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Analyzing the Stock Market Using the Solution of the Fractional Option Pricing Model

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Abstract The aim of this work is to analyze the stock market using the solution of the fractional option pricing model as in literature. First, the Hurst exponent of the stock prices of two different stock index using Detrended Fluctuation Analysis (DFA) method was estimated. A program using MATLAB code was written which is used to calculate the Hurst exponent, the volatility, the discount rate, the call and put options prices efficiently so as to save time and avoid computational errors which may arise through manual computation.

Keywords: *hurst exponent, MATLAB, stock index, DFA. MSC, 98B28*

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1. Introduction

The Black-Scholes model was first developed by Fisher Black and Myron Scholes in 1973. They derived a partial differential equation, now called the Black-Scholes equation, which estimates the price of the option over time. The key idea behind the model is to hedge the option by buying and selling the underlying asset in just the right way and as a consequence, to eliminate risk. Merton [1] was first to publish a paper expanding the mathematical understanding of the options pricing model and coined the term "Black-Scholes option pricing model. Black-Scholes model does not give the value of the option (the price at which it should be sold or bought) directly. In order to determine the value of an economic asset such as stock, one needs to take into account at least two aspects of random variability: the growth of the asset and the stability of stock market price. Osu *et al*. [2] considered a stability analysis of stochastic model of price change at the floor of a stock market. They determined the equilibrium price and growth rate of the stock shares in a particular case by establishing a dynamic stochastic model under certain conditions.

Carr and Madan [3] introduced fast Fourier transform algorithm for the numerical value of the Fourier integral encountered in option pricing. Since then, fast Fourier transform algorithm has become an efficient mathematical tool used in mathematical finance in the valuation problems. A more general pricing framework was developed in Lewis [4] by separating the underlying asset price from the option payoff through the use of Plancheral-Parseval theorem. Dennis [5] showed explicitly how to obtain the solution of the classical Black-Scholes equation for call option pricing using the Green's function for the diffusion equation. Coppex [6] used change of variables to transform the classical Black- Scholes equation into a heat equation. He then applied the heat equation formula to obtain the solution of the Black-Scholes equation.

The arbitrage possibility in the fractional Black-Scholes model depends on the definition of the stochastic integral. More precisely, if one uses the Wick-Ito-Skorohod integral, one obtains an arbitrage-free model. On the other hand, it is easy to give arbitrage examples in continuous time trading with self-financing strategies, if one uses the Riemann-Stieltjes integral. Tommi and Esko [9] discussed the connection between two different notions of self-financing portfolios in the fractional Black-Scholes model by applying the known connection between these two integrals.

Application of Sturm-Liouville Equation to the Solution of the Black –Scholes equation has been presented in Osu *et al*. [7]. An analytical solution of a non-linear Black- Scholes- equation (Option Pricing Model) with transaction cost measure and volatile portfolio risk measure has been presented. The analytical solution was obtained by using variational iteration method in Osu *et al.* [7].

The aim of this study however is to solve Black-Scholes option pricing equation modeled by Fractional Brownian motion with specified boundary conditions. The equation is solved using change of independent variables and Fourier transform.

2. The Model

Given the equation as

$$\frac{\partial \mathbf{V}}{\partial \mathbf{S}}+\mathbf{H}\mathbf{t}^{2\mathbf{H}-1}\mathbf{S}^2\boldsymbol{\sigma}^2\frac{\partial^2 \mathbf{V}}{\partial \mathbf{S}^2}+\mathbf{rS}\frac{\partial \mathbf{V}}{\partial \mathbf{S}}-\mathbf{rV}=0,\mathbf{S}>0,t>0 \quad (2.1)$$

with $V(0,t)=0, V(S,t)\sim S$ as $S\to\infty$, $H\epsilon(0,1), H\neq\frac{1}{2}$, and $V(S,T)=Max\{|S-K|,0\}$.

Where equation (2.1) is called Black-Scholes option pricing equation modeled by Fractional Brownian motion. Equation (2.1) can be reduced to a one-dimensional heat equation of the form [8];

$$\frac{\partial u}{\partial \tau}=p\frac{\partial^2 u}{\partial x^2}. \quad (2.2)$$

Equation (2.2) is a one-dimensional heat equation, which have been solved using Fourier transform with solution given as

$$u(x,\tau)=\frac{1}{\sqrt{4\pi p\tau}}\int_{-\infty}^{\infty}u_0(S)e^{-\frac{(x-S)^2}{4p\tau}}ds. \quad (2.3)$$

THEOREM 2.2: Let the solution of the heat equation be given as in $(\mathbf{2.3})$, then the solution of $(\mathbf{2.1})$ can be obtained from $(\mathbf{2.3})$ in the form

$$V(S,t)=Ke^{-r(T-t)}\phi(d_1)-S\phi(d_2) \quad (2.4)$$

where

$$d_1=\frac{\ln\left(S/K\right)}{\sqrt{\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]\sigma^2(T-t)}}+\sqrt{\frac{\sigma^2(T-t)}{4\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]}}\left(\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]-\frac{2r}{\sigma^2}\right)$$

$$d_2=\frac{\ln\left(S/K\right)}{\sqrt{\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]\sigma^2(T-t)}}-\sqrt{\frac{\sigma^2(T-t)}{4\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]}}\left(\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]+\frac{2r}{\sigma^2}\right)q$$

PROOF:

$$u(x,\tau)=\frac{1}{\sqrt{4\pi p\tau}}\int_{-\infty}^{\infty}u_0(s)e^{-\frac{(x-s)^2}{4p\tau}}ds.$$

We need to transform the initial condition too. From (4.14b), we have:

$$u(x,\tau)=e^{-(\alpha x+\beta\tau)}v(x,\tau).$$

$$u(x,0)=e^{-(\alpha x+\beta(0))}v(x,0)$$

$$u(x,0)=e^{-\left[\left(\frac{p-q}{2p}\right)x-\frac{(p+q)^2}{4p}(0)\right]v(x,0)}$$

$$\therefore u(x,0)=e^{-\left(\frac{p-q}{2p}\right)x}v(x,0).$$

But from (2.5b), we have that

$$v(x,0)=\max\left(e^x-1,0\right)$$

$$\Rightarrow u(x,0)=e^{-\left(\frac{p-q}{2p}\right)x}\max\left(e^x-1,0\right) \quad (2.5a)$$

$$u(x,0)=\max[e^{\left(\frac{p+q}{2p}\right)x}-e^{-\left(\frac{p-q}{2p}\right)x},0].$$

We will make a change of variable in the integration in (2.4) by taking

$$z=\frac{x-s}{\sqrt{2p\tau}}\Rightarrow dz=\frac{-1}{\sqrt{2p\tau}}ds.$$

Thus, the integration becomes:

$$u(x,\tau)=\frac{-1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}u_0(x-z\sqrt{2p\tau})e^{\frac{-z^2}{2}}dz.$$

We will only integrate over the domain where $\boldsymbol{u_0}>0$, that is for $z>\frac{-x}{\sqrt{2p\tau}}$.

On that domain,

$$u_0=e^{\left(\frac{p+q}{2p}\right)\left(x-z\sqrt{2p\tau}\right)}-e^{-\left(\frac{p-q}{2p}\right)\left(x-z\sqrt{2p\tau}\right)}.$$

So, we have:

$$u(x,\tau)=\frac{-1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2p\tau}}}^{\infty}e^{\left(\frac{p+q}{2p}\right)\left(x-z\sqrt{2p\tau}\right)}e^{\frac{-z^2}{2}}dz+\frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2p\tau}}}^{\infty}e^{-\left(\frac{p-q}{2p}\right)\left(x-z\sqrt{2p\tau}\right)}e^{\frac{-z^2}{2}}dz$$

$$u(x,\tau)=\frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2p\tau}}}^{\infty}e^{-\left(\frac{p-q}{2p}\right)\left(x-z\sqrt{2p\tau}\right)}e^{\frac{-z^2}{2}}dz-\frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2p\tau}}}^{\infty}e^{\left(\frac{p+q}{2p}\right)\left(x-z\sqrt{2p\tau}\right)}e^{\frac{-z^2}{2}}dz.$$

$$u(x,\tau)=\boldsymbol{I}_1-\boldsymbol{I}_2 \quad (2.5b)$$

where

$$\mathcal{I}_1 = \frac{1}{\sqrt{2\pi}} \int_{-\frac{x}{\sqrt{2p\tau}}}^{\infty} e^{-\left(\frac{p-q}{2p}\right)\left(x - z\sqrt{2p\tau}\right)} e^{\frac{-z^2}{2}} d \quad (2.5c)$$

$$\mathcal{I}_2 = \frac{1}{\sqrt{2\pi}} \int_{-\frac{x}{\sqrt{2p\tau}}}^{\infty} e^{\left(\frac{p+q}{2p}\right)\left(x - z\sqrt{2p\tau}\right)} e^{\frac{-z^2}{2}} dz.. \quad (2.5d)$$

Evaluation of $\mathcal{I}_1$: To evaluate $\mathcal{I}_1$, consider the exponent; $-\left(\frac{p-q}{2p}\right)\left(x - z\sqrt{2p\tau}\right) - \frac{z^2}{2}$.

