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A Problem of Pursuit Game with Various Constraints on Controls of Players

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Abstract This work introduce a difference controls of players by using a new control method to completing a pursuit game. We study pursuit game problems for controlled partial differential equations of the parabolic type. We proved a theorem on pursuit game with mixed constraints, where pursuers control are subjected to integral (geometric) constraint and geometric (integral) constraint are imposed on evaders control. Moreover, we established the sufficient conditions for which pursuit is possible in the game considered.

Subject Classification: 49N70, 49N75, 91A23

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1. Introduction

Let A be a differential elliptic operator in the space $L_2(\Omega)$, of the form

$$Az = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x) \frac{\partial z}{\partial x_j} \right). \qquad (1)$$

The domain of definition $D(A)$ of the operator A is $\overset{\circ}{C}{}^2(\Omega)$ (space of twice continuously differentiable functions), Ω is a bounded domain with piece-wise smooth boundary in R^n and $a_{ij}(x), x \in \Omega,$ is a measurable bounded function satisfying the conditions $a_{ij}(x) = a_{ji}(x)$ and

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \ge \gamma^2 \sum_{i=1}^{n} \xi_i^2, \text{ for any } (\xi_1, \ldots, \xi_n) \in R^n,$$

where, $\gamma \ne 0$ is a constant. Suppose that,

$$Q_T = \{(x,t) \,|, x \in \Omega\},\ S_T = \{(x,t) \,|, x \in \partial\Omega\},\ t \in (0,T)$$

S_T is the lateral surface of the open cylinder Q_T in R^{n+1}, where the boundary $\partial\Omega$ of the domain Ω is assumed to be piecewise smooth.

Further, recall [1] that $W_2^1(\Omega)$ is the Hilbert space of elements of $L_2(\Omega)$ whose first-order generalized derivatives are square integrable on Ω, $\overset{\circ}{W}{}_2^1(\Omega)$ is the subspace of $W_2^1(\Omega)$ in which smooth compactly supported functions form a dense subset, $W_2^{1,0}(Q_T)$ is the Hilbert space of elements of $L_2(Q_T)$ whose generalized derivatives $z_{x_i}, i = 1, 2, \ldots, n,$ are square integrable on Q_T, and $\overset{\circ}{W}{}_2^{1,0}(Q_T)$ is the subspace of $W_2^{1,0}(Q_T)$ in which smooth functions vanishing near S_T form a dense set.

We consider a controlled system described by the parabolic equation:

$$z_t - Az = -u + v, \quad z|_{t=0} = z_0(x), \quad z|_{S_T} = 0, \qquad (2)$$

It was proved in [1] that if the above-mentioned conditions are satisfied, then problem (2) has a unique solution $z = z(x,t)$ in the class $\overset{\circ}{W}{}_2^1(\Omega)$ for arbitrary $z_0(x) \in S_T$, $u(x,t), v(x,t) \in L_2(Q_T)$, $u = u(x,t), v = v(x,t)$ are the control functions (parameters) of the first player (the pursuer) and the second player (the evader), respectively. The solution has the form

$$z(x,t) = \sum_{k=1}^{\infty} z_k(t)\varphi_k(x), \qquad (3)$$

where the functions $z_k(t),\ 0 < t \le T,\ k = 1, 2, \ldots,$ form a solution of the following infinite system of differential equations and initial conditions:

$$\dot{z}_k(t) = \lambda_k z_k(t) - u_k(t) + v_k(t), z_k(0) = z_k^0, \qquad (4)$$

where $\lambda_1, \lambda_2, \ldots$ are the generalized eigenvalues of the operator A [see [4]], (all these eigenvalues are negative, and $\lambda_k \to -\infty$ as $k \to \infty$), the functions $\varphi_1(x), \varphi_2(x), \ldots$ form an orthonormal complete system of generalized eigenfunctions of A in $L_2(\Omega)$. $u_k(t)$, $v_k(t)$ and z_k^0 are the Fourier coefficients of $u(t,x)$, $v(t,x)$ and $z_0(x)$, respectively, in the system $\{\varphi_i(x)\}$.

