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Enhanced (G'/G)-Expansion Method to Find the Exact Solutions of Nonlinear Evolution Equations in Mathematical Physics

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Abstract In the present paper, we construct the traveling wave solutions involving parameters for the (2+1)-dimensional cubic Klein-Gordon equation (cKG) via Enhanced (G'/G)-expansion method. The efficiency of this method for finding these exact solutions has been demonstrated. As a result, a set of solitary wave solutions are derived, which are expressed by the combinations of rational, hyperbolic and trigonometric functions involving several parameters. It is shown that the method is effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics.

Keywords: *enhanced (G'/G)-expansion method, cKG equation, solitary wave, traveling wave*

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1. Introduction

Nowadays NLEEs have been the subject of all-embracing studies in various branches of nonlinear sciences. A special class of analytical solutions named traveling wave solutions for NLEEs have a lot of importance, because most of the phenomena that arise in mathematical physics and engineering fields can be described by NLEEs. NLEEs are frequently used to describe many problems of protein chemistry, chemically reactive materials, in ecology most population models, in physics the heat flow and the wave propagation phenomena, quantum mechanics, fluid mechanics, plasma physics, propagation of shallow water waves, optical fibers, biology, solid state physics, chemical kinematics, geochemistry, meteorology, electricity etc. Therefore investigation traveling wave solutions is becoming more and more attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the Hirota's bilinear transformation method [1,2], the tanh-function method [3,4], the extended tanh-method [5,6], the Exp-function method [7-14], the Adomian decomposition method [15], the F-expansion method [16], the auxiliary equation method [17], the Jacobi elliptic function method [18], Modified Exp-function method [19], the (G'/G) expansion method [20-29], Weierstrass elliptic function method [30], the homotopy perturbation method [31,32,33,34,35], the homogeneous balance method [36, 37], the Modified simple equation method [38-43], He's polynomial [44], asymptotic methods and nanomechanics [45], the variational iteration method [46,47], casoration formulation [48], frobenius integrable decomposition [49], the extended multiple Riccati equations expansion method [50,51], Enhanced (G'/G)-expansion Method [52] and so on.

Among those approaches, an enhanced (G'/G) -expansion method is a tool to reveal the solitons and periodic wave solutions of NLEEs in mathematical physics and engineering. The main ideas of the enhanced (G'/G) -expansion method are that the traveling wave solutions of NLEEs can be expressed as the combination of rational and irrational functions of (G'/G) , where $G = G(\xi)$ satisfies the second order linear ordinary differential equation $G'' + \mu G = 0$.

The objective of this article is to apply the enhanced (G'/G) -expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the cKG equation. The cKG equation is completely integrable and has N-soliton solutions.

The article is prepared as follows: In section 2, an enhanced (G'/G) -expansion method is discussed. In section 3, we apply this method to the nonlinear evolution equations pointed out above ; in section 4, physical explanations and in section 5 conclusions are given.

2. Methodology

In this section we describe enhanced (G'/G) -expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear evolution equation, say in two independent variables x and t , is given by

$$R(u,u_t,u_x,u_{tt},u_{xx},u_{xt},...............)=0, \qquad (2.1)$$

where $u(\xi)=u(x,t)$ is an unknown function, R is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [52]:

Step 1. Combining the independent variables x and t into one variable ξ , we suppose that

$$u(\xi)=u(x,t), \quad \xi=x\pm\omega t, \qquad (2.2)$$

where $\omega\in\left(R-\{0\}\right)$.

The traveling wave transformation Eq. (2.2) permits us to reduce Eq. (2.1) to the following ODE:

$$R(u,u',u'',\ldots\ \ldots\ldots)=0. \qquad (2.3)$$

Where R is a polynomial in and its derivatives, while $u'(\xi)=\dfrac{du}{d\xi},u''(\xi)=\dfrac{d^2u}{d\xi^2}$ and so on.

