*International Journal of Partial Differential Equations and Applications, 2014, Vol. 2, No. 5, 86-90*
Available online at http://pubs.sciepub.com/ijpdea/2/5/1

DOI:10.12691/ijpdea-2-5-1

# Weak Solutions for a Class of *(p,q)* Laplacian quasilinear Elliptic System with Different Weights

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*Received March 30, 2014; Revised November 09, 2014; Accepted November 16, 2014*

**Abstract** Using variational methods, we study the existence of weak solutions for the degenerate quasilinear elliptic system $\begin{cases} -div\left(h_1(x)\left|\nabla_u\right|^{p-2}.\nabla_u\right)=\lambda F_u(x,u,v) & in\ \Omega \\ -div\left(h_2(x)\left|\nabla_v\right|^{q-2}.\nabla_v\right)=\lambda F_v(x,u,v) & in\ \Omega \\ u=v=0 & on\ \partial\Omega \end{cases}$ where $\Omega\subset\mathbb{R}^N$ is a smooth bounded domain, $\nabla F=(F_u,F_v)$ stands for the gradient of $C^1$ -function $F:\ \Omega\times\mathbb{R}^2\to\mathbb{R}$ the weights $h_i, i=1,2$ are allowed to vanish somewhere, the primitive $F(x;u,v)$ is intimately related to the first eigenvalue of a corresponding quasilinear system

***Keywords:*** *quasilinear elliptic system, Palais-Smale condition, mountain pass theorem, existence*

**Cite This Article:** Tahar Bouali, and Rafik Guefaifiai, "Weak Solutions for a Class of *(p,q)* Laplacian quasilinear Elliptic System with Different Weights." *International Journal of Partial Differential Equations and Applications*, vol. 2, no. 5 (2014): 86-90. doi: 10.12691/ijpdea-2-5-1.

## 1. Introduction

In this paper, we are concerned with the quasilinear elliptic system

$$\begin{cases} -div\left(h_1(x)\left|\nabla_u\right|^{p-2}.\nabla_u\right)=\lambda F_u(x,u,v) & in\ \Omega \\ -div\left(h_2(x)\left|\nabla_v\right|^{q-2}.\nabla_v\right)=\lambda F_v(x,u,v) & in\ \Omega \\ u=v=0 & on\ \partial\Omega \end{cases} \quad (1.1)$$

where $\Omega$ is a smooth bounded domain in $\mathbb{R}^N\ (N\geq 2)$, $\lambda$ nonnegative real number, $1<p<N, 1<q<N$, $(F_u,F_v)=\nabla F$ stands for the gradient of $F$ in the variable $(u,v)\in\mathbb{R}^2$. We point out that in the case, $h_1(x)=h_2(x)=1$, problem (1:1) has been studied in many papers. For more details about this kind of systems, we refer to [4,8,9,11,12,13,15,19], in which the authors used various methods to get the existence of solutions. The degeneracy of this system is considered in the sense that the measurable, non-negative diffusion coefficients $h_1$, $h_2$ are allowed to vanish in $\Omega$ (as well as at the boundary $\partial\Omega$) and/or to blow up in $\bar{\Omega}$. The point of departure for the consideration of suitable assumptions on the diffusion coefficients

Let us introduce the function space $(H)_p$ which consists of functions $h:\Omega\subset\mathbb{R}^N\to\mathbb{R}$, such that $h\in L^1(\Omega)$, $h^{\frac{-1}{p-1}}\in L^1(\Omega)$, and $h^{-s}\in L^1(\Omega)$, for some $p>1, s>\max\left\{\frac{N}{p},\frac{1}{p-1}\right\}$ satisfying $ps\leq N(s+1)$. Then for the weight functions $h_1,h_2$ we assume the following hypothesis:

$(H)$ There exist functions $\mu_1$ in the space $(H)_p$, for some $s_p$ and $\mu_2$ in the space $(H)_q$, for some $s_q$, such that

$$\begin{aligned} &\frac{\mu_1(x)}{C_1}\leq h_1(x)\leq C_1\mu_1(x) \\ and\ &\frac{\mu_2(x)}{C_2}\leq h_2(x)\leq C_2\mu_2(x) \end{aligned} \quad (1.2)$$

a.e. in $\Omega$, for some constants $C_1,C_2>1$.

