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Using Maple to Study Some Differential Problems

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Abstract This paper takes the mathematical software Maple as the auxiliary tool to study the differential problems of four types of functions. We can obtain the infinite series forms of any order derivatives of these functions by using binomial theorem and differentiation term by term theorem, and hence greatly reduce the difficulty of calculating their higher order derivative values. On the other hand, we propose two examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple.

Keywords: *derivatives, infinite series forms, binomial theorem, differentiation term by term theorem, Maple*

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1. Introduction

The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify our previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. As for the instructions and operations of Maple, we can refer to [1-7].

In calculus and engineering mathematics curricula, evaluating the k-th order derivative value $f^{(k)}(c)$ of a function $f(x)$ at $x=c$, in general, needs to go through two procedures: firstly determining the k-th order derivative $f^{(k)}(x)$ of $f(c)$, and then taking $x=c$ into $f^{(k)}(x)$. These two procedures will make us face with increasingly complex calculations when calculating higher order derivative values of this function (i.e. k is large), and hence to obtain the answers by manual calculations is not easy. In this paper, we mainly study the differential problems of the following four types of functions

$$f(x)=\cos[\beta\cosh(rx+s)] \tag{1}$$

$$g(x)=\sin[\beta\cosh(rx+s)] \tag{2}$$

$$p(x)=\cos[\beta\sinh(rx+s)] \tag{3}$$

$$q(x)=\sin[\beta\sinh(rx+s)] \tag{4}$$

where β, r, s are real numbers. We can determine the infinite series forms of any order derivatives of these functions by using binomial theorem and differentiation term by term theorem; these are the major results of this study (i.e., Theorems 1 and 2), and hence greatly reduce the difficulty of calculating their higher order derivative values. For the study of related differential problems can refer to [8-21]. In addition, we provide two examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. Therefore, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce some formulas used in this study.

2.1. Formulas

2.1.1. Taylor Series Expression of the Cosine Function

$$\cos y=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}y^{2n}$$, where y is any real number.

2.1.2. Taylor Series Expression of the Sine Function

$\sin y = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} y^{2n+1}$, where y is any real number.

The followings are two important theorems used in this paper.

2.2. Theorems

2.2.1. Binomial Theorem

Suppose x, y are any real numbers, and n is any positive integer. Then $(x+y)^n = \sum_{m=0}^{n} \binom{n}{m} x^{n-m} y^m$, where

$\binom{n}{m} = \frac{n(n-1)\cdots(n-m+1)}{m!}$ for all positive integers $m \le n$, and $\binom{n}{0} = 1$.

2.2.2. Differentiation Term by Term Theorem ([22])

For all non-negative integers k, if the functions $g_k:(a,b) \to R$ satisfy the following three conditions : (i) there exists a point $x_0 \in (a,b)$ such that $\sum_{k=0}^{\infty} g_k(x_0)$ is convergent, (ii) all functions $g_k(x)$ are differentiable on open interval (a,b), (iii) $\sum_{k=0}^{\infty} \frac{d}{dx} g_k(x)$ is uniformly convergent on (a,b). Then $\sum_{k=0}^{\infty} g_k(x)$ is uniformly convergent and differentiable on (a,b). Moreover, its derivative $\frac{d}{dx} \sum_{k=0}^{\infty} g_k(x) = \sum_{k=0}^{\infty} \frac{d}{dx} g_k(x)$.

Before deriving our first major result, we need a lemma.

2.3. Lemma 1

Suppose r,s are real numbers, and n is any positive integer. Then

$$\cosh^n(rx+s) = \frac{1}{2^n} \cdot \sum_{m=0}^{n} \binom{n}{m} e^{(n-2m)(rx+s)} \quad (5)$$

for all $x \in R$.

Proof

$$\cosh^n(rx+s) = \left[\frac{1}{2}[e^{(rx+s)} + e^{-(rx+s)}]\right]^n$$

$$= \frac{1}{2^n} \cdot \sum_{m=0}^{n} \binom{n}{m} [e^{(rx+s)}]^{n-m} [e^{-(rx+s)}]^m$$

(By binomial theorem)

$$= \frac{1}{2^n} \cdot \sum_{m=0}^{n} \binom{n}{m} e^{(n-2m)(rx+s)}$$

The following is the first major result of this article, we obtain the infinite series forms of any order derivatives of the functions (1) and (2).

