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# Auxiliary Function Method for Nonlinear Partial Differential Equations

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**Abstract** A new auxiliary function method is given, and some exact solutions of the auxiliary function are given too. As an example, Reaction-diffusion equation and BBM-Burgers equation are solved. Obviously, the auxiliary function method can be applied to solve other type of nonlinear partial differential equations as well.

***Keywords:*** *reaction-diffusion equation, BBM-Burgers equation, auxiliary function method, exact solutions*

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## 1. Introduction

In the recent years, seeking exact solutions of nonlinear partial differential equations (NLPDEs) is of great significance, since the nonlinear complex physical phenomena related to the NLPDEs are involved in many fields from physics, mechanics, biology, chemistry and engineering. As mathematical models of the phenomena, the investigation of exact solutions of NLPDEs will help one to understand the mechanism that governs these physical models or to better provide knowledge of the physical problem and possible applications. To this aim, a vast variety of powerful and direct methods for finding the exact significant solutions of NLPDEs though it is rather difficult have been derived. Some of the most important methods are Bäcklund transformation [1,2,3], Hirota's method [4], tanh-function method [5,6,7,8], extended tanh-function method [9,10,11,12], homogeneous balance method [13,14], variational iteration methods [15,16], collocation method [17,18,19], Adomian Padé approximation [20], Jacobi elliptic function expansion method [21], F-expansion method [22], auxiliary equation method [23,24,25], Fan sub-equation method [26,27,28,29,30], extended Fan sub-equation method [31,32], modified extended Fan sub-equation method [33,34,35] and so on. Recently, Sirendaoreji [36,37] proposed a new auxiliary equation method by introducing a new first order nonlinear ordinary differential equation (NLODE) with six-degree nonlinear terms and its solutions to construct exact travelling wave solutions of NLPDEs in a unified way. Later, Zhang and Xia [38] improved this method and obtained new formal solutions of some NLPDEs.

The aim of this letter is to give a new auxiliary function method. And using the auxiliary function method to solve a class of reaction-diffusion equation.

$$u_t - \alpha u_{xx} - \beta(u^3 + au^2 + bu) = 0, \tag{1}$$

and BBM-Burgers equation

$$u_t + u_x + uu_x - \delta u_{xx} - \lambda u_{xxt} = 0. \tag{2}$$

## 2. Auxiliary Function Method

To illustrate the basic concepts of a generalized auxiliary equation method, we consider a given PDE in two independent variables $(x,t)$ and dependent variable $u$:

**Step 1.** We use the wave transformation $u(x,t) = u(\xi), \xi = kx + wt$, and reduce the given NLPDE

$$H(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \cdots) = 0, \tag{3}$$

To the following ODE

$$H(u, u', u'', u''', u^{(4)}, \cdots) = 0, \tag{4}$$

**Step 2.** We seek for the solution of Eq. (4) in the following generalized ansätze

$$u(\xi) = \sum_{i=1}^{n} a_i F^i(\xi) + a_0, \tag{5}$$

with $F(\xi)$ satisfying the following new auxiliary equation

$$F'(\xi) = pF^3(\xi) + rF(\xi), \tag{6}$$

where $F' = \frac{dF}{d\xi}$, the positive integer $n$ can be determined by balancing the highest-order derivative term with the nonlinear terms [37], $a_i (i = 0,1,\cdots,n)$, $k$ and $w$ are constants to be determined and $F(\xi)$ satisfies the variable separated ODE (6), where $p$ and $r$ are real constants. We have found that the auxiliary equation (6) possesses several types of following solutions

$$F_1=[-\frac{r}{2p}-\frac{r}{2p}\tanh(r\xi)]^{\frac{1}{2}},$$

$$F_2=[-\frac{r}{2p}+\frac{r}{2p}\coth(r\xi)]^{\frac{1}{2}},$$

$$F_3=[\frac{ra+ra\tanh(r\xi)}{-(2pa+rb)+rb\tanh(r\xi)}]^{\frac{1}{2}},$$

$$F_4=[\frac{-ra\pm ra\tanh(\pm 2r\xi)}{pa-pa\tanh(\pm 2r\xi)+pb\sec h(\pm 2r\xi)}]^{\frac{1}{2}},$$

$$F_5=[\frac{\mp rab+rab\tanh(\pm 2r\xi)+rb^2\sec h(\pm 2r\xi)}{\pm 2pab-p(a^2+b^2)\sec h(\pm 2r\xi)}]^{\frac{1}{2}},$$