Expanding we have:

$$-\left(\frac{p-q}{2p}\right)x + \left(\frac{p-q}{2p}\right)z\sqrt{2p\tau} - \frac{z^2}{2}$$

$$= \left(-\frac{1}{2}\right)\left\{z^2 - \left(\frac{p-q}{p}\right)\sqrt{2p\tau}z\right\} - \left(\frac{p-q}{2p}\right)x.$$

Completing the square, we have:

$$\left(-\frac{1}{2}\right)\left\{z^2 - \left(\frac{p-q}{p}\right)\sqrt{2p\tau}z + \frac{\tau}{2p}(p-q)^2\right\}$$

$$-\left(\frac{p-q}{2p}\right)x + \frac{\tau}{4p}(p-q)^2$$

$$= \left(-\frac{1}{2}\right)\left\{z - \sqrt{\frac{\tau}{2p}}(p-q)\right\}^2 - \left(\frac{p-q}{2p}\right)x + \frac{(p-q)^2}{4p}\tau$$

Thus $\mathcal{I}_1$ becomes

$$\mathcal{I}_1 = \frac{e^{-\left(\frac{p-q}{2p}\right)x + \frac{(p-q)^2}{4p}\tau}}{\sqrt{2\pi}} \int_{-\frac{x}{\sqrt{2p\tau}}}^{\infty} e^{-\frac{1}{2}\left[z - \sqrt{\frac{\tau}{2p}}(p-q)\right]^2} dz.$$

We will change variables again on the integral. Choose

$$y_1 = z - \sqrt{\frac{\tau}{2p}}(p-q)$$

$$dy_1 = dz$$

$$\mathcal{I}_1 = \frac{e^{-\left(\frac{p-q}{2p}\right)x + \frac{(p-q)^2}{4p}\tau}}{\sqrt{2\pi}} \int_{-\left[\frac{x}{\sqrt{2p\tau}} + \sqrt{\frac{\tau}{2p}}(p-q)\right]}^{\infty} e^{-\frac{y_1^2}{2}} dy_1.$$

This integral can be represented in terms of the cumulative distribution function of a normal random variable, usually denoted by $\boldsymbol{\phi}$.

That is: $\phi(d) = \frac{1}{\sqrt{2\pi}} \int_{-d}^{\infty} e^{-\frac{y^2}{2}} dy,$ see Dunbar (**2014**)

Thus,

$$\mathcal{I}_1 = e^{-\left(\frac{p-q}{2p}\right)x + \frac{(p-q)^2}{4p}\tau} \phi(d_1)$$

where $\boldsymbol{d_1} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{2p\tau}}} + \sqrt{\frac{\boldsymbol{\tau}}{\boldsymbol{2p}}}(\boldsymbol{p} - \boldsymbol{q})$

Evaluation of $\mathcal{I}_2$: To evaluate $\mathcal{I}_2$, consider also the exponent;

$$\left(\frac{p+q}{2p}\right)\left(x - z\sqrt{2p\tau}\right) - \frac{z^2}{2}.$$

Expanding we have:

$$\left(\frac{p+q}{2p}\right)x - \left(\frac{p+q}{2p}\right)\sqrt{2p\tau}z - \frac{z^2}{2}$$

$$= \left(-\frac{1}{2}\right)\left[z^2 + \left(\frac{p+q}{p}\right)\sqrt{2p\tau}z\right] + \left(\frac{p+q}{2p}\right)x.$$

Completing the square, we have:

$$\left(-\frac{1}{2}\right)\left[z^2 + \left(\frac{p+q}{p}\right)\sqrt{2p\tau}z + \frac{\tau}{2p}(p+q)^2\right]$$

$$+\left(\frac{p+q}{2p}\right)x + \frac{\tau}{4p}(p+q)^2$$

$$= \left(-\frac{1}{2}\right)\left[z + \sqrt{\frac{\tau}{2p}}(p+q)\right]^2 + \left(\frac{p+q}{2p}\right)x + \frac{(p+q)^2}{4p}\tau.$$

Thus $\mathcal{I}_2$ becomes

$$\mathcal{I}_2 = \frac{e^{\left(\frac{p+q}{2p}\right)x + \frac{(p+q)^2}{4p}\tau}}{\sqrt{2\pi}} \int_{-\frac{x}{\sqrt{2p\tau}}}^{\infty} e^{-\frac{1}{2}\left[z + \sqrt{\frac{\tau}{2p}}(p+q)\right]^2} dz.$$

We will change variables again on the integral. Choose

$$y_2 = z + \sqrt{\frac{\tau}{2p}}(p+q) \Rightarrow dy_2 = dz$$

$$\mathcal{I}_2 = \frac{e^{\left(\frac{p+q}{2p}\right)x + \frac{(p+q)^2}{4p}\tau}}{\sqrt{2\pi}} \int_{-\left[\frac{x}{\sqrt{2p\tau}} - \sqrt{\frac{\tau}{2p}}(p+q)\right]}^{\infty} e^{-\frac{y_2^2}{2}} dy_2.$$

This integral can be represented in terms of the cumulative distribution function of a normal random variable, usually denoted by $\boldsymbol{\phi}$.

That is:

$$\phi(d) = \frac{1}{\sqrt{2\pi}} \int_{-d}^{\infty} e^{-\frac{y^2}{2}} dy.$$

Thus,

$$\mathcal{I}_2 = e^{\left(\frac{p+q}{2p}\right)x + \frac{(p+q)^2}{4p}\tau} \phi(d_2) \quad (2.7)$$

where $d_2 = \frac{x}{\sqrt{2p\tau}} - \sqrt{\frac{\tau}{2p}}(p+q).$

Thus (2.5b) becomes:

$$u(x,\tau)=e^{-\left(\frac{p-q}{2p}\right)x+\frac{(p-q)^2}{4p}\tau}\phi(d_1)-e^{\left(\frac{p+q}{2p}\right)x+\frac{(p+q)^2}{4p}\tau}\phi(d_2). \tag{2.8}$$

We will unwind each of the changes of variables: Recall

$$v(x,\tau)=e^{\alpha x+\beta\tau}u(x,\tau)$$

$$v(x,\tau)=\left[e^{\left(\frac{p-q}{2p}\right)x-\frac{(p+q)^2}{4p}\tau}\right]\times\left\{\left[e^{-\left(\frac{p-q}{2p}\right)x+\frac{(p-q)^2}{4p}\tau}\right]\phi(d_1)-\left[e^{\left(\frac{p+q}{2p}\right)x+\frac{(p+q)^2}{4p}\tau}\right]\phi(d_2)\right\}$$

$$=\left[e^{-\frac{(p+q)^2}{4p}\tau+\frac{(p-q)^2}{4p}\tau}\right]\phi(d_1)-\left[e^{\left(\frac{p-q}{2p}\right)x+\left(\frac{p+q}{2p}\right)x}\right]\phi(d_2)$$

$$v(x,\tau)=e^{-q\tau}\phi(d_1)-e^{x}\phi(d_2).$$

But $\boldsymbol{x=ln(S/K)}$, $\tau=\frac{\sigma^2}{2}(T-t)$, $q=\frac{2r}{\sigma^2}$, $p=2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}$, $d_1=\frac{x}{\sqrt{2p\tau}}+\sqrt{\frac{\tau}{2p}}(p-q)$,

$$d_2=\frac{x}{\sqrt{2p\tau}}-\sqrt{\frac{\tau}{2p}}(p+q)$$

$$\therefore v(x,\tau)=e^{-r(T-t)}\phi\left\{\sqrt{\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]\sigma^2(T-t)}+\sqrt{\frac{\sigma^2(T-t)}{4\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]}}\left(\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]-\frac{2r}{\sigma^2}\right)\right\} \tag{2.9}$$

$$-\frac{S}{K}\phi\left\{\frac{\ln\left(S/K\right)}{\sqrt{\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]\sigma^2(T-t)}}-\sqrt{\frac{\sigma^2(T-t)}{4\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]}}\left(\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]+\frac{2r}{\sigma^2}\right)\right\}$$

But

$$V(S,t)=Kv(x,\tau)$$

$$=\mathrm{K}e^{-r(T-t)}\phi\left\{\frac{\ln\left(S/K\right)}{\sqrt{\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]\sigma^2(T-t)}}+\sqrt{\frac{\sigma^2(T-t)}{4\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]}}\times\left(\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]-\frac{2r}{\sigma^2}\right)\right\}$$

$$-S\phi\left\{\frac{\ln\left(S/K\right)}{\sqrt{\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]\sigma^2(T-t)}}-\sqrt{\frac{\sigma^2(T-t)}{4\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]}}\times\left(\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]+\frac{2r}{\sigma^2}\right)\right\}.$$

So that

$$V(S,t)=Ke^{-r(T-t)}\phi(d_1)-S\phi(d_2) \tag{2.10}$$

where

$$d_1=\frac{\ln\left(S/K\right)}{\sqrt{\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]\sigma^2(T-t)}}+\sqrt{\frac{\sigma^2(T-t)}{4\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]}}\left(\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]-\frac{2r}{\sigma^2}\right)$$

$$d_2=\frac{\ln\left(S/K\right)}{\sqrt{\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]\sigma^2(T-t)}}-\sqrt{\frac{\sigma^2(T-t)}{4\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]}}\left(\left[2H\left(T-\frac{2\tau}{\sigma^2}\right)^{2H-1}\right]+\frac{2r}{\sigma^2}\right).$$

Hence the proof.