Step 2.We suppose that Eq.(2.3) has the formal solution

$$u(\xi)=\sum_{i=-n}^{n}\left(\frac{a_i(G'/G)^i}{(1+\lambda(G'/G))^i}+b_i(G'/G)^{i-1}\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}\right) \qquad (2.4)$$

where $G=G(\xi)$ satisfy the equation

$$G''+\mu G=0, \qquad (2.5)$$

in which $a_i,b_i(-n\le i\le n;n\in N)$ and λ are constants to be determined later, and $\sigma=\pm1,\mu\ne0$.

Step 3. We determine the positive integer n in Eq. (2.4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (2.3).

Step 4. We substitute Eq. (2.4) into Eq.(2.3) using Eq. (2.5) and then collect all terms of same powers of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$ together, then set each coefficient of them to zero to yield a over-determined system of algebraic equations, solve this system for $a_i,b_i(-n\le i\le n;n\in N)$ and λ,ω .

Step 5. The solution of Eq.(2.5) can be written as follows:

When $\mu<0$, we get

$$\frac{G'}{G}=\sqrt{-\mu}\tanh\left(A+\sqrt{-\mu}\xi\right), \qquad (2.6)$$

and

$$\frac{G'}{G}=\sqrt{-\mu}\coth\left(A+\sqrt{-\mu}\xi\right), \qquad (2.7)$$

Again, when $\mu>0$, the solutions are

$$\frac{G'}{G}=\sqrt{\mu}\tan\left(A-\sqrt{\mu}\xi\right), \qquad (2.8)$$

and

$$\frac{G'}{G}=\sqrt{\mu}\cot\left(A+\sqrt{\mu}\xi\right) \qquad (2.9)$$

where A is an arbitrary constant. Finally, substituting $a_i,b_i(-n\le i\le n;n\in N),\lambda,\omega$ and Eqs.(2.6)-(2.9) into Eq. (2.4) we obtain traveling wave solutions of Eq. (2.1).

3. Application

In this section, we will exert enhanced (G'/G) -expansion method to solve the cKG equation in the form,

$$u_{xx}+u_{yy}-u_{tt}+\alpha u+\beta u^3=0, \qquad (3.1)$$

where α,β are positive constants.

The traveling wave transformation equation $u(\xi)=u(x,y,t),\xi=x+y-\omega t$ reduces Eq. (3.1) to the following ordinary differential equation:

$$(\omega^2-2)u''-\alpha u-\beta u^3=0. \qquad (3.2)$$

Now taking the homogeneous balance between the highest order derivative u'' and the nonlinear term u^3 from Eq.(3.2), we get $n=1$.

Hence for $n=1$ Eq. (2.4) reduces to

$$\begin{aligned}u(\xi)=a_0&+\frac{a_1(G'/G)}{1+\lambda(G'/G)}+\frac{a_{-1}\left[1+\lambda(G'/G)\right]}{(G'/G)}\\&+b_0(G'/G)^{-1}\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}\\&+b_1\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}\\&+b_{-1}(G'/G)^{-2}\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}\end{aligned} \qquad (3.3)$$

where $G=G(\xi)$ satisfies Eq. (2.5).

Substitute Eq. (3.3) along with Eq. (2.5) into Eq. (3.2). As a result of this substitution, we get a polynomial of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}$ From this polynomial, we equate the coefficients of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}$ and setting them to zero, we get a over-determined system that consists of twenty-five algebraic equations. Solving these over determined system of equations, we obtain the following valid sets.

set1:

$$\omega=\pm\sqrt{\left(2+\frac{\alpha}{2\mu}\right)},\lambda=0,a_1=\pm\sqrt{\frac{\alpha}{\beta\mu}},a_0=0,$$
$$a_{-1}=0,b_1=0,b_0=0,b_{-1}=0$$

Set2:

$$\omega=\pm\frac{1}{2}\sqrt{\left(\frac{8\mu-\alpha}{\mu}\right)},\lambda=0,a_1=\mp\sqrt{\frac{-\alpha}{2\mu\beta}},a_0=0,$$
$$a_{-1}=\pm\sqrt{\frac{-\mu\alpha}{2\beta}},b_1=0,b_0=0,b_{-1}=0$$