$$F_6=[\frac{r+2r\tanh(r\xi/2)+r\tanh^2(r\xi/2)}{-4p\tanh(r\xi/2)}]^{\frac{1}{2}},$$

$$F_7=[\frac{rb+r(a+b)\tanh(r\xi)+rb\tanh^2(r\xi)}{-2pa-2pb\tanh(r\xi)}]^{\frac{1}{2}},$$

$$F_8=[\frac{-br\csc h(\pm 2r\xi)}{\mp pa+pa\coth(\pm 2r\xi)+pb\csc h(\pm 2r\xi)}]^{\frac{1}{2}},$$

$$F_9=[\frac{\pm ra^2+ra^2\coth(\pm 2r\xi)+rab\csc h(\pm 2r\xi)}{\mp p(a^2+b^2)-p(a^2-b^2)\coth(\pm 2r\xi)}]^{\frac{1}{2}},$$

$$F_{10}=\left[\frac{\begin{array}{l}(3\alpha^2r^2+\alpha^2-r^3)\\(\alpha+r\tanh(\alpha\xi)+\alpha\tanh^2(\alpha\xi))\end{array}}{\begin{array}{l}\alpha p(r^2-4\alpha^2)\\+p(2\alpha^2r+2\alpha^2-r^3)\tanh(\alpha\xi)\\+\alpha p(r^2-4\alpha^2)\tanh^2(\alpha\xi)\end{array}}\right]^{\frac{1}{2}}$$

$$F_{11}=\left[\frac{\begin{array}{l}rab+rab\tanh(\pm 2r\xi)\\+rb^2\sec h(\pm 2r\xi)\end{array}}{\begin{array}{l}-b(2pa+cr)+rbc\tanh(\pm 2r\xi)\\-(pa^2+rac+pb^2)\sec h(\pm 2r\xi)\end{array}}\right]^{\frac{1}{2}},$$

$$F_{12}=\left[\frac{\pm ra^2+ra^2\coth(\pm 2r\xi)+rab\csc h(\pm 2r\xi)}{\begin{array}{l}\mp(pa^2+rbc+pb^2)\\-(pa^2-rbc-pb^2)\coth(\pm 2r\xi)+rac\csc h(\pm 2r\xi)\end{array}}\right]^{\frac{1}{2}},$$

$$F_{13}=\left[\frac{\begin{array}{l}r(3\alpha^2-4r^2)(2r+\alpha\coth(\alpha\xi)\\+\alpha\csc h(\alpha\xi))\end{array}}{\begin{array}{l}-2pr(5\alpha^2-4r^2)-p\alpha(\alpha^2-4r^2)\coth(\alpha\xi)\\+p\alpha(\alpha^2-4r^2)\csc h(\alpha\xi)\end{array}}\right]^{\frac{1}{2}},$$

$$F_{14}=\left[\frac{\begin{array}{l}ra(1+3\tanh(r\xi/3)+3\tanh^2(r\xi/3)\\+\tanh^3(r\xi/3))\end{array}}{\begin{array}{l}rb-3(2pa+rb)\tanh(r\xi/3)\\+3rb\tanh^2(r\xi/3)-(2pa+rb)\tanh^3(r\xi/3))\end{array}}\right]^{\frac{1}{2}},$$

$$F_{15}=\left[\frac{\begin{array}{l}ra^2+rab\tanh(r\xi/2)\\+ra(2b-3a)\tanh^2(r\xi/2)\\+ra(b-2a)\tanh^3(r\xi/2)\end{array}}{\begin{array}{l}rac-(4rac-rbc+4pa^2)\tanh(r\xi/2)\\+(8pa^2+5rac-2rbc-4pab)\tanh^2(r\xi/2)\\+rc(b-2a)\tanh^3(r\xi/2)\end{array}}\right]^{\frac{1}{2}},$$

$$F_{16}=\left[\frac{\begin{array}{l}ra+ra\tanh(r\xi)\\+ra\tanh^2(r\xi)+ra\tanh^3(r\xi)\end{array}}{\begin{array}{l}rb-(2pa-rb+2rb)\tanh(r\xi)\\+rb\tanh^2(r\xi)-(2pa+rb)\tanh^3(r\xi)\end{array}}\right]^{\frac{1}{2}},$$

$$F_{17}=\left[\frac{(3\alpha^2r+\alpha^2+r^3)(\alpha+r\tan(\alpha\xi)-\alpha\tan^2(\alpha\xi))}{\begin{array}{l}-\alpha p(r^2+4\alpha^2)-(2\alpha^2r+2\alpha^2+r^3)\tan(\alpha\xi)\\+\alpha p(r^2+4\alpha^2)\tan^2(\alpha\xi)\end{array}}\right]^{\frac{1}{2}} \quad (7)$$

where $a,b,c$ and $\alpha$ are undermined constants.