We consider the weighted Sobolev spaces $W_0^{1,p}(\Omega,h_1)$ and $W_0^{1,q}(\Omega,h_2)$ to be defined as the closures of $C_0^\infty$ with respect to the norms

$$\|u\|_{h_1,p}^p = \int_\Omega h_1(x)|\nabla_u|^p dx \quad \text{for all } u \in C_0^\infty(\Omega)$$
$$\|v\|_{h_2,q}^p = \int_\Omega h_2(x)|\nabla_v|^p dx \quad \text{for all } v \in C_0^\infty(\Omega)$$

and set $W = W_0^{1,p}(\Omega, h_1) \times W_0^{1,q}(\Omega, h_2)$. It is clear that $W$ is a reflexive Banach space under the norm

$$\|(u,v)\|_W = \|u\|_{h_{1,p}} + \|u\|_{h_{2,q}} \quad \text{for all } (u,v) \in W.$$

For more details about the space setting we refer to [10] and the references therein. The key in our arguments is the following lemma

**Lemma 1.1** (see [10]). Assume that $\Omega$ is a bounded domain in $\mathbb{R}^N$ and the weight $h$ satisfies $(H)_p$. Then the following embedding hold:

(i) $W_0^{1,p}(\Omega, h) \hookrightarrow L^{p_s^*}(\Omega)$ continuously for $1 < p_s^* < N$, where $p_s^* := \dfrac{N p_s}{N(s+1) - p_s}$.

(ii) $W_0^{1,p}(\Omega, h) \hookrightarrow L^r(\Omega)$ compactly for any $r \in [1, p_s^*)$.

In the sequel we denote by the $p^*$ and $q^*$ the quantities $p_{s_p}^*$ and $q_{s_q}^*$, respectively, where $s_p$ and $s_q$ are induced by condition the $(H)$. The assumptions concerning the coefficient functions of (1.1) are the following:

(A) $a \in L^{\frac{p^*}{p^*-1}}(\Omega)$ and either there exists $\Omega_a^+ \subset \Omega$ of positive Lebesgue measure, i.e, $|\Omega_a^+| > 0$, such that $a(x) > 0$, for all $x \in \Omega_a^+$, neither $a(x) = 0$ in $\Omega$.

(D) $d \in L^{\frac{q^*}{q^*-1}}(\Omega)$ and either there exists $\Omega_d^+ \subset \Omega$ of positive Lebesgue measure, i.e, $|\Omega_d^+| > 0$, such that $d(x) > 0$, for all $x \in \Omega_a^+$, neither $d(x) = 0$ in $\Omega$.

(B) $b(x) \ge 0$, a.e, in $\Omega, b \ne 0$ and $b \in L^w(\Omega)$, where $w = \left[1 - \dfrac{\alpha+1}{p^*} - \dfrac{\beta+1}{q^*}\right]^{-1}$.

Many authors studied the existence of solutions for such problems (equations or systems) ;see for example [5,6,11,15,16,18,19].

Recently in [7], the authors considered the system

$$\begin{aligned} -div(h_1(x)\nabla_u) &= \lambda F_u(x,u,v) \quad in \ \Omega \\ -div(h_2(x)\nabla_v) &= \lambda F_v(x,u,v) \quad in \ \Omega \\ u = v &= 0 \quad on \ \partial\Omega \end{aligned}$$

They are concerned with the nonexistence and multiplicity of nonnegative, nontrivial solutions.