2. Preliminaries

We assume that the pursuit game problem with mixed constraints on controls of players. The controls functions $u(t,x)$ and $v(t,x)$ subject to the following systems of inequalities.

$$\begin{aligned} \|u(\cdot,\cdot)\|^2 &= \sum_{k=1}^{\infty}\int_0^T |u_k(t)|^2\, dt \le \rho, \\ \|v(\cdot,t)\|^2 &= \sum_{k=1}^{\infty} |v_k(t)|^2 \le \sigma,? \end{aligned} \tag{5}$$

$$\begin{aligned} \|u(\cdot,t)\|^2 &= \sum_{k=1}^{\infty} |u_k(t)|^2 \le \rho, \\ \|v(\cdot,\cdot)\|^2 &= \sum_{k=1}^{\infty}\int_0^T |v_k(t)|^2\, dt \le \sigma, \end{aligned} \tag{6}$$

where, $0 \le t \le T$ and ρ, σ are nonnegative constants.

Definition 2.1. In the pursuit game (2), (5), [respectively, (2), (6)], it is possible to complete the pursuit from an initial position $z_0(\cdot)$ if there exists $T = T(z_0(\cdot)) \ge 0$ and a function $u(v,x,t)$, $v \in R^1$, $x \in \Omega$, $t \in [0,T]$, such that the following assertions are valid for an arbitrary function $v_0(x,t) \in L_2(Q_T)$ satisfying the condition $\|v_0(\cdot,t)\| \le \sigma$,

(i) $\|u_0(\cdot,t)\| \le \rho, u = u_0(x,t) \equiv u(v_0(x,t),x,t) \in L_2(Q_T)$;

(ii) the solution $z_0(x,t)$ of problem (2), where $u = u_0(x,t)$ and $v = v_0(x,t)$, satisfies $z_0(x,t) \equiv 0$, for some $t \in [0,T]$. The number T is called the pursuit time.

Let us introduce some additional notation. Suppose that

$$X_k = \left\{ \begin{aligned} & z_0(\cdot) = \left(z_1^0(\cdot), z_2^0(\cdot), z_3^0(\cdot), \ldots\right): \\ & | z_k^0 |^2\, T \le \beta_k^{-2}\alpha_k^{-2}\sigma_k^2 \end{aligned} \right\}, \; X = \bigcup_{k=1}^{\infty} X_k, \tag{7}$$

$$Y = \left\{ \begin{aligned} & z_0(\cdot) : \|z_0(\cdot)\| \\ & + \frac{\sigma}{\sqrt{2|\lambda_1|}}\left(1 + \frac{\sigma}{\sqrt{2|\lambda_1|\varepsilon}}\right) \le \rho T, \varepsilon > 0 \end{aligned} \right\} \tag{8}$$

where, $\alpha_k, \beta_k, \sigma_k$ and σ are positive numbers, $k = 1,2,\ldots$.

System (2), where $u(x,t)$ and $v(x,t)$ satisfy condition (5) [respectively, (6)], will be referred to as the pursuit game (2), (5) [respectively, (2), (6)].). Suppose that

$$u(x,t) = \sum_{k=1}^{\infty} u_k(t)\varphi_k(x), \; v(x,t) = \sum_{k=1}^{\infty} v_k(t)\varphi_k(x). \tag{9}$$

Then the solution $z(x,t)$ of problem (2) is given by

$$z(x,t) = \sum_{k=1}^{\infty} z_k(t)\varphi_k(x). \tag{10}$$

where the functions $z_k(t), 0 \le t \le T, k = 1,2,\ldots$ satisfy the following infinite system of differential equations and initial conditions

$$\dot{z}_k(t) = \lambda_k z_k(t) - u_k(t) + v_k(t), \; z_k(0) = z_k^0. \tag{11}$$

The solution of the system (11) is given by

$$z_k(t) = \Phi_k(t)\begin{pmatrix} z_k^0 - \int_0^t \Phi_k^{-1}(\tau)u_k(\tau)d\tau \\ + \int_0^t \Phi_k^{-1}(\tau)v_k(\tau)d\tau \end{pmatrix}, \tag{12}$$

where, $\Phi_k^{-1}(t) = e^{\lambda_k t}$, $0 < t \le T$, $k = 1,2,\ldots$

3. Pursuit Differential Game

Theorem 2.1. If $\rho > 0$, $\sigma \ge 0$ then in the pursuit game (2), (5) it is possible to complete the pursuit from any initial position $z_0(\cdot) \in X$.

