincides with the mechanical properties of homogenous microvolume ΔV of micromatrix bone.

iii) The fraction of microvolume of the micromatrix bone Δξ is defined as [[30], p. 322]:

$$\Delta\xi = \Delta V / \underline{\Delta V} = \rho / \gamma \tag{2}$$

where ρ is the density of microvolume ΔV, while γ is the density of material (bone) and assume to be constant. From the above it follows 0< Δξ <1.

iv) The porosity that is the mean length of microcraks of the microvolume $\underline{\Delta V}$ alters with mass added /removal to /from micromatrix bone and linearly depends from the history of microstrain [[29], p. 322]. The porosity is characterized by a parameter ê [30]:

$$\hat{e}(t) = \Delta\xi(t) - \Delta\xi_0 \tag{3}$$

where $\Delta\xi_o$ is the initial fraction of the microvolume of micromatrix bone. With other words parameter ê is the change of the mean value of microcracks.

2. The Problem and the Physical Approximation

Assume that someone participates to a volleyball game or training and continues to be exersized, for a long time period. In previous paper we macroscopically study the internal remodeling of tibia due to volleyball [34]. In present paper we want to predict the situation of his /her tibia after a long time, locally at three particularly points as indicated in Figure 1. The lasts are below.

2.1. The Mathematic Formulation of Our Problem

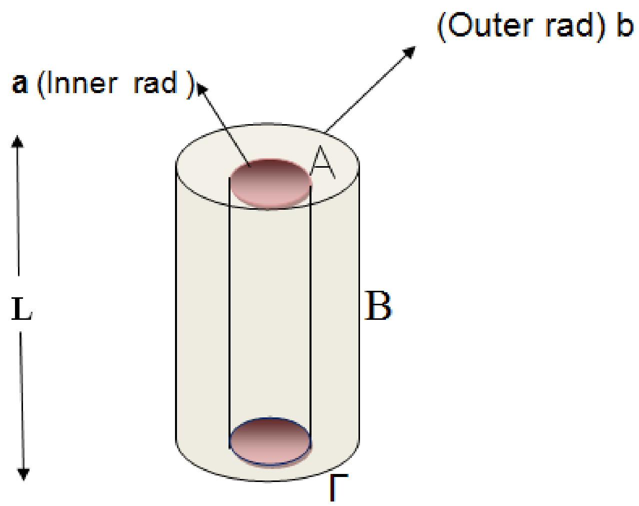
Assume that for t <0 the athlete was normal walking with constant velocity v_o. Therefore the points A, B and Γ of tibia were respectively under a compressive load G_o, P_o

and F_o due to ground reaction force at late stance phase during normal walking. The above are given by:

$$G_o = G - (B_w + B_t), P_o = G - (B_f + B_t / 2) \text{ and } F_o = G - B_w \text{ in units of B.W.} \quad (4)_{1\text{-}2\text{-}3}$$

where G_o is ground reaction force at late stance phase for foot and B_f , B_t are respectively the weights of foot and tibia [35]. There is a statistical relationship between ground reaction force for foot G and velocity v_o [36,37,38]:

$$G = 0.213 v_o + 0.913 \text{ in units of B.W.} \quad (5)$$



A: at the top of inner rad of tibia's diaphysis
B: at the middle of outer rad of tibia's diaphysis and
Γ: at the bottom of middle of distance between inner and outer rad of tibia's diaphysis.

Figure 1. A tibia with length L with inner and outer radii a and b respectively and the points: A, B and Γ

Substituting (5) into $(4)_{1\text{-}2\text{-}3}$ and accounting [35]:

$$B_f = 0.019 \text{B.W and } B_t = 0.044 \text{B.W} \quad (6)_{1\text{-}2}$$

it results:

$$G_o = 0.213 v_o + 0.85, P_o = 0.213 v_o + 0.872 \text{ and } F_o = 0.213 v_o + 0.894 \text{ in units of B.W.} \quad (7)_{1\text{-}2\text{-}3}$$

Therefore the points A, B and Γ of tibia were respectively under costant compressive loads G_o, P_o and F_o.

Also the lasts had the same fraction of element volume $\Delta\xi_o$ and the same mean length of microcracks êo.

2.2. Biomechanical Analysis of Volleyball

At t >0 the athlete starts playing volleyball. During a volleyball game it is possible to observe the followings [34]:

i) The players are running in order to successfully rebut the ball, before it comes to contact with the ground.

ii) The volleyball spiker is vertically jumping as high as possible, in order to hit the ball. Also the players of the opposite team, are simultaneously vertically jumping as high as possible, in order to rebut the hit of the volleyball spiker.