Also, we mention some results concerning the associated eigenvalue problem. Let $\lambda_1$ be the first eigenvalue of the Diruchlet porblem

$$\begin{aligned} -div\left(h_1(x)|\nabla_u|^{P-2}.\nabla_u\right) &= \lambda|\mu|^{\theta-1}|v|^{\delta+1}\mu \quad in \ \Omega \\ -div\left(h_2(x)|\nabla_v|^{q-2}.\nabla_v\right) &= \lambda|\mu|^{\theta+1}|v|^{\delta-1}v \quad in \ \Omega \\ u = v &= 0 \quad on \ \partial\Omega \end{aligned} \tag{1.3}$$

where the functions $h_1$ and $h_2$ satisfy $(H)$, and the exponents $\theta, \delta$ satisfy

$$\frac{\theta+1}{p} + \frac{\delta+1}{q} = 1 \tag{1.4}$$

then, we have that $\lambda_1$ is a positive number, which is characterized variationally by

$$\lambda_1 = \inf_{(u,v)\in W-\{(0,0)\}} \frac{\int_\Omega \begin{pmatrix} \frac{\theta+1}{p} h_1(x)|\nabla_u|^p \\ + \frac{\delta+1}{q} h_2(x)|\nabla_u|^q \end{pmatrix} dx}{\int_\Omega |u|^{\theta+1}|v|^{\delta+1} dx}. \tag{1.5}$$

Moreover, $\lambda_1$ is isolated, the associated eigenfunction $(\varphi_1, \varphi_2)$ is componentwise nonnegative and $\lambda_1$ is the only eigenvalue of (1:2) to which corresponds a componentwise nonnegative eigenfunction. In addition, the set of all eigenfunctions corresponding to the principal eigenvalue $\lambda_1$ forms a one-dimentional manifold $E_1 \subset W$, which is defined by

$$E_1 = \left\{ \left( t_1\varphi_1, t_1^{\frac{p}{q}}\varphi_2 \right) : t_1 \in \mathbb{R} \right\}$$

In the rest of this article, the following assumption is required.

$$\lambda_1 \le \lim_{|(t,s)|\to\infty} \inf \frac{\lambda F(x,t,s)}{|t|^{\theta+1}|s|^{\delta+1}} \tag{1.6}$$

The aim of this work is to extend or complete some of the above results for system (1.1). Our assumptions are as follows: $F(x,t,s)$ are $C^1$ -fonction satisfying the hypotheses below:

$(F.1)$ There exist positive constants $c_1, c_2 > 0$ such that

$$|F_u(x,t,s)| \le c_1|t|^\theta |s|^{\delta+1}, |F_v(x,t,s)| \le c_1|t|^{\theta+1}|s|^\delta$$
$$for \ all \ (t,s) \in \mathbb{R}^2, a.e. x \in \Omega \ and \ some \ \theta, \delta > 0$$

with

$$\frac{\theta+1}{p} + \frac{\delta+1}{q} = 1$$

$(F.2)$ There exist $R > 0$, $0 < \mu < p$ and $0 < \nu < q$ such that

$$\frac{u}{p}F_u(x,u,v) + \frac{v}{q}F_v(x,u,v) - F(x,u,v)$$
$$\ge c\left(|u|^\mu + |v|^\nu\right) \ for \ all \ x \in \bar{\Omega} \ and \ |u| \ge R, |v| \ge R,$$

$(F.3)$ There exists positive constant $C_3$ such that

$$\left|F(x,u,v)\right| \le C_3\left(1+|u|^p+|v|^q\right)$$

$$for\ all\ (u,v)\in\mathbb{R}^2\ and\ \text{a.e. } x\in\Omega;$$

Next, we introduce the functionals $I,J: W\to\mathbb{R}$:

$$I(u,v)=\frac{1}{p}\int_\Omega h_1(x)|\nabla_u|^p dx+\frac{1}{q}\int_\Omega h_2(x)|\nabla_u|^q dx.$$