Set3:

$$\omega=\pm\frac{1}{4}\sqrt{\left(\frac{2(16\mu+\alpha)}{\mu}\right)},\lambda=0,a_1=\mp\frac{1}{2}\sqrt{\frac{\alpha}{\mu\beta}},$$
$$a_0=0,a_{-1}=\pm\frac{1}{2}\sqrt{\frac{\mu\alpha}{\beta}},b_1=0,b_0=0,b_{-1}=0$$

Set4:

$$\omega=\pm\sqrt{\left(\frac{2(\mu+\alpha)}{\mu}\right)},\lambda=0,a_1=\pm\sqrt{\frac{\alpha}{\mu\beta}},a_0=0,$$
$$a_{-1}=0,b_1=\pm\sqrt{\frac{\alpha}{\beta\sigma}},b_0=0,b_{-1}=0$$

Set5:

$$\omega=\pm\sqrt{\left(\frac{(4\mu+\alpha)}{2\mu}\right)},\lambda=\pm2\sqrt{\frac{\beta}{\alpha\mu}},a_1=\mp\frac{4\beta+\alpha}{\sqrt{\alpha\beta\mu}},$$
$$a_0=2,a_{-1}=0,b_1=0,b_0=0,b_{-1}=0$$

Set 6:

$$\omega=\pm\sqrt{\left(\frac{(4\mu+\alpha)}{2\mu}\right)},\lambda=\lambda,a_1=0,a_0=\mp\lambda\sqrt{\frac{\alpha\mu}{\beta}},$$
$$a_{-1}=\pm\sqrt{\frac{\alpha\mu}{\beta}},b_1=0,b_0=0,b_{-1}=0$$

Set7:

$$\omega=\pm\sqrt{\left(\frac{(2\mu-\alpha)}{\mu}\right)},\lambda=\lambda,a_1=0,a_0=0,a_{-1}=0,$$
$$b_1=\pm\sqrt{\frac{-2\alpha}{\beta\sigma}},b_0=0,b_{-1}=0$$

Set8:

$$\omega=\pm\sqrt{\left(\frac{(2\mu+\alpha)}{\mu}\right)},\lambda=\lambda,a_1=0,a_0=\mp\lambda\sqrt{\frac{\alpha\mu}{\beta}},$$
$$a_{-1}=\pm\sqrt{\frac{\alpha\mu}{\beta}},\ b_1=0,b_0=\pm\sqrt{\frac{\alpha\mu}{\beta\sigma}},b_{-1}=0$$

Now for $\mu<0$, substituting the values of $\omega,a_{-1},a_0,a_1,b_{-1},b_0,b_1$ into Eq. (3.3) from the above Set-1 to Set-8, we get the following hyperbolic function solutions of cKG equation.

Family -1:

$$u_1(\xi)=\pm\sqrt{\frac{-\alpha}{\beta}}\tanh(A+\sqrt{-\mu}\xi),$$

$$u_2(\xi)=\pm\sqrt{\frac{-\alpha}{\beta}}\coth(A+\sqrt{-\mu}\xi),$$

where

$$\xi=x+y\mp\sqrt{\left(2+\frac{\alpha}{2\mu}\right)}t.$$

Family-2:

$$u_3=\pm\sqrt{\frac{2\alpha}{\beta}}\ \operatorname{csc}h(2(A+\sqrt{-\mu}\xi)),$$

where

$$\xi=x+y\mp\frac{1}{2}\sqrt{\left(\frac{8\mu-\alpha}{\mu}\right)}t.$$

Family-3:

$$u_4=\mp\frac{1}{2}\sqrt{\frac{-\alpha}{\beta}}(\coth(A+\sqrt{-\mu}\xi)+\tanh(A+\sqrt{-\mu}\xi)),$$

where

$$\xi=x+y\mp\frac{1}{4}\sqrt{\left(\frac{2(16\mu+\alpha)}{\mu}\right)}t.$$

Family-4:

$$u_5=\pm\begin{pmatrix}\sqrt{\frac{-\alpha}{\beta}}\tanh(A+\sqrt{-\mu}\xi)\\+\sqrt{\frac{\alpha}{\beta}}\operatorname{sec}h(A+\sqrt{-\mu}\xi)\end{pmatrix},$$

$$u_6=\pm\sqrt{\frac{-\alpha}{\beta}}\begin{pmatrix}\coth(A+\sqrt{-\mu}\xi)\\+\operatorname{csc}h(A+\sqrt{-\mu}\xi)\end{pmatrix},$$

where

$$\xi=x+y\mp\sqrt{\left(\frac{2(\mu+\alpha)}{\mu}\right)}t.$$

Family-5:

$$u_7=2\mp\frac{\sqrt{\frac{\beta}{\alpha\mu}}(4\beta+\alpha)\sqrt{-\mu}\tanh(A+\sqrt{-\mu}\xi)}{\beta(1\pm2\sqrt{\frac{\beta}{\alpha\mu}}\sqrt{-\mu}\ \tanh(A+\sqrt{-\mu}\xi))},$$

$$u_8=2\mp\frac{\sqrt{\frac{\beta}{\alpha\mu}}(4\beta+\alpha)\sqrt{-\mu}\coth(A+\sqrt{-\mu}\xi)}{\beta(1\pm2\sqrt{\frac{\beta}{\alpha\mu}}\sqrt{-\mu}\ \coth(A+\sqrt{-\mu}\xi))},$$

where

$$\xi = x + y \mp \sqrt{\left(\frac{(4\mu+\alpha)}{2\mu}\right)}t.$$

Family-6:

$$u_9 = \mp\sqrt{\frac{-\alpha}{\beta}}\tanh(A+\sqrt{-\mu}\xi),$$

$$u_{10} = \mp\sqrt{\frac{-\alpha}{\beta}}\coth(A+\sqrt{-\mu}\xi),$$

where

$$\xi = x + y \mp \sqrt{\left(\frac{(4\mu+\alpha)}{2\mu}\right)}t.$$

Family-7:

$$u_{11} = \pm\sqrt{\frac{-2\alpha}{\beta}}\sec h(A+\sqrt{-\mu}\xi),$$

$$u_{12} = \pm\sqrt{\frac{2\alpha}{\beta}}\csc h(A+\sqrt{-\mu}\xi),$$

where

$$\xi = x + y \mp \sqrt{\left(\frac{(2\mu-\alpha)}{\mu}\right)}t.$$

Family-8:

$$u_{13} = \pm\sqrt{\frac{-\alpha}{\beta}}(1+\sec h(A+\sqrt{-\mu}\xi)) \times\coth(A+\sqrt{-\mu}\xi),$$

$$u_{14} = \pm\sqrt{\frac{-\alpha}{\beta}}(1+I\csc h(A+\sqrt{-\mu}\xi)) \times\tanh(A+\sqrt{-\mu}\xi),$$

where

$$\xi = x + y \mp \sqrt{\left(\frac{(2\mu+\alpha)}{\mu}\right)}t.$$

Similarly, for $\mu > 0$; we get the following periodic solutions of cKG equation.

Family -9:

$$u_{15}(\xi) = \pm\sqrt{\frac{\alpha}{\beta}}\tan(A-\sqrt{\mu}\xi),$$

$$u_{16}(\xi) = \pm\sqrt{\frac{\alpha}{\beta}}\cot(A+\sqrt{\mu}\xi),$$

where

$$\xi = x + y \mp \sqrt{\left(2+\frac{\alpha}{2\mu}\right)}t.$$

Family-10:

$$u_{17} = \mp\sqrt{\frac{-2\alpha}{\beta}}\csc(2(A-\sqrt{\mu}\xi)),$$

$$u_{18} = \mp\sqrt{\frac{-2\alpha}{\beta}}\csc(2(A+\sqrt{\mu}\xi)),$$

where

$$\xi = x + y \mp \frac{1}{2}\sqrt{\left(\frac{8\mu-\alpha}{\mu}\right)}t.$$

Family-11:

$$u_{19} = \mp\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}\left(\tan(A-\sqrt{\mu}\xi)-\cot(A-\sqrt{\mu}\xi)\right),$$

$$u_{20} = \mp\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}\left(\cot(A+\sqrt{\mu}\xi)-\tan(A+\sqrt{\mu}\xi)\right),$$

where

$$\xi = x + y \mp \frac{1}{4}\sqrt{\left(\frac{2(16\mu+\alpha)}{\mu}\right)}t.$$

Family-12:

$$u_{21} = \pm\sqrt{\frac{\alpha}{\beta}}\left(\tan(A-\sqrt{\mu}\xi)+\sec(A-\sqrt{\mu}\xi)\right),$$

$$u_{22} = \pm\sqrt{\frac{\alpha}{\beta}}\left(\cot(A+\sqrt{\mu}\xi)+\csc(A+\sqrt{\mu}\xi)\right),$$

where

$$\xi = x + y \mp \sqrt{\left(\frac{2(\mu+\alpha)}{\mu}\right)}t.$$

Family-13:

$$u_{23} = 2\mp\frac{\sqrt{\frac{\beta}{\alpha\mu}}(4\beta+\alpha)\sqrt{\mu}\tan(A-\sqrt{\mu}\xi)}{\beta(1\pm 2\sqrt{\frac{\beta}{\alpha\mu}}\sqrt{\mu}\tan(A-\sqrt{\mu}\xi))},$$

$$u_{24} = 2\mp\frac{\sqrt{\frac{\beta}{\alpha\mu}}(4\beta+\alpha)\sqrt{\mu}\cot(A+\sqrt{\mu}\xi)}{\beta(1\pm 2\sqrt{\frac{\beta}{\alpha\mu}}\sqrt{\mu}\cot(A+\sqrt{\mu}\xi))},$$

where

$$\xi = x + y \mp \sqrt{\left(\frac{(4\mu+\alpha)}{2\mu}\right)}t.$$

Family-14:

$$u_{25} = \pm\sqrt{\frac{\alpha}{\beta}}\cot(A-\sqrt{\mu}\xi),$$

$$u_{26} = \pm\sqrt{\frac{\alpha}{\beta}}\tan(A+\sqrt{\mu}\xi),$$

Where

$$\xi = x + y \mp \sqrt{\left(\frac{(4\mu+\alpha)}{2\mu}\right)}t.$$

Family-15:

$$u_{27} = \sqrt{\frac{-2\alpha}{\beta}}\sec(A - \sqrt{\mu}\xi),$$

$$u_{28} = \sqrt{\frac{-2\alpha}{\beta}}\csc(A + \sqrt{\mu}\xi)),$$

where

$$\xi = x + y \mp \sqrt{\left(\frac{(2\mu-\alpha)}{\mu}\right)}t.$$

Family-16:

$$u_{29} = \pm\sqrt{\frac{\alpha}{\beta}}(1+\sec(A-\sqrt{\mu}\xi))\cot(A-\sqrt{\mu}\xi),$$

$$u_{30} = \pm\sqrt{\frac{\alpha}{\beta}}(1+\csc(A-\sqrt{\mu}\xi))\tan(A-\sqrt{\mu}\xi),$$

where

$$\xi = x + y \mp \sqrt{\left(\frac{(2\mu+\alpha)}{\mu}\right)}t$$

4. Results and Discussion

In this section we will discuss the physical explanations of obtained solutions of cKG equation. It is interesting to point out that the delicate balance between the nonlinearity effect of u^3 and the dissipative effect of u_{xx}, u_{yy} and u_{tt} gives rise to solitons, that after a fully interaction with others, the solitons come back retaining their identities with the same speed and shape. The cKG equation has solitary wave solutions that have exponentially decaying wings. If two solitons of the cKG equation collide, the solitons just pass through each other and emerge unchanged.