Note that the values of $\boldsymbol{S}$ and $\boldsymbol{K}$ can be obtained from daily stock market prices; see Table 1 and Table 2. The values of other parameters can be obtained using the methods given in chapter three.

Equation (2.10) is the solution of the Black-Scholes call option pricing equation modeled by fractional Brownian motion. It is the formula for obtaining the call option price. We will use the put-call parity equation to find the formula for obtaining the corresponding put option price. The put-call parity equation is stated in $(\mathbf{3.38})$ as

$$P = V - S + Ke^{-rT}, \tag{2.11}$$

where $\boldsymbol{P}$ is the put option price; $\boldsymbol{V}$ is the call option price; $\boldsymbol{K}$ is the strike price; $\boldsymbol{r}$ is the discount rate and $\boldsymbol{T}$ is the expiration time.

Substituting $(\mathbf{2.10})$ in $(\mathbf{2.11})$, we have

$$P = Ke^{-r(T-t)}\phi(d_1) - S\phi(d_2) - S + Ke^{-rT}. \tag{2.12}$$

3. Application to the Stock Market

To illustrate how option prices can be calculated, we made use of daily stock price lists of Conoil and Cutix stock index which were obtained online from Cash Craft Asset Management Limited from January 2016 to December 2016. A total of 230 stock prices were obtained for Conoil while a total of 200 stock prices were obtained for Cutix.

Table 1. STOCK PRICE LISTS OF CONOIL FROM JANUARY 2016 TO DECEMBER 2016

S/N	STOCK PRICE	STRIKE PRICE	S/N	STOCK PRICE	STRIKE PRICE	S/N	STOCK PRICE	STRIKE PRICE	S/N	STOCK PRICE	STRIKE PRICE	S/N	STOCK PRICE	STRIKE PRICE
1	24.74	24.74	47	1.58	1.58	93	23.00	23.00	139	23.96	23.96	185	35.90	35.90
2	24.74	24.74	48	18.24	19.15	94	23.00	21.85	140	23.96	23.96	186	35.90	35.90
3	24.74	24.74	49	20.10	20.10	95	23.00	23.00	141	23.96	23.96	187	35.90	35.90
4	24.74	24.74	50	20.10	20.10	96	23.00	23.00	142	23.96	23.96	188	35.90	35.90
5	24.74	24.74	51	20.10	20.10	97	23.00	23.00	143	23.96	23.96	189	35.90	34.11
6	24.74	24.74	52	20.10	20.10	98	23.00	23.00	144	23.96	22.77	190	35.90	34.11
7	24.74	24.74	53	20.10	20.10	99	23.00	23.00	145	22.77	21.65	191	35.90	34.11
8	24.74	24.74	54	20.10	20.10	100	23.00	23.00	146	21.65	21.65	192	35.90	35.90
9	24.74	24.74	55	20.10	20.10	101	23.00	23.00	147	21.65	21.65	193	35.90	35.90
10	24.74	24.74	56	20.10	20.10	102	23.00	23.00	148	21.65	21.65	194	35.90	35.90
11	24.74	23.51	57	20.10	20.10	103	23.00	23.00	149	21.65	21.65	195	35.90	35.90
12	24.74	24.74	58	20.10	20.10	104	23.00	23.00	150	21.65	21.65	196	35.90	35.90
13	24.74	24.74	59	20.10	20.10	105	23.00	23.00	151	21.65	21.65	197	35.90	34.11
14	24.74	24.74	60	20.10	20.10	106	23.00	23.00	152	21.65	21.65	198	35.90	35.90
15	24.74	24.74	61	20.10	19.10	107	23.00	21.85	153	21.65	20.67	199	35.90	35.90
16	24.74	24.74	62	20.10	20.10	108	23.00	21.85	154	20.57	20.57	200	35.90	35.90
17	24.74	24.74	63	20.10	20.10	109	21.85	21.85	155	20.57	20.57	201	35.90	35.90
18	24.74	24.74	64	20.10	20.10	110	21.85	20.96	156	20.57	21.59	202	35.90	34.11
19	24.74	23.51	65	20.10	20.10	111	20.96	20.96	157	20.57	20.57	203	34.11	34.11
20	22.34	22.34	66	20.10	19.10	112	20.96	22.00	158	20.57	20.57	204	34.11	32.41
21	22.34	21.23	67	20.10	20.10	113	22.00	22.00	159	20.57	20.57	205	34.11	34.11
22	22.34	22.34	68	20.10	19.10	114	22.00	22.00	160	20.57	20.59	206	34.11	34.11
23	22.34	21.23	69	16.45	16.45	115	22.00	22.00	161	21.59	22.66	207	34.11	34.11
24	20.25	20.17	70	16.45	16.45	116	22.00	23.10	162	23.79	24.97	208	34.11	34.11
25	19.24	18.33	71	16.45	16.45	117	23.10	24.25	163	26.21	27.52	209	34.11	32.41
26	18.33	18.33	72	16.45	16.45	118	25.45	26.72	164	28.89	30.33	210	34.11	32.41
27	18.33	17.42	73	16.45	17.27	119	25.45	25.45	165	32.84	33.43	211	34.11	34.11
28	17.42	17.42	74	16.45	16.45	120	25.45	25.45	166	35.10	36.85	212	34.11	34.11
29	17.42	17.42	75	16.45	17.27	121	25.45	24.18	167	42.60	44.73	213	34.11	34.11
30	17.42	18.29	76	16.45	17.27	122	25.45	25.34	168	42.50	44.60	214	34.11	32.41
31	17.47	18.34	77	16.45	16.45	123	25.34	24.08	169	37.05	37.05	215	34.11	32.41
32	18.34	18.34	78	16.45	17.27	124	25.22	25.22	170	36.00	37.80	216	34.11	34.11
33	18.34	18.34	79	16.45	17.27	125	25.22	25.22	171	35.91	35.91	217	34.11	34.11
34	18.34	18.34	80	17.27	17.27	126	25.22	25.22	172	37.70	36.50	218	34.11	34.11
35	18.34	18.34	81	17.27	17.27	127	25.22	25.22	173	33.10	33.10	219	34.11	34.11
36	18.34	18.34	82	17.27	17.27	128	23.96	25.22	174	33.10	31.45	220	34.11	34.11
37	18.34	17.43	83	17.27	17.70	129	25.22	25.22	175	33.00	34.65	221	34.11	32.41
38	17.43	16.56	84	17.27	18.10	130	25.22	25.22	176	35.90	35.90	222	34.11	34.11
39	16.56	16.56	85	18.10	17.30	131	25.22	25.22	177	35.90	35.9	223	34.11	34.11
40	16.56	16.56	86	18.16	19.06	132	25.22	25.22	178	35.90	34.11	224	34.11	32.41
41	16.56	16.56	87	19.06	20.01	133	25.22	25.22	179	35.90	35.90	225	34.11	32.41
42	16.56	16.56	88	21.01	22.06	134	25.22	23.96	180	35.90	35.90	226	34.11	32.41
43	16.56	16.56	89	23.00	21.85	135	25.22	25.22	181	35.90	35.90	227	34.11	32.41
44	16.56	16.56	90	23.00	23.00	136	25.22	25.22	182	35.90	35.90	228	35.80	37.59
45	16.56	16.56	91	23.00	23.00	137	25.22	23.96	183	35.90	34.11	229	37.48	37.48
46	16.56	16.56	92	23.00	21.85	138	25.22	23.96	184	35.90	35.90	230	33.48	37.48