2.4. Theorem 1

Suppose β, r, s are real numbers, and k is any positive integer.

(1) If the domain of the function

$$f(x) = \cos[\beta \cosh(rx+s)]$$

is $(-\infty,\infty)$. Then the k-th order derivative of $f(x)$,

$$f^{(k)}(x) = r^k \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{2^{2n}(2n)!} \cdot \sum_{m=0}^{2n} \binom{2n}{m} (2n-2m)^k e^{(2n-2m)(rx+s)} \quad (6)$$

for all $x \in R$.

(2) Suppose the domain of the function

$$g(x) = \sin[\beta \cosh(rx+s)]$$

is $(-\infty,\infty)$. The k-th order derivative of $g(x)$,

$$g^{(k)}(x) = r^k \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n+1}}{2^{2n+1}(2n+1)!} \cdot \sum_{m=0}^{2n+1} \binom{2n+1}{m} (2n-2m+1)^k e^{(2n-2m+1)(rx+s)} \quad (7)$$

for all $x \in R$.

Proof (1) Because

$$f(x) = \cos[\beta \cosh(rx+s)]$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [\beta \cosh(rx+s)]^{2n}$$

(By Formula 2.1.1)

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{(2n)!} \cosh^{2n}(rx+s)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{2^{2n}(2n)!} \cdot \sum_{m=0}^{2n} \binom{2n}{m} e^{(2n-2m)(rx+s)} \quad (8)$$

(By Lemma 1)

By differentiation term by term theorem, differentiating k-times with respect to x on both sides of (8), we obtain

$$f^{(k)}(x) = r^k \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{2^{2n}(2n)!} \cdot \sum_{m=0}^{2n} \binom{2n}{m} (2n-2m)^k e^{(2n-2m)(rx+s)}$$

for all $x \in R$.

(2) Because

$$g(x) = \sin[\beta \cosh(rx+s)]$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [\beta \cosh(rx+s)]^{2n+1}$$

(By Formula 2.1.2)

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n+1}}{(2n+1)!} \cosh^{2n+1}(rx+s)$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n+1}}{2^{2n+1}(2n+1)!} \cdot \sum_{m=0}^{2n+1} \binom{2n+1}{m} e^{(2n-2m+1)(rx+s)} \tag{9}$$

(By Lemma 1)

Using differentiation term by term theorem, differentiating k-times with respect to x on both sides of (9), we have

$$g^{(k)}(x) = r^k \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n+1}}{2^{2n+1}(2n+1)!} \cdot \sum_{m=0}^{2n+1} \binom{2n+1}{m} (2n-2m+1)^k e^{(2n-2m+1)(rx+s)}$$

for all $x \in R$.

To prove the second major result of this study, we also need a lemma.

2.5. Lemma 2

If the assumptions are the same as Lemma 1, then

$$\sinh^n(rx+s) = \frac{1}{2^n} \cdot \sum_{m=0}^{n} (-1)^m \binom{n}{m} \cdot e^{(n-2m)(rx+s)} \tag{10}$$

for all $x \in R$.

Proof

$$\sinh^n(rx+s) = \left[\frac{1}{2}[e^{(rx+s)} - e^{-(rx+s)}]\right]^n$$
$$= \frac{1}{2^n} \cdot \sum_{m=0}^{n} (-1)^m \binom{n}{m} \cdot e^{(n-m)(rx+s)} e^{-m(rx+s)}$$

(By binomial theorem)

$$= \frac{1}{2^n} \cdot \sum_{m=0}^{n} (-1)^m \binom{n}{m} \cdot e^{(n-2m)(rx+s)}$$

Next, we determine the infinite series forms of any order derivatives of the functions (3) and (4).

2.6. Theorem 2

If the assumptions are the same as Theorem 1.

(1) Let the domain of the function

$$p(x) = \cos[\beta \sinh(rx+s)]$$

be $(-\infty, \infty)$. Then the k-th order derivative of $p(x)$,

$$p^{(k)}(x) = r^k \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{2^{2n}(2n)!} \cdot \sum_{m=0}^{2n} (-1)^m \binom{2n}{m} (2n-2m)^k e^{(2n-2m)(rx+s)} \tag{11}$$

for all $x \in R$.