iii) Finally the player who serves the ball, is sometimes jumping as high as possible, but he (she) is not landing to his (her) initial location. This jump can be modeled as an oblique shoot in the plane Oxz (x, z are the horizontal and vertical axons respectively). Particularly the center of the mass of the player is launched from the origin, with a velocity u_o. An angle α is formed between the direction of the vector of velocity u_o and the horizontal axon, as it seems in Figure 2. The maximums high h_M and horizontal displacement S_M are respectively:

$$S_M = u_0^2 \sin^2\alpha / 2g \text{ and } h_M = u_0^2 \sin 2\alpha / g \quad (8)_{1\text{-}2}$$

where g is the acceleration of the gravity

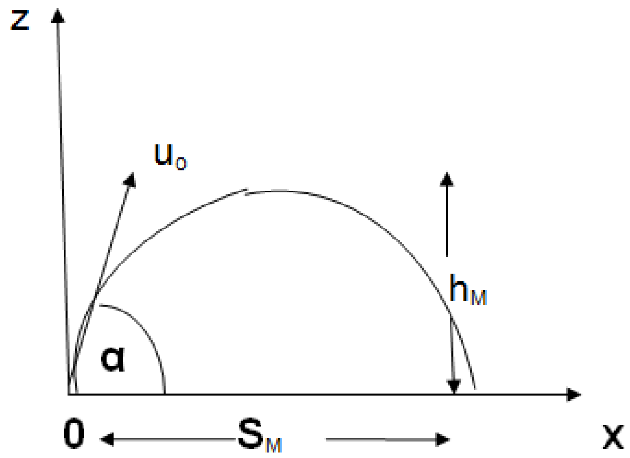


Figure 2. The center of mass of player who serves takes off from the origin of axon and forms an angle α with Oz axon

The server is mainly interested to jump as high as possible than as long as possible, in order to successfully send the ball towards the region of the opposite team. Also the international rules of volleyball forbides the server to leave his (her) area during the service. The last means that his (her) horizontal displacement is always under restriction. Therefore it holds $h_M > S_M$ from which it follows:

$$tg\alpha > 4 \rightarrow \alpha > 76 \rightarrow \sin\alpha > \sin 76 \rightarrow \sin\alpha > 0.97 \approx 1. \quad (9)$$

Since sinα >sin76° = 0.97 ≈1, the jump of the server can approximately be considered as a vertical. Therefore during the volleyball game, the tibia of the athlete is under an axial impact load, due as to running as to vertical jumps. Consequently the points A, B and Γ of tibia were respectively under an axial load G_z, P_z and F_z given by ([34]):

$$G_z = \left[\sum_{i=1}^{x} G_{zi} + \sum_{j=x+1}^{y} G_{zj}\right] / x + y,$$
$$P_z = \left[\sum_{i=1}^{x} P_{zi} + \sum_{j=x+1}^{y} P_{zj}\right] / x + y \quad (10)_{1\text{-}2\text{-}3}$$
$$\text{and } F_z = \left[\sum_{i=1}^{x} F_{zi} + \sum_{j=x+1}^{y} F_{zj}\right] / x + y$$

where G_{zi}, P_{zi} and F_{zi} i= 1, 2, ...,x are respectively: the vertical component of ground reaction force at late stance, during the i time of running at points A, B and Γ of tibia, given by respectively:

$$G_{zi} = G_c - (B_w + B_t),\ P_{zi} = G_c - (B_w + B_t / 2) \quad \text{and } F_{zi} = G_c - B_w \qquad (11)_{1\text{-}2\text{-}3}$$

where G_c is vertical component of ground reaction force at late stance, during phase running for foot. Assume that running velocity is constant.

There is a statistical relationship between ground reaction force G_c and running velocity v [38-44]:

$$G_c = 0.46v + 0.55 \text{ in units of B.W.} \qquad (12)$$

Then (11) $_{1\text{-}2\text{-}3}$ of because of $(6)_{1\text{-}2}$ and (12) take the forms:

$$G_{zi} = 0.46v + 0.487,\ P_{zi} = 0.46v + 0.509 \quad \text{and } F_{zi} = 0.46v + 0.531 \text{ in units of B.W.} \qquad (13)_{1\text{-}2\text{-}3}$$

Finally in $(10)_{1\text{-}2\text{-}3}$ G_{zj}, P_{zj} and F_{zj} where j= x+1, x+2, ψ are respectively the vertical component of ground reaction force at landing phase, during the j time of jumping at points A, B and Γ of tibia. We assume that that the above loads are constant. Therefore it follows that the total axial loads G_z, P_z and F_z given by (10) are constant.

3. A Hollow Circular Cylinder Subjected to an Axial Load

Tibia is modeled as a hollow circular cylinder with lenght L, inner and outer radii a and b respectively, indicated in Figure 1.These radii correspond to endosteal and periosteal surface of bone and are constant due to internal remodeling [45]. Since we deal with microscopic area, we will base upon the density energy theory [31,32,33].