Table 2. STOCK PRICE LISTS OF CUTIX FROM JANUARY 2016 TO DECEMBER 2016

S/N	STOCK PRICE	STRIKE PRICE	S/N	STOCK PRICE	STRIKE PRICE	S/N	STOCK PRICE	STRIKE PRICE	S/N	STOCK PRICE	STRIKE PRICE	S/N	STOCK PRICE	STRIKE PRICE
1	1.66	1.66	41	1.58	1.58	81	1.55	1.58	121	1.62	1.61	161	2.13	2.04
2	1.66	1.66	42	1.58	1.58	82	1.58	1.58	122	1.61	1.61	162	2.13	2.04
3	1.66	1.66	43	1.58	1.58	83	1.58	1.58	123	1.53	1.60	163	2.03	2.13
4	1.66	1.66	44	1.58	1.58	84	1.65	1.64	124	1.60	1.52	164	1.89	1.89
5	1.66	1.58	45	1.58	1.58	85	1.65	1.61	125	1.52	1.59	165	1.89	1.89
6	1.58	1.58	46	1.58	1.60	86	1.58	1.58	126	1.66	1.74	166	1.89	1.89
7	1.58	1.52	47	1.60	1.60	87	1.58	1.58	127	1.75	1.75	167	1.89	1.89
8	1.52	1.52	48	1.60	1.60	88	1.58	1.58	128	1.75	1.75	168	1.89	1.89
9	1.52	1.45	49	1.52	1.52	89	1.58	1.58	129	1.75	1.75	169	1.89	1.89
10	1.45	1.45	50	1.52	1.52	90	1.58	1.58	130	1.75	1.75	170	1.89	1.89
11	1.45	1.38	51	1.52	1.52	91	1.58	1.58	131	1.75	1.67	171	1.89	1.89
12	1.38	1.32	52	1.52	1.52	92	1.58	1.65	132	1.67	1.67	172	1.89	1.89
13	1.32	1.38	53	1.52	1.45	93	1.65	1.65	133	1.67	1.67	173	1.89	1.89
14	1.38	1.44	54	1.45	1.38	94	1.65	1.65	134	1.67	1.72	174	1.89	1.89
15	1.44	1.44	55	1.38	1.32	95	1.65	1.65	135	1.72	1.72	175	1.89	1.89
16	1.44	1.44	56	1.32	1.38	96	1.65	1.65	136	1.72	1.65	176	1.89	1.89
17	1.37	1.37	57	1.30	1.36	97	1.65	1.65	137	1.72	1.65	177	1.89	1.89
18	1.37	1.37	58	1.36	1.42	98	1.65	1.65	138	1.65	1.65	178	1.89	1.89
19	1.44	1.37	59	1.42	1.42	99	1.65	1.58	139	1.65	1.65	179	1.89	1.89
20	1.37	1.38	60	1.42	1.42	100	1.58	1.58	140	1.65	1.57	180	1.89	1.89
21	1.38	1.38	61	1.42	1.35	101	1.58	1.58	141	1.57	1.51	181	1.89	1.89
22	1.38	1.38	62	1.35	1.39	102	1.58	1.58	142	1.57	1.50	182	1.89	1.98
23	1.38	1.38	63	1.39	1.43	103	1.58	1.58	143	1.50	1.50	183	1.98	1.98
24	1.38	1.38	64	1.37	1.39	104	1.58	1.58	144	1.50	1.57	184	1.98	1.98
25	1.38	1.44	65	1.39	1.39	105	1.65	1.65	145	1.57	1.57	185	1.98	1.98
26	1.44	1.44	66	1.44	1.51	106	1.65	1.65	146	1.57	1.57	186	1.98	1.89
27	1.44	1.44	67	1.37	1.35	107	1.65	1.65	147	1.57	1.57	187	1.89	1.89
28	1.44	1.44	68	1.32	1.38	108	1.65	1.65	148	1.57	1.64	188	1.89	1.89
29	1.44	1.51	69	1.43	1.38	109	1.65	1.65	149	1.61	1.61	189	1.89	1.89
30	1.51	1.51	70	1.38	1.38	110	1.65	1.65	150	1.61	1.69	190	1.89	1.89
31	1.51	1.51	71	1.38	1.42	111	1.65	1.70	151	1.69	1.70	191	1.89	1.89
32	1.51	1.52	72	1.44	1.48	112	1.70	1.70	152	1.70	1.75	192	1.89	1.89
33	1.51	1.52	73	1.48	1.48	113	1.70	1.70	153	1.78	1.78	193	1.89	1.89
34	1.51	1.58	74	1.48	1.48	114	1.70	1.62	154	1.85	1.80	194	1.89	1.89
35	1.58	1.58	75	1.48	1.51	115	1.62	1.62	155	1.94	2.00	195	1.89	1.89
36	1.58	1.58	76	1.48	1.51	116	1.62	1.62	156	2.03	2.03	196	1.89	1.89
37	1.58	1.58	77	1.51	1.55	117	1.62	1.62	157	2.13	2.13	197	1.89	1.89
38	1.58	1.58	78	1.55	1.58	118	1.62	1.62	158	2.13	2.13	198	1.89	1.89
39	1.58	1.58	79	1.58	1.55	119	1.62	1.62	159	2.13	2.13	199	1.89	1.89
40	1.58	1.58	80	1.55	1.55	120	1.62	1.62	160	2.13	2.03	200	1.89	1.89

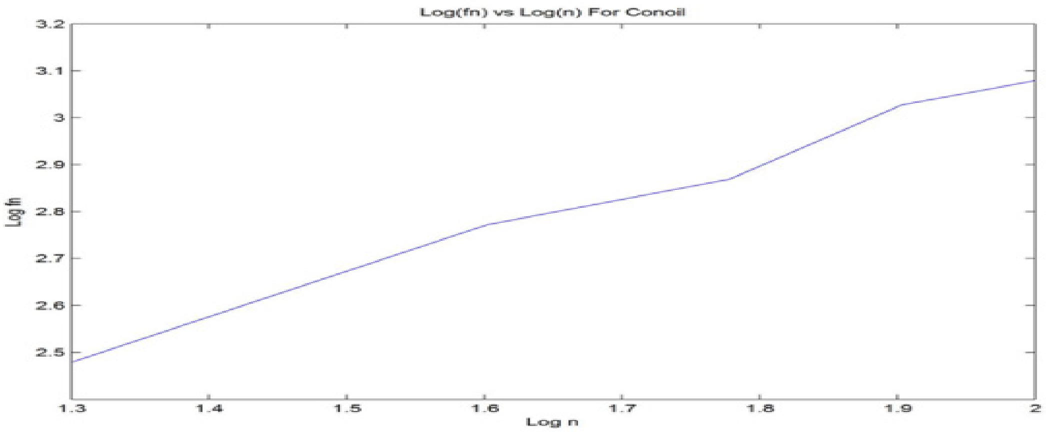

Figure 1. Graph of LogF(n) against logn for Conoil. The slope of the graph is 0.86, thus the Hurst exponent is 0.86

Table 3. THE RESULT OF THE HURST EXPONENT FOR THE STOCK PRICES OF CONOIL USING DETRENDED FLUCTUATION ANALYSIS

n	logn	LogF(n)
20.000	1.301	2.480
40.000	1.602	2.772
60.000	1.778	2.870
80.000	1.903	3.027
100.000	2.000	3.079

Table 4. THE RESULT OF THE HURST EXPONENT FOR THE STOCK PRICES OF CUTIX USING DETRENDED FLUCTUATION ANALYSIS

N	Logn	LogF(n)
20.000	1.301	1.290
40.000	1.602	1.584
60.000	1.778	1.721
80.000	1.903	1.853
100.000	2.000	1.946

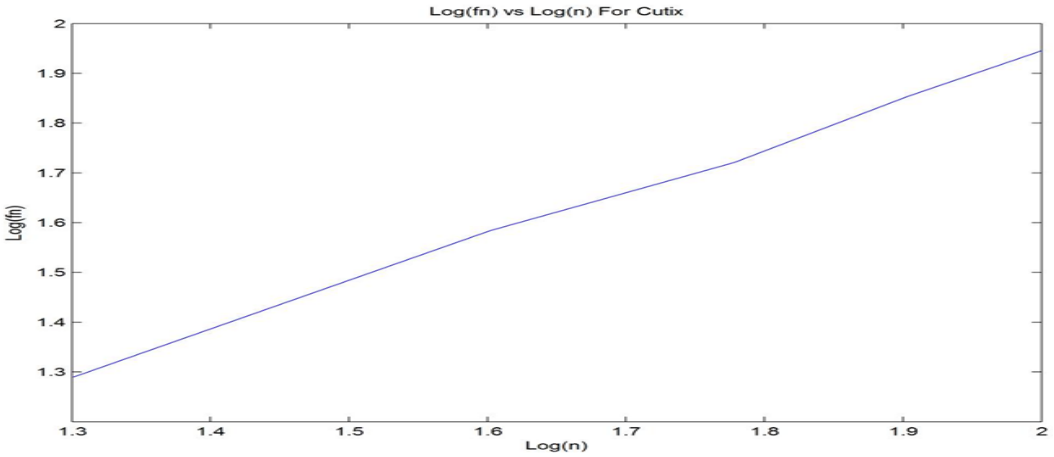

Figure 2. Graph of LogF(n) against logn for Cutix. The slope of the graph is 0.47, thus the Hurst exponent is 0.47

