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Gravity: WEP, Gauge Theory, Quantization, Unification

Hui Peng*

*Corresponding author: davidpeng1749@gmail.com

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Abstract A gauge theory of gravity with an internal symmetry U(1), denoted as Gravito-dynamics, is established, which is dual to the Electrodynamics and complies with Special Relativity. The Gravito-dynamics is quantized and renormalized, denoted as QGD. The Gravito-dynamics is unified with Electrodynamics at classical level, and QGD is unified with QED at quantum level, denoted as Electro-gravity interaction. Following the line of generalizing the U(1) Electrodynamics to Yang-Mills theory, we generalize the U(1) gravity to SU(2) gravity that indicates short-range gravity. Two thought-experiments are proposed to test the underlying physics of the U(1) gravity and to detect the particle nature of gravitational wave that leads to wave-particle duality of gravitational radiation.

Keywords: *gravity, WEP, gauge theory, quantization, renormalization, unification, duality, short-range gravity*

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1. Introduction

To understand and describe all of four forces in one framework is the goal. Two paths of studying gravity are: First path is to treat gravity and other three forces as physical fields; Second path is to treat gravity and other forces as geometric phenomena. Along the first path, other three forces, except gravity, were physically understood, quantized and unified. Along the second path, to geometrize gravity was quite successful. Tremendous efforts have been devoted on establishing geometric theories of other three forces, quantizing geometric theories of all four forces, and unifying four forces in geometric theories.

Facing the difficulties in quantizing geometric theory of gravity and in unifying four forces, we re-take the first path: propose a gauge theory of gravity (denoted as *Gravito-dynamics*), quantize it (denoted as *QGD*) and unify it with Electrodynamics force (denoted as *Electro-gravity force*). The achievements of the gauge theory of gravity of this article are:

1. Gravitational wave is quantized; a thought experiment to test particle nature of gravitational radiation is proposed;
2. Quantization and renormalization of Gravito-dynamics (QGD) are fulfilled;
3. QGD justifies the existence of negative g-charge;
4. Gravity and Electrodynamics are unified at classical and quantum levels in the framework of gauge theory;
5. QGD shows that gravity affects the rest mass at quantum level, which contains that predicted by Einstein at classical level [1,2];
6. Spacetime and internal space of U(1) gravity are dual to each other;
7. Generalize U(1) gravity to SU(2) gravity; gauge bosons of SU(2) automatically carry g-charge (mass), which indicates a short-range gravity;
8. A thought-experiment is proposed to test the underlying physics of the U(1) symmetry of gravity

Therefore, we argue that the gauge theory of gravity of this article provides, at least, a "bridge" between the Newton's theory and an ultimate theory of gravity. Namely the gauge theory of gravity is an approximation of the ultimate theory of Gravity.

2. Internal Symmetry of Gravity

The goal of this article is to describe gravity in terms of physical field. In order to establish a Lorentz-invariant gauge theory of gravity, we need to discover internal symmetry that associates with gravity.

2.1. Physical Origin of Internal Symmetry of Gravity: Original WEP

The Weak Equivalence Principle (WEP) states that the gravitational mass is equal to the inertial mass,

$$m_g = m_{inertial} \; .$$

Where m_g and $m_{inertial}$ are the gravitational mass and inertial mass respectively. Before 1905, the inertial mass is the rest mass. In 1905, Special Relativity (SR) was established and shown that

$$m_{inertial} = \gamma m_0 .$$

Where m_0 is the inertial rest mass, γ is the Lorentz factor.

Taking into account SR, now we have two definitions of WEP:

1. The original definition of WEP (denoted as *Original-WEP*) is,

$$m_g = m_0, \quad (2.1)$$

which leads to 4-current of g-charge and gauge theory of gravity.

2. Second definition of WEP (denoted as *S-WEP*) is,

$$m_g = m_{inertial} = \gamma m_0, \quad (2.2)$$

which leads to energy-momentum tensor as source.

We argue that inertial mass should be conceptually distinct from gravitational mass; the former is defined as mass, the latter as g-charge (Section 2.2). Both active and passive gravitational masses are g-charges and equivalent.

Up to now, all of experiments done have tested and confirmed Original-WEP [3,4,5,6,7,8]. There is no experiment testing S-WEP yet [9] (Section 7.1). Thus, in this article, we adopt the Original-WEP that is the low-speed approximation of S-WEP. We expect that the theory established based on Original-WEP is, at least, the approximation of theories established based on S-WEP.