The required equations of the above theory in cylindrical coordinates are:

i) the microstress equations [[33], p.182]:

$$\tau_{rr,r} + \tau_{r\theta,\theta} / r + \tau_{rz,z} + \tau_{rr} - \tau_{\theta\theta} = \rho(d^2 \acute{u}_r / dt^2)$$
$$\tau_{r\theta,r} + \tau_{\theta\theta,\theta} / r + \tau\theta z, z + 2\sigma r\theta = \rho(d^2 \acute{u}_\theta / dt^2) \qquad (14)_{1\text{-}2\text{-}3}$$
$$\tau_{rz,r} + \tau_{\theta z,z} + 2\tau_{r\theta} + \tau_{zz,z} + \tau_{rz} / r = \rho(d^2{}_z / \acute{u}dt^2)$$

ii) the microstrain-microdisplacement relations [[33], p.179]:

$$e_{rr} = 2\partial \acute{u}_r / \partial_r,\ e_{\theta\theta} = 2(\partial \acute{u}_\theta / \partial_\theta + \acute{u}_r) / r$$
$$e_{zz} = 2\partial \acute{u}_z / \partial_z,\ e_{r\theta} = \partial \acute{u}_\theta / \partial_r - \acute{u}_\theta / r \qquad (15)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$
$$e_{rz} = \partial \acute{u}_{r/}\partial_z + \partial \acute{u}_z / \partial_r$$
$$e_{\theta z} = \partial \acute{u}_\theta / \partial_z + \partial \acute{u}_{z/}\partial_\theta$$

iii) the microstress - microstrain relations:

$$\tau_{rr} = (\lambda_2 + 2\mu_2) e_{rr} + \lambda_2 e_{\theta\theta} + \lambda_1 e_{zz}$$
$$\tau_{\theta\theta} = \lambda_2 e_{rr} + (\lambda_2 + 2\mu_2) e_{\theta\theta} + \lambda_1 e_{zz}$$
$$\tau_{zz} = \lambda_1 (e_{rr} + e_{\theta\theta}) + (\lambda_1 + 2\mu_1) e_{zz} \qquad (16)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$
$$\tau_{r\theta} = 2\mu_{2r} e_{r\theta},\ \tau_{rz} = 2\mu_A e_{rz}$$
$$\text{and } \tau_{\theta z} = 2\mu_A e_{\theta z}$$

where:

$$\lambda_1 = v_A E_A E_T / (1 - v_T) E_A - 2\nu^2 A E_T$$
$$2\mu_1 = \begin{bmatrix} (1 - v_T) E^2{}_A - v_A E_A E_T / (1 - v_T) E_A \\ -2\nu^2 A E_T \end{bmatrix} \qquad (17)_{1\text{-}2\text{-}3\text{-}4}$$
$$\lambda_2 = \begin{bmatrix} v_T E_T E_A + \nu^2{}_A E^2{}_T / (1 - v_T) E_A \\ -2\nu^2{}_A E_T (1 + v_T) \end{bmatrix}$$
$$2\mu_2 = E_T / (1 + v_T)$$

where E_A, E_T and v_A, v_T are Young's modulus and Poisson ratio in transverse and axial direction at macroscopic area.

The above hold because bone continues microscopically to be transversely isotropic material in microscopic area [46].

Finally rate remodeling equation [30] at microscopic area without /and accounting temperature are respectively:

$$d\hat{e} / dt = A(\hat{e}) + A_T(\hat{e})(e_{rr} + e_{\theta\theta}) + A_A(\hat{e}) e_{zz}$$
$$\text{and } d\hat{e}(t) / dt = \begin{bmatrix} A(\hat{e}) + A_T(\hat{e})(e_{rr} + e_{\theta\theta}) \\ + A_A(\hat{e}) e_{zz} + B(\hat{e})\theta \end{bmatrix} \qquad (18)_{1\text{-}2}$$

where $A_T(\hat{e})$, $A_A(\hat{e})$ are rate remodeling coefficients in transverse and axial direction respectively while $B(\hat{e})$ is a rate remodeling coefficient depends from temperature.

The boundary conditions of our problem are:

i) at point A:

$$\tau_{rr} = \tau_{\theta\theta} = \tau_{rz} = \tau_{\theta z} = \tau_{r\theta} = 0$$
$$\tau_{zz} = -G_z B / (b^2 - a^2) \text{ at } r = a \text{ and } z = L \qquad (19)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$

ii) at point B:

$$\tau_{zz} = -P_z B / (b^2 - a^2) \text{ at } r = b \text{ and } z = L/2 \qquad (20)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$

iii) at point Γ:

$$\tau_{rr} = \tau_{\theta\theta} = \tau_{rz} = \tau_{\theta z} = \tau_{r\theta} = 0$$
$$\tau_{zz} = P_o B / (b^2 - a^2) \qquad (21)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$
$$\text{at } r = (a + b)/2 \text{ and } z = 0$$

where B is body's weight of the runner in units of B.W. and assumed to be constant during training.