(2) Suppose the domain of the function

$$q(x) = \sin[\beta \sinh(rx+s)]$$

is $(-\infty, \infty)$. Then

$$q^{(k)}(x) = r^k \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n+1}}{2^{2n+1}(2n+1)!} \cdot \sum_{m=0}^{2n+1} (-1)^m \binom{2n+1}{m} (2n-2m+1)^k e^{(2n-2m+1)(rx+s)} \tag{12}$$

for all $x \in R$.

Proof (1) Because

$$p(x) = \cos[\beta \sinh(rx+s)]$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [\beta \sinh(rx+s)]^{2n}$$

(By Formula 2.1.1)

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{(2n)!} \sinh^{2n}(rx+s)$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{2^{2n}(2n)!} \cdot \sum_{m=0}^{2n} (-1)^m \binom{2n}{m} e^{(2n-2m)(rx+s)} \tag{13}$$

(By Lemma 2)

Also, by differentiation term by term theorem, differentiating k-times with respect to x on both sides of (13), we have

$$p^{(k)}(x) = r^k \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n}}{2^{2n}(2n)!} \cdot \sum_{m=0}^{2n} (-1)^m \binom{2n}{m} (2n-2m)^k e^{(2n-2m)(rx+s)}$$

for all $x \in R$.

(2) Because

$$q(x) = \sin[\beta \sinh(rx+s)]$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [\beta \sinh(rx+s)]^{2n+1}$$

(By Formula 2.1.2)

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n+1}}{(2n+1)!} \sinh^{2n+1}(rx+s)$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n+1}}{2^{2n+1}(2n+1)!} \cdot \sum_{m=0}^{2n+1} (-1)^m \binom{2n+1}{m} e^{(2n-2m+1)(rx+s)} \tag{14}$$

(By Lemma 2)

Using differentiation term by term theorem, differentiating k-times with respect to x on both sides of (14), then

$$q^{(k)}(x) = r^k \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \beta^{2n+1}}{2^{2n+1}(2n+1)!} \cdot \sum_{m=0}^{2n+1} (-1)^m \binom{2n+1}{m} (2n-2m+1)^k e^{(2n-2m+1)(rx+s)}$$

for all $x \in R$.

3. Examples

In the following, for the differential problems of the four types of functions in this study, we provide two examples and use Theorems 1,2 to determine the infinite series forms of any order derivatives and some higher order derivative values of these functions. On the other hand, we employ Maple to calculate the approximations of these higher order derivative values and their solutions for verifying our answers.

3.1. Example 1

3.1.1. Suppose the Domain of the Function

$$f(x)=\cos[2\cosh(4x-3)] \tag{15}$$

is $(-\infty,\infty)$. Then by (6) of Theorem 1, we obtain any *k*-th order derivative of *f(x)*,

$$f^{(k)}(x)=4^k\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{2^{2n}(2n)!}\cdot\sum_{m=0}^{2n}\binom{2n}{m}(2n-2m)^k e^{(2n-2m)(4x-3)} \tag{16}$$

for all $x\in R$.

Thus, we can evaluate the 5-th order derivative value of *f(x)*at *x=1/4*,

$$f^{(5)}\left(\frac{1}{4}\right)=4^5\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{2^{2n}(2n)!}\cdot\sum_{m=0}^{2n}\binom{2n}{m}(2n-2m)^5 e^{-4(n-m)} \tag{17}$$

Next, we use Maple to verify the correctness of (17).

>f:=x->cos(2*cosh(4*x-3));
>evalf((D@@5)(f)(1/4),28);
5.511455125745335740513334·10^5
>evalf(4^5*sum((-1)^n*2^(2*n)/(2^(2*n)*(2*n)!)*sum((2*n)!/(m!*(2*n-m)!)*(2*n-2*m)^5*exp(-4*(n-m)), m=0.2*n),n=0.infinity),28);
5.511455125745335740513345·10^5

3.1.2. If the Domain of the Function

$$g(x)=\sin[3\cosh(5x-6)] \tag{18}$$

is $(-\infty,\infty)$. Then by (7) of Theorem 1, we obtain

$$g^{(k)}(x)=5^k\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 3^{2n+1}}{2^{2n+1}(2n+1)!}\cdot\sum_{m=0}^{2n+1}\binom{2n+1}{m}(2n-2m+1)^k e^{(2n-2m+1)(5x-6)} \tag{19}$$

for all $x\in R$.