2.2. $U(1)_g$ Internal Symmetry of Gravity

Let's start with the non-perfect duality between Coulomb's law and Newton's law, which describe static electric force and Newton's force, respectively,

$$F_e = \frac{kq_{e1}q_{e2}}{r^2}$$

$$F_g = \frac{GM_{01}M_{02}}{r^2}$$

where, according to Original-WEP, M_{01} and M_{02} are rest mass of objects 1 and object 2. In SI unit, the electric charge q_e has the unit $[q_e]$ = [Coulomb], "k" is Coulomb's constant and has the unit [k] = [N m^2 C^{-2}], "G" is Newton constant and has unit [G] = [N m^2 kg^{-2}], "N" is the unit "Newton", "m" is the unit "meter", M_{01} has the unit $[M_0]$ =[kg].

Convert to Heaviside–Lorentz units (HLU) system. The Coulomb's law becomes

$$F_e = \frac{Q_{e1}Q_{e2}}{r^2} \quad (2.3)$$

Where the electric charge (e-charge), $Q_e \equiv \sqrt{k}q_e$, has the unit

$$unit\left[Q_e\right] = \sqrt{N}m. \quad (2.4)$$

For keeping the duality, writing the Newton's law in the same form,

$$F_g = \frac{Q_{g1}Q_{g2}}{r^2}. \quad (2.5)$$

Following the e-charge conjugation, let's introduce the gravitational charges (g-charge) as:

$$Q_{g+} \equiv +\sqrt{G}M_0, \quad (2.6)$$

$$Q_{g-} \equiv -\sqrt{G}M_0. \quad (2.7)$$

We always have positive rest mass $M_0 > 0$, and M_0 is conserved. Thus, g-charge is conserved. There are two situations: (1) if an elementary particle carries a positive g-charge Q_{g+}, then its antiparticle carries a negative g-charge Q_{g-}; (2) if an elementary particle carries a negative g-charge Q_{g-}, then its antiparticle carries a positive g-charge Q_{g+}. The negative g-charge will be justified in quantum gravity in Section 4.6.

The g-charge has the unit,

$$unit\left[Q_g\right] = \sqrt{N}m. \quad (2.8)$$

Thus, in HLU system, e-charge Q_e and g-charge Q_g have the same unit. We, now, have a perfect duality between the Coulomb's and Newton's Laws.

The quanta of a long-range physical field must have zero rest mass. Since Newton's time, gravity had been considered as a long-range force. Namely if gravity is a physical field, then the quanta of gravitational field must carry zero rest mass, namely carry zero g-charge. Therefore, a physical theory for long-range gravity must be an Abelian gauge theory.

As the conservation of electric charge leads to the $U(1)_e$ internal symmetry of electrodynamics, the conservation of g-charge leads to the $U(1)_g$ internal symmetry of gravity,

$$U(1)_e : \exp\left(\frac{-iQ_e\theta_e}{\hbar}\right), \quad (2.9)$$

$$U(1)_g : \exp\left(\frac{-iQ_g\theta_g}{\hbar}\right). \quad (2.10)$$

Let's compare the units of both internal spaces. The requirement that $\frac{Q_e\theta_e}{\hbar}$ and $\frac{Q_g\theta_g}{\hbar}$ must be dimensionless leads to the unit of both internal spaces,

$$unit\left[\theta_e\right] = s\sqrt{N}, \quad (2.11)$$

$$unit\left[\theta_g\right] = s\sqrt{N}, \quad (2.12)$$

i.e., the internal spaces, θ_e and θ_g, for electrodynamics and gravity, have the same unite. In other word, since the e-charge and g-charge have the same unit, the internal spaces of electrodynamics and gravity must have the same unit. Thus, we argue that the internal space θ_e of electrodynamics is the dual of the internal space θ_g of gravity,

$$\theta_g \leftrightarrow \theta_e, \quad (2.13)$$

and the internal group $U(1)_e$ of electrodynamics is the dual of the internal group $U(1)_g$ of gravity, and vice versa,

$$\exp\left(\frac{-iQ_g\theta_g}{\hbar}\right) \leftrightarrow \exp\left(\frac{-iQ_e\theta_e}{\hbar}\right). \quad (2.14)$$

Namely, under the e-charge/g-charge conversion

$$Q_g \leftrightarrow Q_e, \quad (2.15)$$

the internal symmetry groups of electrodynamics and gravity converts to each other.

2.3. Conversion between $U(1)_g$ Internal Space and Spacetime

As a consequence of $U(1)_g$ of gravity, we discover a relation between internal symmetry group and spacetime symmetry group, i.e., one can be converted to other [10]. For showing this, one way is to compare the units of internal space with that of spacetime.