$$J(u,v)=\int_\Omega F(x,u,v)dx.$$

**Lemma 1.2.** The functionals *I*, *J*, are well defined. Moreover, *I* is continuous and *J* are compact.

We say that $(u,v)$ is a weak solution of problem (1.1) if $(u,v)$ is a critical point of the functionals

$$\varphi(u,v)=I(u,v)-\lambda J(u,v)$$

$$\int_\Omega h_1(x)|\nabla_u|^p.\nabla\phi dx=\lambda\int_\Omega F_u(x,u,v)\phi dx$$

$$\int_\Omega h_2(x)|\nabla_u|^q.\nabla\psi dx=\lambda\int_\Omega F_v(x,u,v)\psi dx$$

$$for\ any\ (\phi,\psi)\in W.$$

The main results of this paper are the following two theorems.

## 2. Proof of the Main Result

Theorem 2.1. 2.1. *Let (F.1) hold, such that (1.4) satisfait, then there exist* $\lambda_0>0$, *such that system (1.1) possesses a weak solution for all* $0\le\lambda\le\lambda_0$.

Lemma 2.2. 2.2. *Let* $\{w_m\}$ *be a sequence weakly converging to* $w$ *in* $W$. *There we have*

(i) $\varphi(w)\le\liminf_{m\to\infty}\varphi(w_m)$

(ii) $\lim_{m\to\infty} I(w_m)=I(w)$

*Proof. (i)* Let $\{w_m\}=\{(u_m,v_m)\}$ be a sequence that converges weakly to $w=(u,v)\in W$. By the weak lower semicontinuity of the norm in the space $W_0^{1,p}(\Omega,h_1)$ and $W_0^{1,q}(\Omega,h_2)$, we deduce that

$$\liminf_{m\to\infty}\int_\Omega h_1(x)\left|\nabla_{u_m}\right|^p dx+\int_\Omega h_2(x)\left|\nabla_{v_m}\right|^q dx$$
$$\ge\int_\Omega h_1(x)|\nabla_u|^p dx+\int_\Omega h_2(x)|\nabla_v|^q dx.$$

The compactness of operator *J*, by lemma 2.2, imply the conclusion.

Lemma 2.3. 2.3. *The functional* $\varphi$ *is coercive and bounded from below.*

*Proof*. By $(F.1)$ there exists $c_1',c_2'$ such that for all $(t,s)\in\mathbb{R}^2$ and $x\in\Omega$ we deduce that

$$\left|F(x,s,t)\right|\le c_1'|t|^{\theta+1}|s|^{\delta+1},$$

Applying Young's inequality, we obtain

$$\begin{aligned}&\int_\Omega F(x,u,v)dx\le c_1'|u|^{\theta+1}|v|^{\delta+1}dx\\&\le c_1'\left(\frac{\theta+1}{p}\int_\Omega|u|^p dx+\frac{\delta+1}{q}\int_\Omega|v|^q dx\right)\\&\le c_1'\left(\frac{\theta+1}{p}s_1\int_\Omega h_1(x)|\nabla_u|^p dx+\frac{\delta+1}{q}s_2\int_\Omega h_2(x)|\nabla_v|^q dx\right)\\&\le c\left(\frac{\theta+1}{p}\|u\|_{h_1,p}^p+\frac{\delta+1}{q}\|u\|_{h_1,q}^q\right)\end{aligned}\tag{2.1}$$

where $s_1,s_2$ are the embedding constants of $W_0^{1,p}(\Omega,h_1)\hookrightarrow L^p(\Omega)$, $W_0^{1,q}(\Omega,h_1)\hookrightarrow L^q(\Omega)$ and $c=\max\{c_1's_1,c_1's_2\}$,

Consequently

$$\varphi(u,v)\ge\left(\frac{1}{p}-\lambda c\frac{\theta+1}{p}\right)\|u\|_{h_1,p}^p+\left(\frac{1}{q}-\lambda c\frac{\delta+1}{q}\right)\|v\|_{h_1,q}^q.$$

Tayking $\lambda_0>0$ such that $\min\left\{\begin{matrix}1-\lambda(\theta+1)c,\\1-\lambda(\delta+1)c\end{matrix}\right\}>0$ for all $0\le\lambda<\lambda_0$, it follows that for $0\le\lambda<\lambda_0,\varphi$ is coercive, indeed $\varphi(u,v)\to\infty$ as $\|(u,v)\|_W\to\infty$.