Our problem has a unique solution [[33], p.186] and assume that microdisplacements are of the form:

$$\check{u}_r = A(t) r + B(t) / r \check{u}_\theta = 0 \quad \text{and } \check{u}_z = C(t) z \qquad (22)_{1\text{-}2\text{-}3}$$

where A(t), B(t), C(t) are unknowns. Then microstrains (15) are written as:

$$e_{rr} = 2A(t) - 2B(t) / r^2$$
$$e_{\theta\theta} = 2A(t) + 2B(t) / r^2 \qquad (23)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$
$$e_{zz} = 2C(t),\ e_{r\theta} = 0,$$
$$e_{rz} = 0 \text{ and } e_{\theta z} = 0.$$

Therefore (16) because of (23) take the forms:

$$\begin{aligned}
&\tau_{rr} = 4(\lambda_2+\mu_2)A(t) - 4\mu_2 B(t)/r^2 + 2\lambda_1 C(t)\\
&\tau_{\theta\theta} = 4(\lambda_2+\mu_2)A(t) + 4\mu_2 B(t)/r^2 + 2\lambda_1 C(t)\\
&\tau_{zz} = 4\lambda_1 A(t) + 2(\lambda_1+2\mu_1)C(t)\\
&\tau_{r\theta} = 0,\ \tau_{rz} = 0 \text{ and } \tau_{\theta z} = 0.
\end{aligned} \qquad (24)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$

Applying (19)–(20)–(21) into (24) it is possible to obtain:

i) at point A:

$$\begin{aligned}
&A_1(t) = \lambda_1 G_z B/4\pi(b^2-a^2)f,\ B_1(t) = 0\\
&C_1(t) = -2(\lambda_2+\mu_2)G_z B/4\pi(b^2-a^2)f
\end{aligned} \qquad (25)_{1\text{-}2\text{-}3}$$

ii) at point B:

$$\begin{aligned}
&A_2(t) = \lambda_1 F_z B/4\pi(b^2-a^2)f\\
&B_2(t) = 0\\
&C_2(t) = -2(\lambda_2+\mu_2)F_z B)]/4\pi(b^2-a^2)f
\end{aligned} \qquad (26)_{1\text{-}2\text{-}3}$$

iii) at point Γ:

$$\begin{aligned}
&A_3(t) = \lambda_1 P_z B/4\pi(b^2-a^2)f\\
&B_3(t) = 0 \text{ and}\\
&C_3(t) = -2(\lambda_2+\mu_2)BP_z/4\pi(b^2-a^2)f
\end{aligned} \qquad (27)_{1\text{-}2\text{-}3}$$

where

$$f = (\lambda_2+\mu_2)(\lambda_1+2\mu_1) - \lambda_1^2. \qquad (28)$$

Employing (25), (26) and (27) into (22) it possible to obtain the microdisplacements at points A, B and Γ respectively. Also employing (25), (26) and (27) into (23), it is possible to obtain the corresponding microstrains. Thus:

i) at point A:

$$\begin{aligned}
&e^1{}_{rr} = e^1{}_{\theta\theta} = \lambda_1 G_f B/2\pi(b^2-a^2)f\\
&e^1{}_{zz} = -(\lambda_2+\mu_2)G_f B/\pi(b^2-a^2)f
\end{aligned} \qquad (29)_{1\text{-}2\text{-}3}$$

ii) at point B:

$$\begin{aligned}
&e^2{}_{rr} = e^2{}_{\theta\theta} = \lambda_1 F_z B/2\pi(b^2-a^2)f\\
&e^2{}_{zz} = -(\lambda_2+\mu_2)F_z B)/\pi(b^2-a^2)f
\end{aligned} \qquad (30)_{1\text{-}2\text{-}3}$$

iii) at point Γ:

$$\begin{aligned}
&e^3{}_{rr} = e^3{}_{\theta\theta} = \lambda_1 P_z B/2\pi(b^2-a^2)f\\
&e^3{}_{zz} = -(\lambda_2+\mu_2)P_z B/\pi(b^2-a^2).
\end{aligned} \qquad (31)_{1\text{-}2\text{-}3}$$

At continuity we distinguish the following cases:

i) Internal remodeling of tibia does depends upon temperature:

Then substituting (29), (30), (31) into $(18)_1$ it follows:

i) at point A:

$$d\hat{e}/dt = A(\hat{e}) + \begin{bmatrix} A_T(\hat{e})\lambda_1 \\ -A_A(\hat{e})(\lambda_2+\mu_2) \end{bmatrix} G_z B/\pi(b^2-a^2)F \quad (32)$$

ii) at point B:

$$d\hat{e}/dt = A(\hat{e}) + \begin{bmatrix} A_T(\hat{e})\lambda_1 \\ -A_A(\hat{e})(\lambda_2+\mu_2) \end{bmatrix} F_z B/\pi(b^2-a^2)f \quad (33)$$

iii) at point Γ:

$$d\hat{e}/dt = A(\hat{e}) + \begin{bmatrix} A_T(\hat{e})\lambda_1 \\ -A_A(\hat{e})(\lambda_2+\mu_2) \end{bmatrix} P_z B/\pi(b^2-a^2)f. \quad (34)$$