We make graphs of obtained solutions, so that they can represent the importance of each obtained solution and physically interpret the consequence of parameters as well. Some of our obtained traveling wave solutions are represented in Figure 1-Figure 4 with the aid of Maple software:

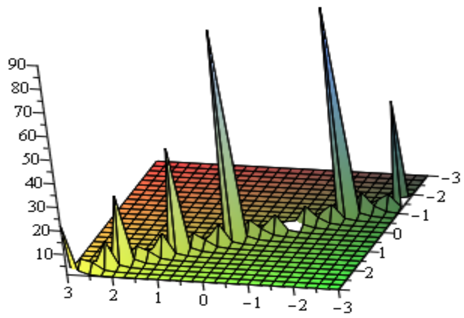

Figure 1. (Family 3) Shape of $\mu_4=(\xi)$ for μ=-1, α=2, β=1, A=1, y=0 in interval -3≤x,t≤3

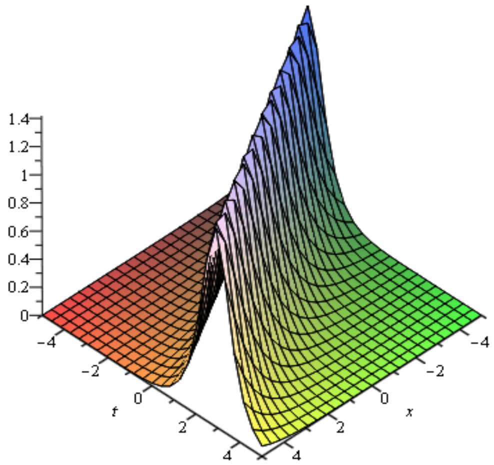

Figure 2. (Family 7) Shape of $\mu_{11}=(\xi)$ for μ=-1, α=1, β=1, A=0, y=0 in interval -5≤x,t≤5

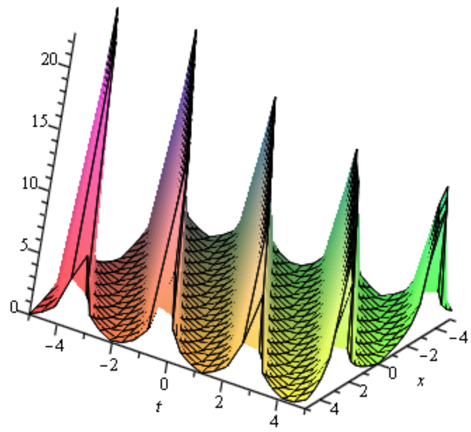

Figure 3. (Family 12) Shape of $\mu_{21}=(\xi)$ for μ=1, α=1, β=1, A=1, y=0 in interval -5≤x,t≤5

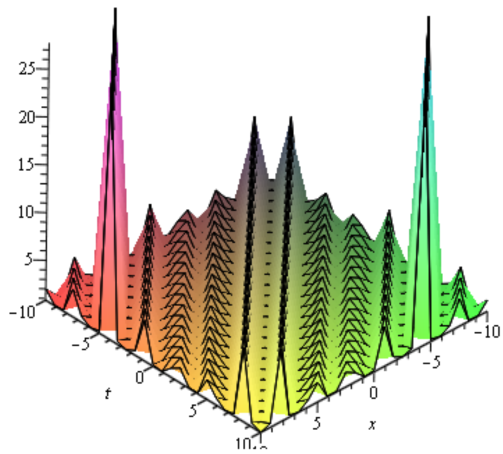

Figure 4. (Family 15) Shape of $\mu_{27}=(\xi)$ for μ=1, α=1, β=3, A=0, y=0 in interval -10≤x,t≤10

5. Conclusion

In short, we have illustrated the Enhanced (G'/G)-expansion method and utilized it to find the exact solutions of nonlinear equations with the help of Maple 13. We have successfully obtained some solitons, singular solitons and plane periodic solutions of the cKG equation involving parameters. When the parameters are taken as special values, the solitary wave solutions and the periodic wave solutions are obtained. Taken as a whole, it is worthwhile to mention that this method is effective for solving other nonlinear evolution equations in mathematical physics.

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