**Proof of theorem 2.1** The coerciveness of $\varphi$ and the weak sequential lower semi continuity are enough in order to prove that $\varphi$ attaints its infimum, so the system (1.1) has at least one weak solution.

**Lemma 2.4.** Let $(u_n,v_n)$ be a bounded sequence in $W$ such that $\varphi(u_n,v_n)$ is bounded and $\varphi'(u_n,v_n)\to 0$ as $n\to\infty$, Then $(u_n,v_n)$ has a convergent subsequence.

Theorem 2.4. *2.5. In addition to (F.2) and (F.3), then there exist* $\lambda_0>0$ *such that system (1.1) possesses a weak nontrivial solution for all* $0\le\lambda<\lambda_0$.

*Proof.* To prove the existence of a weak nontrivial solution we apply a version of the Mountain Pass theorem due to Ambrosetti and Rabinowitz [1]. For this purpose we verify that $\varphi$ satisfies:

*(i)* the mountain pass type geometry,

*(ii)* the $(PS)_c$ condition.

*(i)* d'apres

$$\varphi(u,v)\ge\left(\frac{1}{p}-\lambda c\frac{\theta+1}{p}\right)\|u\|_{h_1,p}^p+\left(\frac{1}{q}-\lambda c\frac{\delta+1}{q}\right)\|v\|_{h_1,q}^q$$

Hence, there exists *r > 0*, small enough, such that

$$\inf_{\|(u,v)\|=r}\varphi(u,v)>0=\varphi(0,0).$$

On the other hand by using (1.4), we have

$$\begin{aligned}\varphi\left(t^{\frac{1}{p}}\varphi_1,t^{\frac{1}{q}}\varphi_2\right)&\le\frac{t}{p}\int_\Omega h_1(x)|\nabla\varphi_1|^p dx+\int_\Omega h_2(x)|\nabla\varphi_2|^q dx\\&-(\lambda_1+\varepsilon)\int_\Omega\left(\left|t^{\frac{1}{p}}\varphi_1\right|^{\theta+1}\left|t^{\frac{1}{q}}\varphi_2\right|^{\delta+1}\right)dx\\&=-t\varepsilon\int_\Omega\left(|\varphi_1|^{\theta+1}|\varphi_2|^{\delta+1}\right)dx.\end{aligned}$$

Thus, we conclude that ther exists $t>0$, large enough, such that for $k=\left(t^{\frac{1}{p}}\varphi_1, t^{\frac{1}{q}}\varphi_2\right)$, we have $\|k\|>r$ and $\varphi(k)<0$.

*(ii)* According to Lemma (2.4), it is sufficient to prove that the sequence $\{(u_n,v_n)\}$ is bounded in $W$.

Let $\{(u_n,v_n)\}$ be such a $(PS)_c$ sequence, that is, $\varphi(u_n,v_n)\to c$ and $\varphi'(u_n,v_n)\to 0$ as $n\to\infty$. We obtain

$$\varepsilon_n + c \geq \varphi(u_n,v_n) - \varphi'(u_n,v_n)\left(\frac{u_n}{p},\frac{v_n}{q}\right)$$

$$= \lambda\int_\Omega \begin{bmatrix} \frac{u_n}{p}F_u(x,u_n,v_n) \\ +\frac{v_n}{q}F_v(x,u_n,v_n) \\ -F(x,u_n,v_n)\end{bmatrix} dx$$

$$\geq \lambda c_4 \int_\Omega\left(|u_n|^\mu + |v_n|^\nu\right)dx$$

$$\geq \sigma \int_\Omega\left(|u_n|^\mu + |v_n|^\nu\right)dx \quad where\ \sigma = \lambda c_4.$$

which shows from $(F.2)$ that

$$\int_\Omega\left(|u_n|^\mu + |v_n|^\nu\right)dx \leq c_5 \quad for\ all\ n. \tag{2.2}$$