Since the living bone is continually remodeling [47,48] we assume that Young's modulus and Poisson's ratio depends upon to ê [34,49,50,51]. Then from (17) it results that λ_1, λ_2, μ_1, μ_2 depend also upon ê. At continuity we impose:

$$\begin{aligned}
&A_T(\hat{e}) = \alpha_T + \hat{e}\alpha_T,\ A_T(\hat{e}) = \alpha_A + \hat{e}\alpha_A\\
&\lambda_1(\hat{e}) = \Lambda_1 + \hat{e}\Lambda_1,\ \lambda_2(\hat{e}) = \Lambda_2 + \hat{e}\Lambda_2\\
&\mu_1(\hat{e}) = M_1 + \hat{e}M_1,\ \mu_2(\hat{e}) = M_2 + \hat{e}M_2
\end{aligned} \qquad (35)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6\text{-}7}$$

Therefore (32)-(33)-(34) conclude to the following form:

$$d\hat{e}/dt = \alpha^{(i)}(\hat{e}^2 - 2\beta^{(i)}\hat{e} + \gamma^{(i)}),\ i = 1,2,3 \qquad (36)$$

where:

i) at point A:

$$\begin{aligned}
&\alpha^{(1)} = c_2,\ \beta^{(1)} = -c_1/2c_2\\
&\gamma^{(1)} = \left[c_o + \begin{bmatrix} \Lambda_1\alpha_T \\ -(\Lambda_2+M_2)\alpha_A \end{bmatrix} G_z B \right] / c_2\pi(b^2-a^2)F
\end{aligned} \qquad (37)_{1\text{-}2\text{-}3}$$

ii) at point B:

$$\begin{aligned}
&\alpha^{(2)} = c_2,\ \beta^{(2)} = -c_1/2c_2 \text{ and}\\
&\gamma^{(2)} = \left[c_o + \begin{bmatrix} \Lambda_1\alpha_T \\ -(\Lambda_2+M_2)\alpha_A \end{bmatrix} F_z B \right] / c_2\pi(b^2-a^2)F
\end{aligned} \qquad (38)_{1\text{-}2\text{-}3}$$

iii) at point Γ:

$$\begin{aligned}
&\alpha^{(3)} = c_3,\ \beta^{(3)} = -c_1/2c_2 \textit{ and}\\
&\gamma^{(3)} = \left[c_o + \begin{bmatrix} \Lambda_1\alpha_T \\ -(\Lambda_2+M_2)\alpha_A \end{bmatrix} P_z B \right] / c_2\pi(b^2-a^2)F
\end{aligned} \qquad (39)_{1\text{-}2\text{-}3}$$

where:

$$F = (\Lambda_2+M_2)(\Lambda_1+2M_1)^2 - \Lambda_1^2 \qquad (40)$$

Initially the mean length of the pre-existing microcracks [52,53] at points A, B and Γ of tibia was

$$e(0) = e_o. \qquad (41)$$

The solutions of (36) satisfying (41) are:

$$\begin{aligned}
&e^{(i)}{}_1 = \beta^{(i)} + \sqrt{\left(\beta^{(i)^2} - \gamma^{(i)}\right)}\\
&\text{and } e^{(i)}{}_2 = \beta^{(i)} - \sqrt{\left(\beta^{(i)^2} - \gamma^{(i)}\right)}
\end{aligned} \qquad (42)_{1\text{-}2}$$

The acceptable solutions are given in Table 1. [34]. Accor-dingly to these results after a long time period, the tibia of athlete locally at points A, B and Γ will be strenghted, that is the mean lenght of their microcracks will be decreased. The last can be explained as follows: i) the mean lenght of pre-existing microcracks [52,53] wil be decreased or ii) some of preexisting microcracks [52,53] will be closed or iii) a combination of both previous cases will be arised.

Table 1. The acceptable solutions of our problem

Cases:	The solution and its physic-cal meaning
$-\xi_o < e_2 < eo < e_1 < 1-\xi_o$ and $\alpha<0$	lime(t)= $e_1 > e_o$. Tibia at points A, B and Γ has been strenghted.
$-\xi_o < e_o < e_1 < e_2 < 1-\xi_o$ and $\alpha>0$	lime(t)= $e_1 > e_o$. Tibia at points A, B and Γ has been strenghted
$-\xi_o < e_o < e_2 < 1-\xi_o < e_1$ and $\alpha>0$	lime(t)= $e_2 > e_o$. Tibia at points A, B and Γ has been strenghted.
$e_2 < -\xi_o < e_o < e_1 < 1-\xi_o$ and $\alpha<0$	lime(t)= $e_2 > e_o$. Tibia at points A, B and Γ has been strenghted.