Proof: Let $\rho > \sigma$, $v(x,t)$ is an arbitrary control of the second player.

We use the control method proposed in [5], we shall seek the pursuers control as in the form

$$u_k(t) = \Phi_k^{-1}(t)c_k + v_k(t), \; 0 < t \le T_0, \tag{13}$$

where, $c_k \ne 0$, $k = 1,2,\ldots$ are constants and T_0 satisfies the following:

$$0 < T_0 \le T, \; \rho > \sigma\sqrt{T_0}. \tag{14}$$

Define

$$R_k(t) = \int_0^t \Phi_k^{-1}(\tau)\Phi_k^{-1}(\tau)d\tau, \; 0 < t \le T_0, \; k = 1,2,\ldots \tag{15}$$

If we set $c_k = R_k^{-1}(T_0)z_k^0$, $k = 1,2,\ldots$ in (13), we get

$$u_k(t) = \Phi_k^{-1}(t)R_k^{-1}(T_0)z_k^0 + v_k(t), \; 0 < t \le T_0. \tag{16}$$

Suppose that $z_0(\cdot) = \left(z_1^0(\cdot), z_2^0(\cdot), z_3^0(\cdot), \ldots\right)$ is an arbitrary point of the set X . We now show that the control (16) ensure that $z(\cdot, T_0) = 0$ for all $k = 1,2,\ldots$.

By substituting the control (16) into the solution (12), we get

$$z_k(t) = \Phi_k(t)\begin{pmatrix} z_k^0 - R_k(t)R_k^{-1}(T_0)z_k^0 \\ - \int_0^t \Phi_k^{-1}(\tau)v_k(\tau)d\tau \\ + \int_0^t \Phi_k^{-1}(\tau)v_k(\tau)d\tau \end{pmatrix} \tag{17}$$

Hence, at $t = T_0$

$$z_k(T_0) = \Phi_k(T_0)\left(z_k^0 - R_k(T_0)R_k^{-1}(T_0)z_k^0\right) = 0,\ k = 1,2,\ldots \quad (18)$$

Therefore, in the pursuit game problem (2), (5), it is possible to complete the pursuit from the initial state $z_0(\cdot) \in X$ for $t = T_0$.

Now, let us prove the admissibility the control (16). In the pursuit game (2), (5), the controls $u(x,t)$ and $v(x,t)$ satisfying inequalities (5). We have,

$$\|v(\cdot,t)\| \le \sigma,\ 0 \le t \le T \quad (19)$$

It follows that,

$$\|v(\cdot,\cdot)\|_{T_0}^2 = \int_0^{T_0} \|v(\cdot,t)\|^2\, d\tau \le \sigma^2 T_0,\ 0 < T_0 \le T. \quad (20)$$

Let,

$$\alpha_k = \sup_{0<t\le T_0} \Phi_k^{-1}(t),\ \beta_k = R_k^{-1}(T_0),$$
$$\sigma_k = \frac{\left|z_k^0\right|}{\|z_0(\cdot)\|}\left(\rho - \sigma\sqrt{T_0}\right),\ k = 1,2,\ldots \quad (21)$$

By setting $u(x,t) = 0$ on $\Omega \times (T_0, T]$ and taking into account (16), we see that

$$\|u(\cdot,\cdot)\| = \left(\sum_{k=1}^{\infty}\int_0^{T_0} |u_k(\tau)|^2\, d\tau\right)^{1/2} = \left(\sum_{k=1}^{\infty}\int_0^{T_0} \left|\Phi_k^{-1}(\tau)R_k^{-1}(T)z_k^0 + v_k(\tau)\right|^2 d\tau\right)^{1/2} \quad (22)$$

Applying the Minkowski inequality, we get

$$\|u(\cdot,\cdot)\| \le \left(\sum_{k=1}^{\infty}\int_0^{T_0} \left|\Phi_k^{-1}(t)R_k^{-1}(T_0)z_k^0\right|^2 d\tau\right)^{1/2} + \left(\sum_{k=1}^{\infty}\int_0^{T_0} |v_k(\tau)|^2\, d\tau\right)^{1/2}. \quad (23)$$