Next, we use the following interpolation inequality: let $0<e_1<e_2<e_3$ and suppose that for some measurable function $u:\Omega\to\mathbb{R}$ we have that

$$\int_\Omega\left(|u|^{e_1}\right)dx < \infty \quad and \quad \int_\Omega\left(|u|^{e_3}\right)dx < \infty,$$

Then

$$\int_\Omega\left(|u|^{e_2}\right)dx \leq \int_\Omega\left(|u|^{e_1}\right)^{\frac{e_3-e_2}{e_3-e_1}}dx + \int_\Omega\left(|u|^{e_3}\right)^{\frac{e_3-e_2}{e_3-e_1}}dx \tag{2.3}$$

We use (2.3) for $0<\mu<p<p^*$ and $0<\nu<q<q^*$, we get

$$\int_\Omega\left(|u_n|^p\right)dx \leq \int_\Omega\left(|u_n|^\mu\right)^{\frac{p^*-p}{p^*-\mu}}dx.\int_\Omega\left(|u_n|^{p^*}\right)^{\frac{p-\mu}{p^*-\mu}}dx \tag{2.4}$$

$$\int_\Omega\left(|v_n|^p\right)dx \leq \int_\Omega\left(|v_n|^\mu\right)^{\frac{q^*-q}{q^*-\nu}}dx.\int_\Omega\left(|v_n|^{q^*}\right)^{\frac{q-\nu}{q^*-\nu}}dx \tag{2.5}$$

Using (2.2), we obtain

$$\int_\Omega\left(|u_n|^p\right)dx \leq c_6\int_\Omega\left(|u_n|^{p^*}\right)^{\frac{p-\mu}{p^*-\mu}}dx \tag{2.6}$$

and

$$\int_\Omega\left(|v_n|^p\right)dx \leq c_7\int_\Omega\left(|u_n|^{q^*}\right)^{\frac{q-\nu}{q^*-\nu}}dx \tag{2.7}$$

By Lemma 1.1, it follows that

$$\int_\Omega\left(|u_n|^{p^*}\right)^{\frac{p-\mu}{p^*-\mu}}dx \leq c_8\|u_n\|_{h_1,p}^{\tilde{p}} \tag{2.8}$$

and

$$\int_\Omega\left(|v_n|^{q^*}\right)^{\frac{q-\nu}{q^*-\nu}}dx \leq c_9\|v_n\|_{h_2,p}^{\tilde{q}} \tag{2.9}$$

where $\tilde{p}=\frac{p-\mu}{p^*-\mu}p^*$ and $\tilde{q}=\frac{q-\nu}{q^*-\nu}q^*$. On the other hand, by $(F.2)$ and (2.4) -(2.9),we get

$$(u_n,v_n) \geq \frac{1}{p}\|u_n\|_{h_1,p}^p + \frac{1}{q}\|v_n\|_{h_1,q}^q - c_{10}\left(\|u_n\|_{h_1,p}^{\tilde{p}} + \|v_n\|_{h_1,q}^{\tilde{q}}\right)$$

Since $\varphi(u_n,v_n)$ is bounded and $\tilde{p}<p, \tilde{q}<q$, it follows that $(u_n,v_n)$ is bounded in $W$. By Lemma 2:4, we obtain that the functional $\varphi(u,v)$ satisfies the $(PS)_c$ condition (compactness condition).

The assumptions of the mountain pass theorem in [3] are satisfied. Then the functional $\varphi$ admits a nontrivial critical point in W and thus system (1.1) has a nontrivial weak solution. The proof of Theorem 2.1 is complete.

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