Therefore, the 7-th order derivative value of *g(x)*at *x=1*,

$$g^{(7)}(1)=5^7\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 3^{2n+1}}{2^{2n+1}(2n+1)!}\cdot\sum_{m=0}^{2n+1}\binom{2n+1}{m}(2n-2m+1)^7 e^{-(2n-2m+1)} \tag{20}$$

>g:=x->sin(3*cosh(5*x-6));
>evalf((D@@7)(g)(1),24);
3.11674791628708010692687·10^9
>evalf(5^7*sum((-1)^n*3^(2*n+1)/(2^(2*n+1)*(2*n+1)!)*sum((2*n+1)!/(m!*(2*n-m+1)!)*(2*n-2*m+1)^7*exp(-(2*n-2*m+1)),m=0.(2*n+1)),n=0.infinity),24);
3.11674791628708010692539·10^9

3.2. Example 2

3.2.1. Assume the Domain of the Function

$$p(x)=\cos[4\sinh(8x+6)] \tag{21}$$

is $(-\infty,\infty)$. Using (11) of Theorem 2, we obtain

$$p^{(k)}(x)=8^k\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 4^{2n}}{2^{2n}(2n)!}\cdot\sum_{m=0}^{2n}(-1)^m\binom{2n}{m}(2n-2m)^k e^{(2n-2m)(8x+6)} \tag{22}$$

for all $x\in R$.

Thus, the 6-th order derivative value of *p(x)* at *x=–1/2*,

$$p^{(6)}\left(-\frac{1}{2}\right)=8^6\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 4^{2n}}{2^{2n}(2n)!}\cdot\sum_{m=0}^{2n}(-1)^m\binom{2n}{m}(2n-2m)^6 e^{4(n-m)} \tag{23}$$

>p:=x->cos(4*sinh(8*x+6));
>evalf((D@@6)(p)(-1/2),24);
-1.85432625222336228 9·10^{12}
>evalf(8^6*sum((-1)^n*4^(2*n)/(2^(2*n)*(2*n)!)*sum((-1)^m*(2*n)!/(m!*(2*n-m)!)*(2*n-2*m)^6*exp(4*(n-m)), m=0.2*n),n=0.infinity),24);
1.85432625222336226 9·10^{12}

3.2.2. If the Domain of the Function

$$q(x)=\sin[7\sinh(2x-5)] \tag{24}$$

is $(-\infty,\infty)$. Using (12) of Theorem 2, we obtain

$$q^{(k)}(x)=2^k\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 7^{2n+1}}{2^{2n+1}(2n+1)!}\cdot\sum_{m=0}^{2n+1}(-1)^m\binom{2n+1}{m}(2n-2m+1)^k e^{(2n-2m+1)(2x-5)} \tag{25}$$

for all $x\in R$.

Hence, we can determine the 6-th order derivative value of *q(x)* at *x=3*,

$$q^{(k)}(x)=2^k\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 7^{2n+1}}{2^{2n+1}(2n+1)!}\cdot\sum_{m=0}^{2n+1}(-1)^m\binom{2n+1}{m}(2n-2m+1)^k e^{(2n-2m+1)(2x-5)} \tag{26}$$

>q:=x->sin(7*sinh(2*x-5));
>evalf((D@@6)(q)(3),20);
-9.468885110895329·10^7
>evalf(2^6*sum((-1)^n*7^(2*n+1)/(2^(2*n+1)*(2*n+1)!)*sum((-1)^m*(2*n+1)!/(m!*(2*n-m+1)!)*(2*n-2*m+1)^6*exp(2*n-2*m+1),m=0.(2*n+1)),n=0.infinity),20);
-9.468885110895331·10^7

4. Conclusion

In this article, we provide a new technique to determine any order derivatives of some types of functions. We hope this technique can be applied to solve another differential problems. On the other hand, the binomial theorem and the differentiation term by term theorem play significant roles in the theoretical inferences of this study. In fact, the applications of these two theorems are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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