By using (20) and (21), we conclude that

$$\|u(\cdot,\cdot)\| \le \left(\sum_{k=1}^{\infty}\int_0^{T_0} \alpha_k^2\beta_k^2 \left|z_k^0\right|^2 d\tau\right)^{1/2} + \sigma\sqrt{T_0}. \quad (24)$$

Taking into account (7), we have

$$\|u(\cdot,\cdot)\| \le \left(\sum_{k=1}^{\infty}\int_0^{T_0} \frac{\sigma_k^2}{T_0}\, d\tau\right)^{1/2} + \sigma\sqrt{T_0}$$

And finally, (21) implies that

$$\|u(\cdot,\cdot)\| \le \left(\sum_{k=1}^{\infty} \frac{\left|z_k^0\right|^2}{\|z_0(\cdot)\|^2}(\rho - \sigma\sqrt{T_0})^2\right)^{1/2} + \sigma\sqrt{T_0} = \rho - \sigma\sqrt{T_0} + \sigma\sqrt{T_0} = \rho$$

From the above formula, we conclude that the control $u(x,t)$ of the first player is admissible control. Thus, the pursuit in the pursuit game (2), (5), is completed from an arbitrary initial position $z_0(\cdot) = \left(z_1^0(\cdot), z_2^0(\cdot), z_3^0(\cdot), \ldots\right) \in X$. This complete the proof of theorem 1.

Theorem 2.2. If $\rho > 0$, $\sigma \ge 0$ and $\varepsilon > 0$, then in the pursuit game (2), (6), the pursuit is completed in the ε -neighborhood of zero (which means the solution $z_0(x,t)$) of the pursuit game (2), (6), satisfies the inequality $\|z(\cdot,t)\| \le \varepsilon$ for some $t \in [0,T]$) it is possible to complete the pursuit from any initial position $z_0(\cdot) \in Y$.

Proof: Consider the pursuit game (2), (6). Suppose that $\rho > \sigma$, $z_0(\cdot)$ is an arbitrary point of the set Y, $v(x,t)$ is an arbitrary control of the second player.

Let $\varepsilon > 0$ be an arbitrary real number such that $\|z_0(\cdot)\| \le \varepsilon$. We define the pursuer control by

$$u_k(t) = \Phi_k^{-1}(t)R_k^{-1}(T_1)z_k^0,\ 0 < t \le T_1,\ T_1 = \frac{\|z_0(\cdot)\|}{\rho}. \quad (25)$$

where,

$$R_k(t) = \int_0^t \Phi_k^{-1}(t)\Phi_k^{-1}(\tau)d\tau,\quad 0 < t \le T_1,\ k = 1,2,\ldots. \quad (26)$$

In accordance with (12), (25) and (26), we obtain

$$z_k(t) = \Phi_k(t)\begin{pmatrix} z_k^0 - R_k(t)R_k^{-1}(T_1)z_k^0 \\ +\int_0^t \Phi_k^{-1}(\tau)v_k(\tau)d\tau \end{pmatrix}. \quad (27)$$

In view of (27), we have

$$z_k(T_1) = \Phi_k(T_1)\int_0^{T_1} \Phi_k^{-1}(\tau)v_k(\tau)d\tau, \quad (28)$$

But,

$$|z_k(T_1)| = \left|\Phi_k(T_1)\int_0^{T_1} \Phi_k^{-1}(\tau)v_k(\tau)d\tau\right| \le \int_0^{T_1} \left|\Phi_k(T_1)\Phi_k^{-1}(\tau)\right| |v_k(\tau)|\, d\tau.$$

By using the cauchy-schwarz inequality, we conclude that

$$|z_k(T_1)| \le \left(\int_0^{T_1} \left|\Phi_k(T_1)\Phi_k^{-1}(\tau)\right|^2 d\tau\right)^{1/2} \left(\int_0^{T_1} |v_k(\tau)|^2\, d\tau\right)^{1/2}. \quad (29)$$