From the other hand we distinguish the following cases:

i) $G_z \geq F_z \geq P_z$ then from $(37)_3$, $(38)_3$ and $(39)_3$ it is possible to obtain: $\gamma^{(1)} \geq \gamma^{(2)} \geq \gamma^{(3)}$ and because of $(42)_2$ it follows:

$$e^{(1)}{}_1 \leq e^{(2)}{}_1 \leq e^{(3)}{}_1. \quad (43)$$

We observe that the decrease of porosity of tibia at points A, B, Γ due only to mechanic loads. Particularly it analogically depends upon the magnitude of them.

ii) $G_z \leq F_z \leq P_z$ then from $(37)_3$, $(38)_3$ and $(39)_3$ it is possible to obtain: $\gamma^{(1)} \leq \gamma^{(2)} \leq \gamma^{(3)}$ and because of $(42)_1$ it follows:

$$e^{(1)}{}_1 \geq e^{(2)}{}_1 \geq e^{(3)}{}_1. \quad (44)$$

We observe that the decrease of porosity of tibia at points A, B ,Γ due only to mechanic loads. Particularly it revengelly depends upon the magnitude of them.

ii) The internal remodeling of tibia depends upon temperature:

Then substituting (29),(30),(31) into $(18)_2$ it follows:

i) at point A:

$$d\hat{e}/dt = A(\hat{e}) + \begin{bmatrix} A_T(\hat{e})\lambda_1 \\ -A_A(\hat{e})(\lambda_2+\mu_2) \end{bmatrix} G_f B/\pi(b^2-a^2)f + B(\hat{e})\theta \quad (45)$$

ii) at point B:

$$d\hat{e}/dt = A(\hat{e}) + \begin{bmatrix} A_T(\hat{e})\lambda_1 \\ -A_A(\hat{e})(\lambda_2+\mu_2) \end{bmatrix} P_f B/\pi(b^2-a^2)f + B(\hat{e})\theta \quad (46)$$

iii) at point Γ:

$$d\hat{e}/dt = A(\hat{e}) + \begin{bmatrix} A_T(\hat{e})\lambda_1 \\ -A_A(\hat{e})(\lambda_2+\mu_2) \end{bmatrix} F_f B/\pi(b^2-a^2)f + B(\hat{e})\theta. \quad (47)$$

Finally substituting (35) and

$$B(\hat{e}) = d + d\hat{e} \quad (48)$$

into (45)-(46) -(47) we again conclude to (36) where:

i) at point A:

$$\alpha^{(1)} = c_2,\ \beta^{(1)} = -(2d+c_1)/2c_2 \ \textit{and}$$
$$\gamma^{(1)} = \frac{c_o + \left[\Lambda_1\alpha_T - (\Lambda_2+M_2)\alpha_A\right]G_f B}{c_2\left[\pi(b^2-a^2)F+d\right]} \quad (49)_{1\text{-}2\text{-}3}$$

ii)at point B:

$$\alpha(2) = c_2,\ \beta^{(2)} = -(2d+c_1)/2c_2 \text{ and}$$
$$\gamma^{(2)} = \frac{c_o + \left[\Lambda_1\alpha_T - (\Lambda_2+M_2)\alpha_A\right]P_f B}{c_2[\pi(b^2-a^2)F+d]} \quad (50)_{1\text{-}2\text{-}3}$$

iii) at point Γ:

$$\alpha^{(3)} = c_2,\ \beta^{(3)} = -(2d+c_1)/2c_2$$
$$\gamma^{(3)} = \frac{c_o + \left[\Lambda_1\alpha_T - (\Lambda_2+M_2)\alpha_A\right]F_f B}{c_2[\pi(b^2-a^2)F+d]}. \quad (51)_{1\text{-}2\text{-}3}$$

The solutions of (36) satisfying (41) are given by $(42)_{1\text{-}2}$ and the same as at previous case are valid.

4. Discussion -Conclusion

Our model at both cases accounting and neglecting temperature coincide with the result of corresponding problem at macroscopic area [34,54-62]. From the above we conclude that temperature plays no one role to growth and propagation of microcracks in a tibia due to volley ball activity. In contrast with the above phenomenon exclusively due to mechanical loads.

References

[1] Burstein, A., Reilly, D. and Martens, M. (1976). "Aging of bone tissue mechanical properties. J. Bone Joint Surg." A58, 82-86.

[2] Thompson, D. (1980): "Age changes in bone mineralization, cortical thickness, and haversian canal area." Calcif Tissue Int. 31, 5-11.

[3] Grynpas, M. D.and Holmyard, D. (1988): "Changes inquality of bone mineral on aging and in disease." Scan Microsc. 2, 1045-1054.

[4] Hui, S. L., Slemenda, C. W. and Johnston, C.C. (1988): Age and bone mass as predictors of fracture in a prospective study. J. Clin. Invest. 81, 1804-1809.

[5] Kiebzak G. M. (1991). "Age-related bone changes." Exp. Gerontol. 26, 171-187.

[6] Simmons, E. D., Pritzker, K. P. and Grynpas, M. D. (1991). "Age - related changes in the human femoral cortex." J. Orthop. Res.9, 155-167.

[7] Melvin JW. "Fracture mechanics of bone." J. Biomech. Eng. 1993. Nov; 115(4B):549-554.

[8] Currey, J. D., Brear, K. and Zioupos, P. (1996). "The effects of aging and changes in mineral content in degrading the toughness of human femora." J. Biomech. 29, 257-260.

[9] Francis, R.M. (1996). "Low bone mineral content is common but osteoporotic fractures are rare in elderly rural Gambian women." J. Bone Miner. Res.11, 1019-1025.