Table 5

S/N	S	T	r	V	P	S/N	S	T	R	V	P
1	24.74	0.004	0.02	24.132	23.634	116	22.00	0.504	2.19	7.227	-12.184
2	24.74	0.009	0.04	23.684	22.733	117	23.10	0.509	2.21	7.576	-12.857
3	24.74	0.013	0.06	23.248	21.853	118	25.45	0.513	2.23	8.337	-14.229
4	24.74	0.017	0.08	22.824	20.992	119	25.45	0.517	2.25	7.917	-14.837
5	24.74	0.022	0.10	22.411	20.152	120	25.45	0.522	2.27	7.909	-14.896
6	24.74	0.026	0.11	22.009	19.330	121	25.45	0.526	2.29	7.490	-15.494
7	24.74	0.030	0.13	21.618	18.527	122	25.45	0.530	2.31	7.859	-15.054
8	24.74	0.035	0.15	21.237	17.742	123	25.34	0.535	2.33	7.447	-15.527
9	24.74	0.039	0.17	20.866	16.974	124	25.22	0.539	2.34	7.814	-14.975
10	24.74	0.043	0.19	20.505	16.224	125	25.22	0.543	2.36	7.810	-15.024
11	24.74	0.048	0.21	19.149	13.487	126	25.22	0.548	2.38	7.808	-15.071
12	24.74	0.052	0.23	19.811	14.773	127	25.22	0.552	2.40	7.807	-15.116
13	24.74	0.057	0.25	19.478	14.071	128	23.96	0.557	2.42	7.397	-15.682
14	24.74	0.061	0.27	19.153	13.385	129	25.22	0.561	2.44	7.807	-15.201
15	24.74	0.065	0.28	18.837	12.715	130	25.22	0.565	2.46	7.808	-15.241
16	24.74	0.070	0.30	18.528	12.059	131	25.22	0.570	2.48	7.810	-15.279
17	24.74	0.074	0.32	18.228	11.417	132	25.22	0.574	2.50	7.814	-15.316
18	24.74	0.078	0.34	17.936	10.790	133	25.22	0.578	2.51	7.818	-15.351
19	24.74	0.083	0.36	16.770	8.437	134	25.22	0.583	2.53	7.410	-15.898
20	22.34	0.087	0.38	15.687	8.647	135	25.22	0.587	2.55	7.829	-15.415
21	22.34	0.091	0.40	14.673	6.600	136	25.22	0.591	2.57	7.836	-15.445
22	22.34	0.096	0.42	15.205	7.598	137	25.22	0.596	2.59	7.428	-15.985
23	22.34	0.100	0.44	14.225	5.625	138	25.22	0.600	2.61	7.436	-16.011
24	20.25	0.104	0.45	13.314	5.874	139	23.96	0.604	2.63	7.469	-14.750
25	19.24	0.109	0.47	11.916	4.100	140	23.96	0.609	2.65	7.479	-14.773
26	18.33	0.113	0.49	11.742	4.623	141	23.96	0.613	2.67	7.490	-14.794
27	18.33	0.117	0.51	10.993	3.118	142	23.96	0.617	2.68	7.501	-14.814
28	17.42	0.122	0.53	10.836	3.676	143	23.96	0.622	2.70	7.514	-14.832
29	17.42	0.126	0.55	10.681	3.329	144	22.77	0.626	2.72	7.127	-15.328
30	17.42	0.130	0.57	11.058	4.012	145	22.77	0.630	2.74	6.789	-14.577
31	17.47	0.135	0.59	10.933	3.670	146	21.65	0.635	2.76	6.828	-13.444
32	18.34	0.139	0.61	10.779	2.456	147	21.65	0.639	2.78	6.842	-13.456
33	18.34	0.143	0.63	10.631	2.121	148	21.65	0.643	2.80	6.857	-13.466
34	18.34	0.148	0.64	10.487	1.793	149	21.65	0.648	2.82	6.872	-13.475
35	18.34	0.152	0.66	10.346	1.472	150	21.65	0.652	2.84	6.888	-13.483
36	18.34	0.157	0.68	10.209	1.158	151	21.65	0.657	2.85	6.905	-13.490
37	18.34	0.161	0.70	9.572	-0.104	152	21.65	0.661	2.87	6.923	-13.496
38	17.43	0.165	0.72	8.977	-0.376	153	21.65	0.665	2.89	6.600	-13.897
39	16.56	0.170	0.74	8.865	0.232	154	20.57	0.670	2.91	6.613	-12.831
40	16.56	0.174	0.76	8.753	-0.029	155	20.57	0.674	2.93	6.632	-12.833
41	16.56	0.178	0.78	8.644	-0.283	156	20.57	0.678	2.95	7.008	-12.423
42	16.56	0.183	0.80	8.537	-0.532	157	20.57	0.683	2.97	6.671	-12.835
43	16.56	0.187	0.81	8.433	-0.776	158	20.57	0.687	2.99	6.691	-12.834
44	16.56	0.191	0.83	8.332	-1.014	159	20.57	0.691	3.01	6.713	-12.833
45	16.56	0.196	0.85	8.233	-1.248	160	20.57	0.696	3.02	7.098	-12.416
46	16.56	0.200	0.87	8.137	-1.477	161	21.59	0.700	3.04	7.476	-13.027
47	1.58	0.204	0.89	0.767	-0.162	162	23.79	0.704	3.06	8.266	-14.348
48	18.24	0.209	0.91	9.198	-1.306	163	26.21	0.709	3.08	9.143	-15.795