Since $\Phi_k^{-1}(t) = e^{\lambda_k t}$ and $\left(1 - e^{2\lambda_k T_1}\right) \le 1$, $k = 1,2,\ldots$ it is easy to show that

$$\begin{aligned}&\int_0^{T_1}\left|\Phi_k(T_1)\Phi_k^{-1}(\tau)\right|^2 d\tau\\&=\int_0^{T_1}\left|\Phi_k(T_1-\tau)\right|^2 d\tau=\int_0^{T_1}e^{2\lambda_k(T_1-\tau)}d\tau\\&=\frac{1-e^{2\lambda_k T_1}}{-2\lambda_k}\le\frac{1}{-2\lambda_k}\end{aligned}\tag{30}$$

From (29) and (30), it follows that

$$\left|z_k(T_1)\right|^2\le\frac{1}{2|\lambda_k|}\int_0^{T_1}\left|v_k(\tau)\right|^2 d\tau.\tag{31}$$

Since, $0<(-\lambda_1)\le(-\lambda_2)\le\ldots(-\lambda_k)\ldots$ this implies that

$$\begin{aligned}\|z(\cdot,T_1)\|^2&=\sum_{k=1}^{\infty}|z_k(T_1)|^2\le\frac{1}{2|\lambda_1|}\sum_{k=1}^{\infty}\int_0^{T_1}|v_k(\tau)|^2 d\tau\\&=\frac{1}{2|\lambda_1|}\int_0^{T_1}\|v(\cdot,\tau)\|^2 d\tau=\frac{1}{2|\lambda_1|}\sigma_1^2,\end{aligned}\tag{32}$$

where,

$$\sigma_1^2=\int_0^{T_1}\|v(\cdot,\tau)\|^2 d\tau.\tag{33}$$

Now, if $\|z(\cdot,T_1)\|\le\varepsilon$, then by the definition the pursuit in the pursuit game (2), (6), it is possible to complete from the initial state $z_0(\cdot)$ for $t=T_1$.

In the case $\|z(\cdot,T_1)\|>\varepsilon$, it follows from (32) that

$$2|\lambda_1|\varepsilon^2<2|\lambda_1|\|z(\cdot,T_1)\|^2<\sigma_1^2.\tag{34}$$

We assume that $z_0(\cdot)=z(\cdot,T_1)$, and define the pursuers control by:

$$\begin{aligned}&u_k(t)=\frac{\Phi_k^{-1}(t)z_k(T_1)}{\Phi_k(T_1)\left(R_k(T_1+T_2)-R_k(T_1)\right)},\\&0<t\le T_2,\quad T_2=\frac{\|z(\cdot,T_1)\|}{\rho}.\end{aligned}\tag{35}$$

Moreover, according to (26) and (35), it is easy to see that

$$\begin{aligned}&z_k(T_1)-\Phi_k(T_1)\int_{T_1}^{T_1+T_2}\Phi_k^{-1}(\tau)u_k(\tau)d\tau\\&=0,\ k=1,2,\ldots\end{aligned}\tag{36}$$

By a direct computation using (27), (28) and (36), we get

$$\begin{aligned}&z_k(T_1+T_2)\\&=\Phi_k(T_2)\left(\Phi_k(T_1)\int_{T_1}^{T_1+T_2}\Phi_k^{-1}(\tau)v_k(\tau)d\tau\right).\end{aligned}\tag{37}$$

In the case $\|z(\cdot,T_1+T_2)\|\le\varepsilon$. By the definition the pursuit in the pursuit game (2), (6) is completed from the initial state $z_0(\cdot)$ for $t=T_1+T_2$. In the case $\|z(\cdot,T_1+T_2)\|>\varepsilon.$

By setting $z_0(\cdot)=z(\cdot,T_1+T_2)$ and use the previous argument, we conclude that

$$2|\lambda_1|\varepsilon^2<2|\lambda_1|\|z(\cdot,T_1+T_2)\|^2<\sigma_2^2\tag{38}$$

where,

$$\sigma_2^2=\int_{T_1}^{T_1+T_2}\|v(\cdot,\tau)\|^2 d\tau.\tag{39}$$

Now, we will prove that the pursuit in the pursuit game (2), (6) is complete from the initial state $z_0(\cdot)$ in finitely many steps.