[10] Aspray, T. J., Prentice, A., Cole, T. J., Sawo, Y., Reeve, J. and Francis RM. (1996). "Low bone mineral content is comon but osteoporotic fractures are rare in elderly rural Gambian women." J. Bone Miner. Res.11, 1019-1025

[11] Yeni, Y. N. and Norman, T. L. (2000). “Fracture toughness of human femoral neck: Effect of microstructure, composition and age.” Bone 26, 499-504.

[12] Wang, X., Shen, X., Li, X. and Agrawal C. M. (2002). “Age- related changes in the collagen network and toughness of bone.” Bone 31, 1-7.

[13] Akkus, O., Adar, F. and Schaffler, M. B. (2004). “Age-related changes in physicochemical properties of mineral crystals are related to impaired mechanical function of cortical bone.” Bone 34, 443-453.

[14] Ritchie. R, Kinney H., Kruzic R., and Nalla R. (2005). “A fracture mechanics and mechanistic approach to the failure of cortical bone.” Fatigue Fract. Engng Mater Struct. Fatigue Fract. Engng Mater Struct 28, 345-37

[15] Behiri, JC. and Bonfield W. (1989): “Orientation depen-dence of the fracture mechanics of cortical bone.” J. Biomech,. 22, 863-872.

[16] Yeni, Y. N., Brown, C. U., Wang, Z. and Norman, T. L. (1997). “The influence of bone morphology on fracture toughness of the human femur and tibia.” Bone 21, 453-459.

[17] Yeni, Y. N., Brown, C.U. and Norman, T.L. (1998). “Influence of bone composition and apparent density on fracture toughness of the human femur and tibia.” Bone 22, 79-84.

[18] Feng, Z., Rho, J., Han, S. and Ziv, I. (2000). “Orientation and loading condition dependence of fracture toughness in cortical bone.” Mater. Sci. Engng CC11, 41-46.

[19] Brown, C. U., Yeni, Y. N. and Norman, TL.(2000). “Fracture toughness is dependent on bone location-A study of the femoral neck, femoral shaft and the tibial shaft.” J. Biomed. Mater. Res.49, 380-389.

[20] Phelps, J. B., Hubbard, G. B., Wang, X. and Agrawal C. M. (2000): “Microstructural heterogeneity and the fracture toughness of bone.” J. Biomed. Mater. Res.51, 735-471.

[21] Yeni, Y. N. and Norman, T. L. (2000). “Fracture toughness of human femoral neck: Effect of microstructure, composition and age.” Bone 26, 499-504.

[22] Seeman, E. (1999). “The structural basis of bone fragility in men.” Bone 25, 143-147.

[23] Rimnac, C. M., Petko, A. A., Santners, T. J. and Wright, T. M (1993). “The effect of temperature, stress and microstructure on the creep of compact bovine bone.” J. Biomech.26, 219-228.

[24] Ford C.M. and Keaveny, T.M. (1996). “The dependence of shear failure properties of trabecular bone on apparent density and trabecular orientation.” J. Biomech. 29, 1309-1317.

[25] Carter, D. R. and Hayes, W. C. (1976): “Fatigue life of compact bone-I. Effects of stress amplitude, temperature and density.” J. Biomech.9, 27-34, Biomech. 26, 219-228.

[26] Norman, T.L., Nivargikar, S. V. and Burr, D. B. (1996). “Resistance to crack growth in human cortical bone is greater in shear than in tension.” J. Biomech.29, 1023-1031.

[27] Feng, Z., Rho, J., Han, S. and Ziv, I. (2000). “Orientation and loading condition dependence of fracture toughness in cortical bone.” Mater. Sci. Engng CC11, 41-46.

[28] Feng X. and McDonald J. (2011). “Disorders of Bone Remodeling,” Annu Rev Pathol.; 6: 121-145.

[29] Cowin S. and Hegedus D. (1976). “Bone remodeling I: Theory of adaptive elasticity” J. Elastic. 6, pp. 313-326.

[30] Hegedus D. και Cowin S. (1976). “Bone remodeling II: Theory of adaptive elasticity” J. Elastic. 6, pp. 337-352.

[31] Sih G.C. (1985). “Mechanics and Physics of energy density theory”, Theoret., Appl., Fract. , Mech., 44, pp. 157-173.

[32] Sih G.C (1972-1982). “Mechanics of fracture, Introductory chapters”, Vol. I- VII, edited by G.C. Sih, Martinus Nijhoff, The Hague.

[33] Sih GC(1988). “Thermomechanics of solids: nonequilibrium and irreversibility”, Theoretical and Applied Fracture Mechanics, 9, pp. 175-198.

[34] Tsili M. (2008). “The hyperthrophy of tibia induced by the volleyball” in: www.ispub.com/journal-of-internet journal of bioengineering , Volume .4. number 1.

[35] Harless E. (1860). “The static moments of human limbs (in German). Treatsises of the Math-Phys. Class of the Royal Acad. of Sci. of Bavaria, 8, pp.69 και 257.