S/N	S	T	r	V	P	S/N	S	T	R	V	P
49	20.10	0.213	0.93	9.542	-2.590	164	28.89	0.713	3.10	10.113	-17.402
50	20.10	0.217	0.95	9.436	-2.845	165	32.84	0.717	3.12	11.154	-20.198
51	20.10	0.222	0.97	9.333	-3.094	166	35.10	0.722	3.14	12.378	-21.113
52	20.10	0.226	0.98	9.232	-3.338	167	42.60	0.726	3.16	15.083	-25.601
53	20.10	0.230	1.00	9.134	-3.577	168	42.50	0.730	3.18	15.096	-25.529
54	20.10	0.235	1.02	9.038	-3.811	169	37.05	0.735	3.19	12.524	-22.997
55	20.10	0.239	1.04	8.945	-4.040	170	36.00	0.739	3.21	12.896	-21.573
56	20.10	0.243	1.06	8.409	-5.056	171	35.91	0.743	3.23	12.232	-22.251
57	20.10	0.248	1.08	8.765	-4.483	172	37.70	0.748	3.25	12.428	-23.849
58	20.10	0.252	1.10	8.678	-4.697	173	33.10	0.752	3.27	11.364	-20.470
59	20.10	0.257	1.12	8.594	-4.907	174	33.10	0.757	3.29	10.760	-21.159
60	20.10	0.261	1.14	8.512	-5.113	175	33.00	0.761	3.31	12.064	-19.659
61	20.10	0.265	1.15	8.007	-6.054	176	35.90	0.765	3.33	12.474	-22.128
62	20.10	0.270	1.17	8.353	-5.511	177	35.90	0.770	3.35	12.524	-22.102
63	20.10	0.274	1.19	8.277	-5.704	178	35.90	0.774	3.36	11.850	-22.862
64	20.10	0.278	1.21	8.203	-5.892	179	35.90	0.778	3.38	12.625	-22.048
65	20.10	0.283	1.23	8.130	-6.077	180	35.90	0.783	3.40	12.676	-22.020
66	20.10	0.287	1.25	7.654	-6.951	181	35.90	0.787	3.42	12.728	-21.991
67	20.10	0.291	1.27	7.991	-6.434	182	35.90	0.791	3.44	12.779	-21.962
68	20.10	0.296	1.29	7.525	-7.284	183	35.90	0.796	3.46	12.077	-22.742
69	16.45	0.300	1.31	6.432	-5.546	184	35.90	0.800	3.48	12.880	-21.903
70	16.45	0.304	1.32	6.380	-5.681	185	35.90	0.804	3.50	12.931	-21.874
71	16.45	0.309	1.34	6.329	-5.814	186	35.90	0.809	3.52	12.980	-21.844
72	16.45	0.313	1.36	6.280	-5.944	187	35.90	0.813	3.53	13.030	-21.815
73	16.45	0.317	1.38	6.547	-5.549	188	35.90	0.817	3.55	13.078	-21.786
74	16.45	0.322	1.40	6.185	-6.195	189	35.90	0.822	3.57	12.336	-22.599
75	16.45	0.326	1.42	6.450	-5.806	190	35.90	0.826	3.59	12.375	-22.577
76	16.45	0.330	1.44	6.404	-5.931	191	35.90	0.830	3.61	12.414	-22.556
77	16.45	0.335	1.46	6.053	-6.551	192	35.90	0.835	3.63	13.261	-21.678
78	16.45	0.339	1.48	6.315	-6.172	193	35.90	0.839	3.65	13.304	-21.654
79	16.45	0.343	1.49	6.273	-6.289	194	35.90	0.843	3.67	13.344	-21.632
80	17.27	0.348	1.51	6.227	-7.227	195	35.90	0.848	3.69	13.382	-21.611
81	17.27	0.352	1.53	6.186	-7.339	196	35.90	0.852	3.71	13.417	-21.592
82	17.27	0.357	1.55	6.147	-7.448	197	35.90	0.857	3.72	12.603	-22.467
83	17.27	0.361	1.57	6.264	-7.310	198	35.90	0.861	3.74	13.479	-21.563
84	17.27	0.365	1.59	6.370	-7.191	199	35.90	0.865	3.76	13.506	-21.553
85	18.10	0.370	1.61	6.042	-8.579	200	35.90	0.870	3.78	13.528	-21.546
86	18.16	0.374	1.63	6.630	-7.769	201	35.90	0.874	3.80	13.546	-21.544
87	19.06	0.378	1.65	6.922	-8.263	202	35.90	0.878	3.82	12.673	-22.472
88	21.01	0.383	1.66	7.590	-9.228	203	34.11	0.883	3.84	12.890	-20.478
89	23.00	0.387	1.68	7.462	-11.463	204	34.11	0.887	3.86	12.035	-21.384
90	23.00	0.391	1.70	7.823	-10.968	205	34.11	0.891	3.88	12.888	-20.508
91	23.00	0.396	1.72	7.784	-11.085	206	34.11	0.896	3.89	12.876	-20.533
92	23.00	0.400	1.74	7.350	-11.799	207	34.11	0.900	3.91	12.857	-20.565
93	23.00	0.404	1.76	7.710	-11.313	208	34.11	0.904	3.93	12.829	-20.606
94	23.00	0.409	1.78	7.281	-12.011	209	34.11	0.909	3.95	11.895	-21.586
95	23.00	0.413	1.80	7.640	-11.530	210	34.11	0.913	3.97	11.839	-21.654
96	23.00	0.417	1.82	7.607	-11.634	211	34.11	0.917	3.99	12.685	-20.788
97	23.00	0.422	1.83	7.576	-11.736	212	34.11	0.922	4.01	12.612	-20.872
98	23.00	0.426	1.85	7.545	-11.835	213	34.11	0.926	4.03	12.525	-20.971
99	23.00	0.430	1.87	7.516	-11.932	214	34.11	0.930	4.05	11.483	-22.055
100	23.00	0.435	1.89	7.488	-12.027	215	34.11	0.935	4.06	11.352	-22.196
101	23.00	0.439	1.91	7.461	-12.119	216	34.11	0.939	4.08	12.155	-21.375
102	23.00	0.443	1.93	7.435	-12.209	217	34.11	0.943	4.10	11.987	-21.553
103	23.00	0.448	1.95	7.410	-12.296	218	34.11	0.948	4.12	11.793	-21.758
104	23.00	0.452	1.97	7.386	-12.381	219	34.11	0.952	4.14	11.567	-21.995
105	23.00	0.457	1.99	7.364	-12.464	220	34.11	0.957	4.16	11.305	-22.267
106	23.00	0.461	2.00	7.342	-12.545	221	34.11	0.961	4.18	10.008	-23.600
107	23.00	0.465	2.02	6.944	-13.154	222	34.11	0.965	4.20	10.644	-22.948
108	23.00	0.470	2.04	6.925	-13.227	223	34.11	0.970	4.22	10.228	-23.374
109	21.85	0.474	2.06	6.920	-12.136	224	34.11	0.974	4.23	8.725	-24.911
110	21.85	0.478	2.08	6.613	-12.607	225	34.11	0.978	4.25	8.133	-25.512
111	20.96	0.483	2.10	6.607	-11.771	226	34.11	0.983	4.27	7.415	-26.238
112	20.96	0.487	2.12	6.931	-11.370	227	34.11	0.987	4.29	6.521	-27.141
113	22.00	0.491	2.14	6.907	-12.484	228	35.80	0.991	4.31	7.610	-27.680
114	22.00	0.496	2.16	6.894	-12.545	229	37.48	0.996	4.33	5.179	-31.802
115	22.00	0.500	2.17	6.882	-12.605	230	33.48	1.000	4.35	0.000	-32.990

Table 6. THE RESULT OF THE CALL AND PUT OPTION PRICES FOR CUTIX

S/N	S	t	r	V	P	S/N	S	T	R	V	P
1	1.66	0.005	0.003	0.423	0.418	101	1.58	0,505	0.247	0.191	-0.152
2	1.66	0.010	0.006	0.420	0.410	102	1.58	0,510	0.250	0.190	-0.156
3	1.66	0.015	0.008	0.416	0.403	103	1.58	0,515	0.252	0.189	0.161
4	1.66	0.020	0.011	0.413	0.396	104	1.58	0,520	0.255	0.188	-0.165
5	1.66	0.025	0.013	0.357	0.257	105	1.65	0,525	0.257	0.195	-0.177
6	1.58	0.030	0.016	0.386	0.363	106	1.65	0,530	0.260	0.193	-0.181
7	1.58	0.035	0.018	0.345	0.258	107	1.65	0,535	0.262	0.192	-0.185
8	1.52	0.040	0.021	0.366	0.335	108	1.65	0,540	0.265	0.191	-0.190
9	1.52	0.045	0.023	0.318	0.216	109	1.65	0,545	0.267	0.189	-0.194
10	1.45	0.050	0.025	0.343	0.308	110	1.65	0,550	0.269	0.188	-0.198
11	1.45	0.055	0.028	0.296	0.189	111	1.65	0,555	0.272	0.206	-0.145
12	1.38	0.060	0.030	0.284	0.185	112	1.70	0,560	0.274	0.191	-0.213
13	1.32	0.065	0.033	0.338	0.354	113	1.70	0.565	0.275	0.190	-0.218
14	1.38	0.070	0.035	0.349	0.360	114	1.70	0.570	0.278	0.150	-0.321
15	1.44	0.075	0.038	0.327	0.275	115	1.62	0.575	0.280	0.179	-0.216
16	1.44	0.080	0.040	0.325	0.269	116	1.62	0.580	0.283	0.178	-0.220
17	1.37	0.085	0.042	0.306	0.250	117	1.62	0.585	0.285	0.176	-0.224
18	1.37	0.090	0.045	0.304	0.245	118	1.62	0.590	0.287	0.175	-0.228
19	1.44	0.095	0.047	0.274	0.141	119	1.62	0.595	0.290	0.174	-0.232
20	1.37	0.100	0.050	0.305	0.249	120	1.62	0.600	0.292	0.173	-0.236
21	1.38	0.105	0.052	0.299	0.230	121	1.62	0.605	0.295	0.167	-0.252
22	1.38	0.110	0.055	0.297	0.224	122	1.61	0.610	0.297	0.169	-0.243
23	1.38	0.115	0.057	0.294	0.219	123	1.53	0.615	0.300	0.185	-0.158
24	1.38	0.120	0.060	0.292	0.213	124	1.60	0.620	0.302	0.127	-0.348
25	1.38	0.125	0.062	0.321	0.295	125	1.52	0.625	0.304	0.182	-0.164
26	1.44	0.130	0.064	0.300	0.211	126	1.66	0.630	0.307	0.199	-0.180
27	1.44	0.135	0.067	0.298	0.205	127	1.75	0.635	0.309	0.178	-0.286
28	1.44	0.140	0.069	0.295	0.200	128	1.75	0.640	0.312	0.177	-0.290
29	1.44	0.145	0.072	0.328	0.295	129	1.75	0.645	0.314	0.176	-0.294
30	1.51	0.150	0.074	0.305	0.198	130	1.75	0.650	0.317	0.175	-0.299
31	1.51	0.155	0.077	0.302	0.192	131	1.75	0.655	0.319	0.135	-0.400
32	1.51	0.160	0.079	0.305	0.201	132	1.67	0.660	0.321	0.164	-0.293
33	1.51	0.165	0.082	0.303	0.195	133	1.67	0.665	0.324	0.163	-0.297
34	1.51	0.170	0.084	0.330	0.274	134	1.67	0.670	0.326	0.181	-0.247
35	1.58	0.175	0.086	0.307	0.177	135	1.72	0.675	0.329	0.166	-0.315
36	1.58	0.180	0.089	0.304	0.172	136	1.72	0.680	0.331	0.131	-0.402
37	1.58	0.185	0.091	0.302	0.166	137	1.72	0.685	0.334	0.130	-0.406
38	1.58	0.190	0.094	0.300	0.160	138	1.65	0.690	0.336	0.156	-0.314
39	1.58	0.195	0.096	0.297	0.154	139	1.65	0.695	0.338	0.155	-0.318
40	1.58	0.200	0.099	0.295	0.148	140	1.65	0.700	0.341	0.114	-0.418
41	1.58	0.205	0.101	0.293	0.143	141	1.57	0.705	0.343	0.117	-0.381
42	1.58	0.210	0.103	0.291	0.137	142	1.57	0.710	0.346	0.110	-0.397
43	1.58	0.215	0.106	0.289	0.131	143	1.50	0.715	0.348	0.137	-0.303
44	1.58	0.220	0.108	0.286	0.126	144	1.50	0.720	0.351	0.160	-0.233
45	1.58	0.225	0.111	0.284	0.120	145	1.57	0.725	0.353	0.141	-0.325
46	1.58	0.230	0.113	0.292	0.142	146	1.57	0.730	0.355	0.140	-0.328
47	1.60	0.235	0.116	0.284	0.111	147	1.57	0.735	0.358	0.139	-0.332
48	1.60	0.240	0.118	0.281	0.105	148	1.57	0.740	0.360	0.162	-0.263
49	1.52	0.245	0.121	0.265	0.094	149	1.61	0.745	0.363	0.140	-0.348
50	1.52	0.250	0.123	0.263	0.089	150	1.61	0.750	0.365	0.166	-0.269
51	1.52	0.255	0.125	0.261	0.084	151	1.69	0.755	0.368	0.149	-0.362
52	1.52	0.260	0.128	0.260	0.079	152	1.70	0.760	0.370	0.163	-0.327
53	1.52	0.265	0.130	0.220	-0.026	153	1.78	0.765	0.372	0.150	-0.402
54	1.45	0.270	0.133	0.206	-0.034	154	1.85	0.770	0.375	0.131	-0.479
55	1.38	0.275	0.135	0.198	-0.027	155	1.94	0.775	0.377	0.183	-0.384
56	1.32	0.280	0.138	0.246	0.130	156	2.03	0.780	0.380	0.167	-0.472
57	1.30	0.285	0.140	0.241	0.125	157	2.13	0.785	0.382	0.174	-0.501
58	1.36	0.290	0.143	0.249	0.122	158	2.13	0.790	0.385	0.172	-0.506
59	1.42	0.295	0.145	0.230	0.040	159	2.13	0.795	0.387	0.171	-0.511
60	1.42	0.300	0.147	0.229	0.036	160	2.13	0.800	0.389	0.118	-0.635
61	1.42	0.305	0.150	0.190	-0.066	161	2.13	0.805	0.392	0.122	-0.627
62	1.35	0.310	0.152	0.232	0.078	162	2.13	0.810	0.394	0.121	-0.632
63	1.39	0.315	0.155	0.237	0.074	163	2.03	0.815	0.397	0.190	-0.406
64	1.37	0.320	0.157	0.224	0.043	164	1.89	0.820	0.399	0.144	-0.476
65	1.39	0.325	0.160	0.216	0.013	165	1.89	0.825	0.402	0.143	-0.480