First, let k be the smallest positive integer number such that

$$2|\lambda_1|\varepsilon^2 k>\sigma^2.\tag{40}$$

In the kth step, we have the following two cases:

1) $\|z(\cdot,T_1+T_2+\ldots+T_k)\|\le\varepsilon;$

2) $\|z(\cdot,T_1+T_2+\ldots+T_k)\|>\varepsilon.$

In case 2), by setting $z_0(\cdot)=z(\cdot,T_1+T_2+\ldots+T_k)$, where

$$T_k=\frac{\|z(\cdot,T_1+T_2+\ldots+T_{k-1})\|}{\rho}.$$

Continuing and use in the previous arguments, we conclude that

$$2|\lambda_1|\varepsilon^2<2|\lambda_1|\|z(\cdot,T_1+T_2+\ldots+T_k)\|^2<\sigma_k^2,\tag{41}$$

where,

$$\sigma_k^2=\int_{T_1+T_2+\ldots+T_{k-1}}^{T_1+T_2+\ldots+T_k}\|v(\cdot,\tau)\|^2 d\tau\tag{42}$$

By using (34), (38) and (41), we obtain

$$\begin{aligned}&\sigma_1^2+\sigma_2^2+\ldots+\sigma_k^2>2|\lambda_1|\varepsilon^2+2|\lambda_1|\varepsilon^2+\ldots+2|\lambda_1|\varepsilon^2\\&=2|\lambda_1|\varepsilon^2 k\end{aligned}$$

It follows from (40) that

$$\sigma_1^2+\sigma_2^2+\ldots+\sigma_k^2>\sigma^2\tag{43}$$

On the other hand by using (33),(39) and (42), we get

$$\begin{aligned}&\sigma_1^2+\sigma_2^2+\ldots+\sigma_k^2\\&=\int_0^{T_1+T_2+\ldots+T_k}\|v(\cdot,\tau)\|d\tau\le\int_0^T\|v(\cdot,\tau)\|d\tau\le\sigma^2,\end{aligned}\tag{44}$$

which contradicts the inequality (43), hence, the inequality $\|z(\cdot,T_1+T_2+\ldots+T_k)\|>\varepsilon$ does not hold. Therefore, the inequality $\|z(\cdot,T_1+T_2+\ldots+T_k)\|>\varepsilon$ in the case 1), holds.

Thus, by the definition it is possible to complete the pursuit (2), (6) from an initial position $z_0(\cdot)\in Y$ before the $(k+1)st$ step at time at $T(z_0(\cdot))=T_1+T_2+\ldots+T_k$.

Finally, we will estimate pursuit time. The above discussion shows that

$$\begin{aligned}T(z_0(\cdot))&=T_1+T_2+\ldots+T_k\\&=\frac{\|z_0(\cdot)\|}{\rho}+\frac{\|z(\cdot,T_1)\|}{\rho}+\cdots\\&+\frac{\|z(\cdot,T_1+T_2+\ldots+T_{k-1})\|}{\rho}\end{aligned}$$

Thus, we deduce from (34), (38) and (41), that

$$T(z_0(\cdot)) \le \frac{1}{\rho}\left(\|z_0(\cdot)\| + \frac{\sigma}{\sqrt{2|\lambda_1|}}[\sigma_1 + \sigma_2 + \cdots + \sigma_{k-1}] \right)$$

A simple calculation by using (40) yields

$$T(z_0(\cdot)) \le \frac{1}{\rho}\left(\|z_0(\cdot)\| + \frac{\sigma}{\sqrt{2|\lambda_1|}}\left[1 + \frac{\sigma}{\sqrt{2|\lambda_1|\varepsilon}}\right] \right)$$

This ends the proof of the theorem.

4. Conclusion

By using a new control method, we have studied pursuit game problem with dynamics described by a partial differential equation of first order. We state and prove a theorem on pursuit with mixed constraints on control of players. Integral (geometric) constrain is imposed on the control of the pursuer whereas, that of the evader is subject to geometric (integral) constraint. In this theorem, we established the sufficient conditions for which pursuit is possible in the game considered.

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