[36] Andriachi T., Ogle J. and Galante J. (1977). “Walking speed as a base for normal and abnormal gait measurement.” J. Biomech., 10, pp. 261-268.

[37] Rohrle H., Scholten R., Sigolloto C. et., al., (1984). “Joint forces in the human pelvis - leg skeleton during walking.” J. Biomech., 17, pp. 409-424.

[38] Whalen R., Carter D. and Steele C. (1988). “Influence of the physical activity on the regulation of bone activity.” J. Biomech., 21., pp., 825-837.

[39] Alexander N. and Jayes A. (1980). “Fourier analysis on forces exerted in walking and running.” J. Biomech. 13. pp. 383-390.

[40] Bates B., Osterning L. and Sawhill J. (1983). “An assessment of subject variability, subject -shoe interaction and the evaluation of running shoes using ground reaction force data.” J. Biomech., 16., pp.181-191.

[41] Cavagna P. (1964). “Mechanical work in running” J. Appl. Physiol. 19., pp. 249-256

[42] Cavagna P. and LaFortune M. (1980). “Ground reaction forces in distance -running.” J. Biomech., 13. pp. 397-406.

[43] Fukanaga T., Matsuo A., Yuasa K., et., al., (1980). “Effect of running velocity on external mechanical output.” Ergonomics, 23, pp.,123-136

[44] Winter D. (1983). “Moments of forces and mechanical power in jogging.” J.Biomech., 16, pp., 91-97.

[45] Frost H.M (1964). “Dynamics of bone remodeling in bone biodynamics.” (edited by Frost H.M) Little and Brown 316, Boston.

[46] Reilly D.T. and Burstein A. (1975). J. Biomech., 8., p.393.

[47] Cowin S. and Van -Burskirk W. (1978). “Internal bone remodeling induced by a medullary pin.” J. Biomech. 11, pp. 269-275.

[48] Wolff. J. (1884). “Das gesetz der transformation der inneren architecture knocken bei pathologism veranderungen der aussen knochenform.” Sitz Ber. Preuss Acad. d. Wiss 22, Sitz Physik-Math. K_1.

[49] Wolff J. (1892). Das gesetz der transformation knocken hirschald, Berlin.

[50] Tsili M. (2000). “Theoretical solutions for internal bone remodeling of diaphyseal shafts using adaptive elasticity theory.” J. Biomech., 33 pp. 235-239.

[51] Tsili M. (2008): “Internal bone remodeling induced by the distance - running and the unkown remodeling coefficients” in: www.ispub.com/journal- of- internet journal of bioengineering vol. 4. number 2.

[52] Grifith A. (1921). “The phenomena of rupture and flow in solids” Philosophical Transactions of the Royal Society of London A 221, pp., 163-198

[53] Griffith A. (1924). “The theory of rupture.” In: Proc., Ist., Int., Congr., Appl., Mech. Biereno, C.B. Burgers, J.M(eds). Delft: Tech. Boekhandel en Drukkerij. J. Waltman Jr., pp. 54-63.

[54] Calbet A., Diaz Herrera P and Rodrignex L. (1999). “High bone mineral density in male elite professional volleyball players.” Osteopor., Inter, 10, pp. 468-474.

[55] Fehling P., Alekel L. and Classey J., et., al., (1995). “A comparison of bone mineral densities among female athletes in impact loading and active loading sports.” Bone 17., pp. 205-210.

[56] Ito M., Nakamura T., Ikesa S., et., al., (2001). “Effects of life time volleyball exersize on bone mineral densities in lum barspine, calcaneus and tibia for pro, peri- and postmenopausal women.” Ostepor. Intern., 12, pp. 104-111.

[57] Rittweger J., Beller G., Ehrig G., et., al., (2000). “Bone- muscle strenght indices for the human lower leg.” Bone , 27. pp. 319-326.

[58] Alfredson N., Nordsatorm P. and Lorentzon R., (1997). “Bone mass in female volleyball players. A comparison of total and regional bone mass in female volleyball players and nonactive females.” Calcif., Tissue, Int., 60, pp. 338-342.

[59] Hara S., Yanagi H., Amagai H., et., al., (2001). “Effect of physical activity during teenage years, based on type of sport and duration of exersize on bone mineral density of young, premenopausal Japanese women.” Calcif., Tissue Int., 68., pp. 23-30.

[60] Nikander R., Sievanen H., Heinonen A. et., al., (2005). “Femoral neck structure in adult female athlete, subjected to different loading modalities.” J. Bone Miner., Res. 20., pp. 520-528.

[61] Nikander R., Sievanen H., Heinonen A. et., al., (2006). “Loading modalities and bone structures at nonweight-bearing upper extremity and weight - bearing lower extremity” A poct- study of adult female athletes.” Bone, 39, pp. 886-894.

[62] Nichols L., Raugh M., Barrack M. (2007). “Bone mineral density in female high school athletes: Interactions of mentrual function and type of mechanical loading.” Bone 41., pp. 371-377.