**Step 3.** Substitute ansatz (5) along with Eq. (6) into (4) and equate the coefficients of all powers of $F(\xi)$ to zero yields a set of algebraic equations for unknowns $a_i(i=0,1,\cdots,n), p,r,k$ and $w$.

**Step 4.** Solve the set of algebraic equations by use of MAPLE can permit obtention of explicit expressions of $a_i(i=0,1,\cdots,n), p,r,k$ and $w$.

**Step 5.** Obtain exact solutions. By using the results obtained in the above steps, we can derive a series of travelling wave solutions of Eq. (3) depending on the solution $F(\xi)$ of Eq. (6). Selecting appropriate $F_i(\xi)$ $(i=1,2,\cdots,17)$ and substituting it into the travelling wave solutions Eq. (5), we can obtain exact solutions of Eq. (3).

# 3. Application

As an application, we use the new auxiliary function method to solve a class of reaction-diffusion equation and BBM-Burgers equation.

## 3. 1. Reaction-diffusion Equation

We make transformation $u(x,t)=u(\xi)=u(kx+wt)$ and change Eq. (1) into the form

$$wu'-\alpha k^2u''-\beta u^3-a\beta u^2-b\beta u=0, \quad (8)$$

Now balancing $u''$ and $u^3$, we obtain $n=2$. This suggests that

$$u(\xi)=a_0+a_1F(\xi)+a_2F^2(\xi), \tag{9}$$

where $a_0, a_1$ and $a_2$ are constants to be determined. Substitution of (9) into (8), we have:

$$w\begin{bmatrix} a_1F'(\xi)+2a_2F(\xi)F'(\xi)] \\ -ak^2[F''(\xi)+2a_2F'(\xi)F'(\xi) \\ +2a_2F(\xi)F''(\xi)] \\ -\beta[a_0+a_1F(\xi)+a_2F^2(\xi)]^3 \\ -a\beta[a_0+a_1F(\xi)+a_2F^2(\xi)]^2 \\ -b\beta[a_0+a_1F(\xi)+a_2F^2(\xi) \end{bmatrix}=0, \tag{10}$$

Substitution of (6) into (10), and equate the coefficients of all powers of $F(\xi)$ to zero, we get a set of algebraic equation as following:

$$-8\alpha a_2k^2p^2-\beta a_2^3=0,$$

$$-3\alpha a_1k^2p^2-3\beta a_1a_2^2=0,$$

$$-12\alpha a_2k^2pr-3\beta(a_0a_2^2+a_1^2a_2)$$
$$-\beta aa_2^2+2a_2wp=0,$$

$$-4\alpha a_1k^2pr-\beta(a_1^3+6a_0a_1a_2)-2\beta aa_1a_2+awp=0,$$

$$-4\alpha a_2k^2r^2-3\beta(a_0a_1^2+a_0^2a_2)$$
$$-\beta a(a_1^2+2a_0a_2)-\beta ba_2+2a_2wr=0,$$

$$-\alpha a_1k^2r^2-3\beta a_0^2a_1-2\beta aa_0a_1-\beta ba_1+a_1wr=0,$$

$$-\beta a_0^3-\beta aa_0^2-\beta ba_0=0. \tag{11}$$

Solving the resulting algebraic system (11), we get the two group solutions

**Case 1.**

$$a_0=-\frac{1}{2}a+\frac{1}{2}\sqrt{a^2-4b}, a_1=0,$$

$$a_2=\frac{p(3a^2-12b-a\sqrt{a^2-4b})}{r(-a+3\sqrt{a^2-4b})},$$

$$k=\pm\frac{\sqrt{-2\alpha\beta(a^2-4b)}}{4r\alpha},$$

$$w=-\frac{\beta a\sqrt{a^2-4b}}{4r}. \tag{12}$$

Substituting (12) and (7) into (9), we get the solutions of Eq. (1)

$$u_{1,i}(\xi)=-\frac{1}{2}a+\frac{1}{2}\sqrt{a^2-4b}$$
$$+\frac{p(3a^2-12b-a\sqrt{a^2-4b})}{r(-a+3\sqrt{a^2-4b})}F_i^2(\xi) \tag{13}$$
$$(i=1,2,\cdots,17).$$