S/N	S	t	r	V	P	S/N	S	T	R	V	P
66	1.44	0.330	0.162	0.252	0.098	166	1.89	0.830	0.404	0.142	-0.485
67	1.37	0.335	0.164	0.200	-0.023	167	1.89	0.835	0.406	0.140	-0.489
68	1.32	0.340	0.167	0.226	0.076	168	1.89	0.840	0.409	0.139	-0.494
69	1.43	0.345	0.169	0.191	-0.072	169	1.89	0.845	0.411	0.137	-0.498
70	1.38	0.350	0.172	0.207	-0.009	170	1.89	0.850	0.414	0.135	-0.503
71	1.38	0.355	0.174	0.223	0.038	171	1.89	0.855	0.416	0.134	-0.508
72	1.44	0.360	0.177	0.230	0.033	172	1.89	0.860	0.419	0.132	-0.512
73	1.48	0.365	0.179	0.217	-0.023	173	1.89	0.865	0.421	0.131	-0.517
74	1.48	0.370	0.182	0.216	-0.028	174	1.89	0.870	0.423	0.129	-0.522
75	1.48	0.375	0.184	0.227	0.006	175	1.89	0.875	0.426	0.127	-0.526
76	1.48	0.380	0.186	0.226	0.001	176	1.89	0.880	0.428	0.125	-0.531
77	1.51	0.385	0.189	0.233	0.008	177	1.89	0.885	0.431	0.124	-0.536
78	1.55	0.390	0.191	0.233	-0.010	178	1.89	0.890	0.433	0.122	-0.541
79	1.58	0.395	0.194	0.208	-0.093	179	1.89	0.895	0.436	0.120	-0.546
80	1.55	0.400	0.196	0.217	-0.057	180	1.89	0.900	0.438	0.118	-0.551
81	1.55	0.405	0.199	0.228	-0.024	181	1.89	0.905	0.440	0.116	-0.556
82	1.58	0.410	0.201	0.218	-0.068	182	1.89	0.910	0.443	0.142	-0.475
83	1.58	0.415	0.204	0.216	-0.072	183	1.98	0.915	0.445	0.117	-0.593
84	1.65	0.420	0.206	0.220	-0.093	184	1.98	0.920	0.448	0.114	-0.598
85	1.65	0,425	0.208	0.204	-0.137	185	1.98	0.925	0.450	0.112	-0.604
86	1.58	0,430	0.211	0.212	-0.086	186	1.98	0.930	0.453	0.055	-0.721
87	1.58	0,435	0.213	0.210	-0.091	187	1.89	0.935	0.455	0.101	-0.588
88	1.58	0,440	0.216	0.209	-0.095	188	1.89	0.940	0.457	0.098	-0.593
89	1.58	0,445	0.218	0.207	-0.100	189	1.89	0.945	0.460	0.095	-0.599
90	1.58	0,450	0.221	0.206	-0.104	190	1.89	0.950	0.462	0.092	-0.605
91	1.58	0,455	0.223	0.205	-0.109	191	1.89	0.955	0.465	0.089	-0.611
92	1.58	0,460	0.225	0.231	-0.029	192	1.89	0.960	0.467	0.085	-0.618
93	1.65	0,465	0.228	0.211	-0.123	193	1.89	0.965	0.470	0.081	-0.625
94	1.65	0,470	0.230	0.209	-0.127	194	1.89	0.970	0.472	0.077	-0.632
95	1.65	0,475	0.233	0.208	-0.132	195	1.89	0.975	0.474	0.072	-0.640
96	1.65	0,480	0.235	0.207	-0.136	196	1.89	0.980	0.477	0.067	-0.648
97	1.65	0,485	0.238	0.205	-0.141	197	1.89	0.985	0.479	0.061	-0.657
98	1.65	0,490	0.240	0.204	-0.145	198	1.89	0.990	0.482	0.054	-0.667
99	1.65	0,495	0.243	0.169	-0.239	199	1.89	0.995	0.484	0.045	-0.678
100	1.58	0,500	0.245	0.193	-0.148	200	1.89	1.000	0.487	0.033	-0.693

3.2.1. The Graphical Representation of the Call and Put Option Prices

The results of the call and put option prices for Conoil and Cutix stock index are represented in the following graphs:

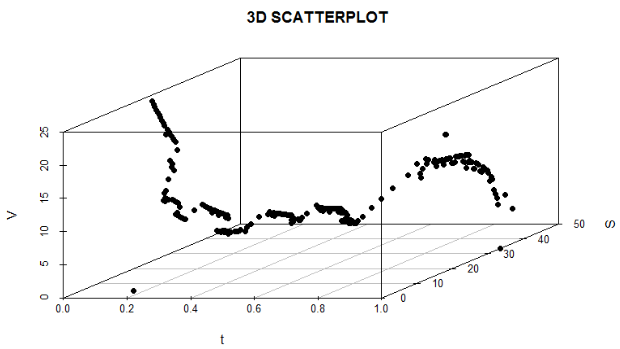

Figure 3. Graph of call option prices for conoil against the stock prices, s and the time to maturity, t with varying strike prices,k and varying discount rate,r.(chosen parameters: Hurst exponent,H=0.86, volatility, σ=4.346, length of the period in years, τ=0.003)

From the graph, we notice that the call option values move downward and upward in similar pattern, thus it will be possible to predict the pattern the subsequent call option values will take.

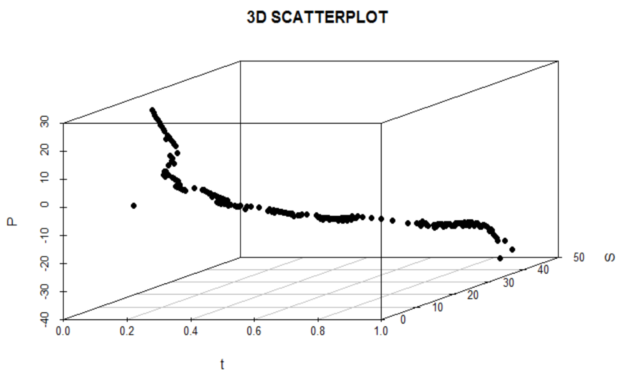

Figure 4. graph of put option prices for conoil against the stock prices, S and the time to maturity, t with varying strike prices, k and varying discount rate,r. (chosen parameters: Hurst exponent, $H = 0.86$, volatility, $\sigma = 4.346$, length of the period in YEARS, $\tau = 0.003$)

From the Figure 4 above, we notice that the put option values move downward and upward in similar pattern too, thus it will be possible to predict the pattern subsequent put option values will take.