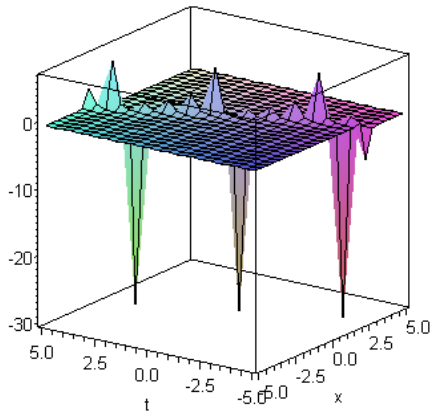


**Figure 1.** $u_{1,8}$ as $a=2, b=0.9, p=1.6, r=1, k=1.8, w=0.8$

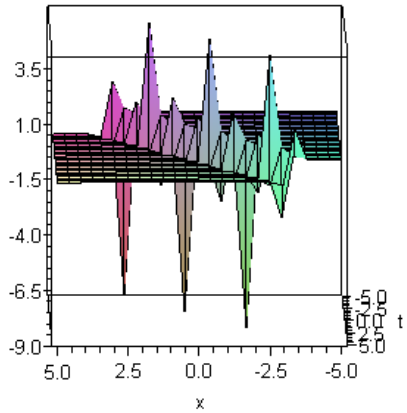


**Figure 2.** $u_{1,9}$ as $a=2, b=0.9, p=1.4, \ r=2, k=1.6, w=1$

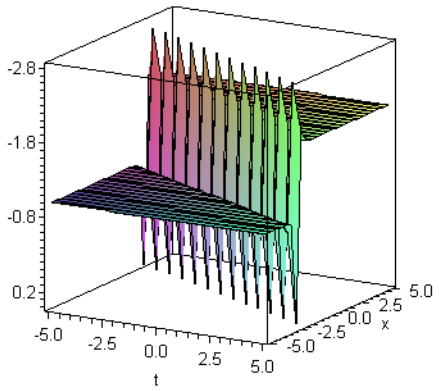


**Figure 3.** $u_{1,12}$ as $a=2.2, b=1.2, p=1.6$ , $r=2, k=1.6$ , $c=2, w=0.8$

**Case 2.**

$$a_0=-\frac{1}{2}a-\frac{1}{2}\sqrt{a^2-4b}, a_1=0,$$

$$a_2=\frac{p(3a^2-12b+a\sqrt{a^2-4b})}{r(-a-3\sqrt{a^2-4b})},$$

$$k=\pm\frac{\sqrt{-2\alpha\beta(a^2-4b)}}{4r\alpha}, \tag{14}$$

$$w=\frac{\beta a\sqrt{a^2-4b}}{4r}.$$

Substituting (14) and (7) into (9), we get the solutions of Eq. (1)

$$u_{2,i}(\xi) = -\frac{1}{2}a - \frac{1}{2}\sqrt{a^2-4b} + \frac{p(3a^2-12b+a\sqrt{a^2-4b})}{r(-a-3\sqrt{a^2-4b})}F_i^2(\xi) \quad (15)$$
$$(i = 1, 2, \cdots, 17).$$

## 3.2. BBM-Burgers Equation

We make transformation $u(x,t) = u(\xi) = u(kx+wt)$ and change Eq. (2) into the form

$$(w+k)u - \frac{1}{2}ku^2 - \delta k^2 u' - \lambda k^2 w u'' = 0, \quad (16)$$

Now balancing $u''$ and $u^3$, we obtain $n = 4$. This suggests that

$$u(\xi) = a_0 + a_1F(\xi) + a_2F^2(\xi) + a_3F^3(\xi) + a_4F^4(\xi), \quad (17)$$

where $a_0, a_1, a_2, a_3$ and $a_4$ are constants to be determined. Substitution of (17) into (16), we have:

$$\begin{aligned}
&(w+k)\begin{bmatrix} a_0 + a_1F(\xi) + a_2F^2(\xi) \\ +a_3F^3(\xi) + a_4F^4(\xi) \end{bmatrix} \\
&-\frac{1}{2}k\begin{bmatrix} a_0 + a_1F(\xi) + a_2F^2(\xi) \\ +a_3F^3(\xi) + a_4F^4(\xi) \end{bmatrix}^2 \\
&-\delta k^2\begin{bmatrix} a_1F'(\xi) + 2a_2F(\xi)F'(\xi) \\ +3a_3F^2(\xi)F'(\xi) + 4a_4F^3(\xi)F'(\xi) \end{bmatrix} \\
&-\lambda k^2 w\begin{bmatrix} a_1F''(\xi) + 2a_2F(\xi)F''(\xi) \\ +3a_3F^2(\xi)F''(\xi) + 4a_4F^3(\xi)F''(\xi) \\ +2a_2F'(\xi)F'(\xi) + 6a_3F(\xi)F'(\xi)F'(\xi) \\ +12a_4F^2(\xi)F'(\xi)F'(\xi) \end{bmatrix} = 0,
\end{aligned} \quad (18)$$