From the graph of Figure 5, we notice that the movement of the call option values is completely random, thus it will be difficult to make predictions on subsequent call option values

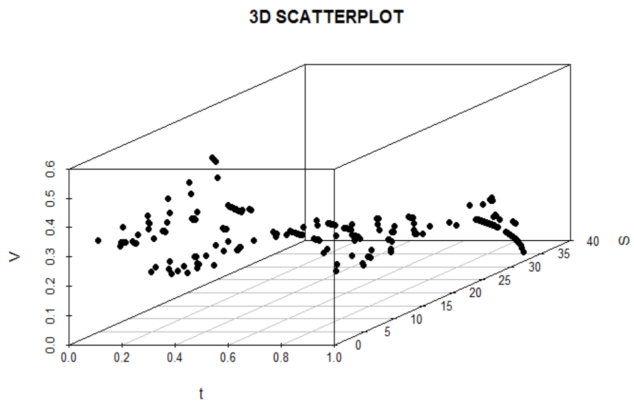

Figure 5. Graph of call option prices for cutix against the stock prices, S and the time to maturity, t WITH varying strike prices, K and varying discount rate,r. (Chosen parameters: Hurst exponent, $H = 0.47$, volatility, $\sigma = 0.488$, length of the period in years,$\tau = 0.003$).

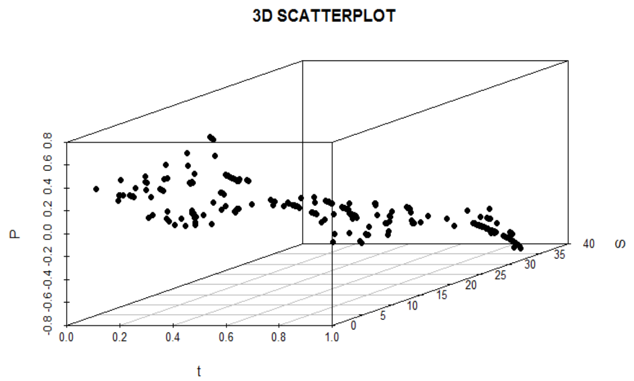

Figure 6. Graph of put option prices for Cutix against the stock prices, S and the time to maturity,t with varying strike prices,K and varying discount rate,r. (Chosen parameters: Hurst exponent, $H = 0.47$, volatility, $\sigma = 0.488$, length of the period in years, $\tau = 0.003$)

Figure 6 shows that the movement of the put option values is completely random as that of the call option, thus it will be difficult to make predictions on subsequent put option values.

4. Conclusion

In this work, we have used the formula we have obtained by solving the Black-Scholes option pricing equation modeled by fractional Brownian motion using change of variables and Fourier transform to analyze and study option pricing. Increase in volatility increases option value (both call and put options) while decrease in volatility decreases option values. Furthermore, increase in the value of the Hurst exponent also increases the option values, while decrease in the value of the Hurst exponent also decreases the option values. We discovered too that the higher the value of the Hurst exponent, the easier it is to predict option values. Prediction of option values can be very difficult when the value of the Hurst exponent is very small

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Appendix A

MATLAB PROGRAM ON HOW TO CALCULATE HURST EXPONENT, VOLATILITY,CALL AND PUT OPTION PRICES

```
%% This part reads the files for stock and strike and assigns them to S and K respectively
data = xlsread('StockStrikeConoil.xlsx'); % To compute for Cutix, simply replace StockStrikeConoil.xlsx with StockStrikeCutix.xlsx
%% This part computes X,Y,Z,F(n),log(n) and log(F(n))
xx = (1:length(data(:,2)))';
n= 20:20:100;% values of n to be used
logF=[];logn=[];F=[];% empty matrix to store logF,logn and F(n)
for rr = 1:length(n)
numberofy = length(xx)-rem(length(xx),n(rr));
U = [];
for i = 1:length(data(1:numberofy,2))
   U(i,1) = data(i,2) - mean(data(1:numberofy,2));
end
nn=numberofy/n(rr);
y=data(1:numberofy,2);
x=[];xxx=(1:n(rr))';
b=[];a=[];
for k=1:nn
   x = [(1:n(rr))';x];
   yyy = y(k*n(rr)-n(rr)+1:k*n(rr));
   b(1,k) = ((n(rr)*sum(xxx.*yyy))-(sum(xxx)*sum(yyy)))/(n(rr)*sum(xxx.^2)-(sum(xxx))^2);
   a(1,k) = mean(yyy)- b(1,k)*mean(xxx);

end
```

```
z=[];ll=0;

for j=1:numberofy
   if rem(j,n(rr))==1
      ll=ll+1;
   end
   z(j,1)=a(1,ll)*x(j)+b(1,ll);
end

f(1,rr) = sqrt((1/numberofy)*sum((U(1:numberofy,1)-z).^2));% PROBLEM HERE
logn(1,rr) = log10(n(1,rr));
logf(1,rr) = log10(f(1,rr));
end

fprintf('%9 s  %15 s %21 s \n \n','n','Log (n)','log (F(n))')% prints the tittles n,logn and log(F(n))
fprintf('%9 .3f  %15 .3f %21 .3f \n',[n;logn;logf])% prints the values of n,logn and logF
plot(logn,logf)
coefficients = polyfit(logn, log(F(n)), 1);
disp(['The Slope is: ',num2str(coefficients(1))])
disp(' ')

%% PART TWO
%% Constants used are:
  ii = data(:,1); H = coefficients(1); K = data(:,3); S = data(:,2); tau = 1/365; T = 1; t = ii./length(ii);
d1=[];d2=[];CDF1=[];CDF2=[];V=[]; U=[];P=[];

%% This part computes zigma, m, P and V

for i = 2:length(S)
   U(i,1) = log(S (i,1)/S(i-1,1));
end
U(1,:)=[];U=[U;0];
U_mean _sqrt=[];% used to compute zigma
for i = 1:length(U)
   U_mean _sqrt(i,1) = (U(i,1)-mean(U))^2;
end
U_mean _sqrt _sum = sum(U_mean _sqrt); % used to compute zigma
zigma = (sqrt((1/(length(U)-1))*U_mean _sqrt _sum))/sqrt(tau); % zigma value
alpha = abs (S(length(S),1) - S(1,1)) ./ sum(S);
r = alpha + zigma.*t; % Computes the values of r

%% Continue
for tt = 1:length(ii)
   d1(tt,1) = (((log(S (tt,1)./K(tt,1)))./(sqrt((2.*H.*(T-((2.*tau)./(zigma.^2))).^(2.*H-1)).*(zigma.^2).*(T-t(tt,1))))) + (sqrt(((zigma.^2).*(T-t(tt,1)))./(4.*(2.*H.*(T-((2.*tau)./(zigma.^2))).^(2.*H-1))))).*((2.*H.*(T-((2.*tau)./(zigma.^2))).^(2.*H-1))-((2.*R(tt,1))./(zigma.^2))));
   d2(tt,1) = (((log(S (tt,1)./K(tt,1)))./(sqrt((2.*H.*(T-((2.*tau)./(zigma.^2))).^(2.*H-1)).*(zigma.^2).*(T-t(tt,1))))) - (sqrt(((zigma.^2).*(T-t(tt,1)))./(4.*(2.*H.*(T-((2.*tau)./(zigma.^2))).^(2.*H-1))))).*((2.*H.*(T-((2.*tau)./(zigma.^2))).^(2.*H-1))+((2.*R(tt,1))./(zigma.^2))));
   CDF1(tt,1)=cdf('norm',d1(tt,1));
   CDF2(tt,1)=cdf('norm',d2(tt,1));
   V(tt,1) = K (tt,1).*exp (-R(tt,1).*(T-t(tt,1))).*CDF1(tt,1) - S (tt,1).*CDF2(tt,1);
   P(tt,1) = V(tt,1) - S(tt,1) + K (tt,1).*exp(-R(tt,1).*T);
end
%% This part concatenates  S,K,V and P and then saves them as excel file
Solution = [S,K,V,P];
save('Cutix.mat','Solution')
save('Cutix.xlsx','Solution')

%% This part prints the columns of S,k,V and P
fprintf('%9 s  %15 s %20 s %25 s \n \n','S','K','V','P')
fprintf('%11 .3f  %12 .3f %22 .3f %26 .3f \n',[S';K';V';P'])
```