Substitution of (6) into (18), and equate the coefficients of all powers of $F(\xi)$ to zero, we get a set of algebraic equation, and solving the algebraic system, we get the four group solutions

**Case 1.**

$$a_0 = 0, a_1 = 0, a_2 = \frac{2p(5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})}{5\lambda r}, a_3 = 0,$$

$$a_4 = \frac{2p^2[12\lambda\delta^2 + (6\delta^2 - 5\lambda)(5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})]}{5\lambda r^2[12\delta^2 - (5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})]},$$

$$k = \frac{5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2}}{24\lambda\delta r},$$

$$w = \frac{\delta(5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})(-7\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})}{20\lambda^2 r[12\delta^2 - (5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})]} \quad (19)$$

Substituting (19) and (7) into (17), we get the solutions of Eq. (2)

$$u_{1,i}(\xi) = \frac{2p}{5\lambda r}(5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})F_i^2 + \frac{2p^2[12\lambda\delta^2 + (6\delta^2 - 5\lambda)(5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})]}{5\lambda r^2[12\delta^2 - (5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})]}F_i^4 \quad (20)$$
$$(i = 1, 2, \cdots, 17).$$

**Case 2.**

$$a_0 = \frac{-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2}}{5\lambda}, a_1 = 0,$$

$$a_2 = \frac{2p(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})}{5\lambda r}, a_3 = 0,$$

$$a_4 = \frac{2p^2}{5\delta r^2}[\delta + \frac{5}{12\delta}(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})] (\frac{5}{2} \pm \frac{1}{2\lambda}\sqrt{25\lambda^2 + 24\lambda\delta^2}),$$

$$k = \frac{-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2}}{24\lambda\delta r},$$

$$w = -\frac{(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})}{120\lambda\delta r} [\frac{5}{2} \pm \frac{1}{2\lambda}\sqrt{25\lambda^2 + 24\lambda\delta^2})]. \quad (21)$$

Substituting (21) and (7) into (17), we get the solutions of Eq. (2)

$$u_{2,i}(\xi) = a_0 + a_2F_i^2(\xi) + a_4F_i^4(\xi)(i = 1, 2, \cdots, 17). \quad (22)$$

**Case 3.**

$$a_0 = \frac{2\delta^2[12\delta - 5(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})]}{5\lambda\delta(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})},$$

$$a_1 = 0, a_2 = 0, a_3 = 0,$$

$$a_4 = \frac{p^2(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})}{5\lambda r^2},$$

$$k = -\frac{5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2}}{24\lambda\delta r},$$

$$w = -\frac{\delta}{10\lambda r}. \quad (23)$$

Substituting (23) and (7) into (17), we get the solutions of Eq. (2)

$$u_{3,i}(\xi) = \frac{2\delta^2[12\delta - 5(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})]}{5\lambda\delta(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})} + \frac{p^2(-5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})}{5\lambda r^2}F_i^4(\xi) \quad (24)$$
$$(i = 1, 2, \cdots, 17).$$

**Case 4.**

$$a_0 = 0, a_1 = 0, a_2 = 0, a_3 = 0,$$

$$a_4 = -\frac{p^2(5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})}{5\lambda r^2},$$

$$k = \frac{5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2}}{24\lambda\delta r},$$

$$w = -\frac{\delta}{10\lambda r}. \tag{25}$$

Substituting (25) and (7) into (17), we get the solutions of Eq. (2)

$$u_{4,i}(\xi) = -\frac{p^2(5\lambda \pm \sqrt{25\lambda^2 + 24\lambda\delta^2})}{5\lambda r^2} F_i^4 (i = 1, 2, \cdots, 17). \tag{26}$$

# 4. Conclusion

In this letter, a new auxiliary function method is given, and some exact solutions of the auxiliary function are given too. Reaction-diffusion equation and BBM-Burgers equation are solved by using the new auxiliary function method. Obviously, the auxiliary function method can be applied to solve other type of nonlinear partial differential equations as well.

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