Now let's study whether there is a relation between the gravity internal space θ_g with unit "$s\sqrt{N}$", and spacetime x_μ with unit of meter "m". For this aim, we consider the spacetime translation group T^4:

$$T^4 : \exp\left(\frac{-ix_\mu p^\mu}{\hbar}\right) \tag{2.16}$$

Where x_μ is spacetime coordinate, p^μ is the four-momentum of an object,

$$p^\mu = MU^\mu = \gamma M(c, v_x, v_y, v_z),$$

where M is the rest mass of the object, U^μ is 4-velocity.

For studying the relation between gravity internal space and spacetime, we suggest to convert the rest mass M in T^4 to g-charge $Q_g = \sqrt{G}M$. We have,

$$x_\mu p^\mu = x_\mu (MU^\mu) = \frac{x_\mu(\sqrt{G}MU^\mu)}{\sqrt{G}} = Q_g\left[\frac{x_\mu U^\mu}{\sqrt{G}}\right]$$

which suggest us to define a new "coordinate x'",

$$x' \equiv \frac{x_\mu U^\mu}{\sqrt{G}} = \left(\frac{U^\mu}{\sqrt{G}}\right)x_\mu, \tag{2.17}$$

which has the unit,

$$\textit{unit}\left[x'\right] = s\sqrt{N}, \tag{2.18}$$

which is the same as that of the internal space θ_g of gravity.

The spacetime translation group T^4 becomes,

$$T^4 : \exp\left(\frac{-ix_\mu p^\mu}{\hbar}\right) \equiv \exp\left(\frac{-iQ_g x'}{\hbar}\right). \tag{2.19}$$

Comparing T^4 symmetry in terms of Q_g with $U(1)_g$ group,

$$U(1)_g : \exp\left(\frac{-iQ_g\theta_g}{\hbar}\right)$$

we obtain the duality between spacetime translation symmetry group and internal symmetry group. Namely, by converting the rest mass to g-charge, and converting x' to θ_g, the spacetime translation symmetry T^4 is converted to the internal $U(1)_g$ symmetry of gravity, and vice versa,

$$\exp\left(\frac{-ix_\mu p^\mu}{\hbar}\right) \leftrightarrow \exp\left(\frac{-iQ_g x'}{\hbar}\right) \leftrightarrow \exp\left(\frac{-iQ_g\theta_g}{\hbar}\right). \tag{2.20}$$

The internal space θ_g is dual to x' that is the function of spacetime x_μ, thus, θ_g is the function of spacetime x_μ,

$$\theta_g \leftrightarrow x' = \left(\frac{U^\mu}{\sqrt{G}}\right)x_\mu. \tag{2.21}$$

Moreover, since the internal space θ_e of $U(1)_e$ of electrodynamics is dual of the internal space θ_g of $U(1)_g$ of gravity, the internal symmetry group $U(1)_e$ is dual to the spacetime translation symmetry T^4,

$$\exp\left(\frac{-iQ_e\theta_e}{\hbar}\right) \leftrightarrow \exp\left(\frac{-iQ_g\theta_g}{\hbar}\right) \leftrightarrow \exp\left(\frac{-iQ_g x'}{\hbar}\right). \tag{2.22}$$

And the internal space θ_e is the function of spacetime x_μ,

$$\theta_e \leftrightarrow x' = \left(\frac{U^\mu}{\sqrt{G}}\right)x_\mu, \tag{2.23}$$

We conclude that, under the conversion between invariant mass M and g-charge Q_g, the following corresponding variables convert to each other, denoted as *Gravity Internal Space/Spacetime duality*:

1. Spacetime translation group T^4 is dual to the gravity internal group $U(1)_g$, and thus, is dual to the Electrodynamics internal group $U(1)_e$;
2. 4-dimension spacetime x_μ corresponds to x' that is dual to one-dimension Gravity internal space θ_g^a of $U(1)_g$ and thus, is dual to Electrodynamics internal space θ_e^a of $U(1)_e$. Therefore, internal space θ_g^a of $U(1)_g$ and internal space θ_e^a of $U(1)_e$ are function of spacetime, which implies that the $U(1)_g$ is a local internal symmetry group;
3. The g-charge (rest mass) conservation represented by $U(1)_g$ corresponds to the energy conservation and momentum conservation represented by T^4; mass conservation, energy conservation and momentum conservation have equal footing in classical mechanics.

3. $U(1)_g$ Gauge Theory of Gravity: Gravito-dynamics

The symmetry dictates interaction. Let's derive the $U(1)_g$ gauge theory of gravity by following the same procedure used for constructing an Abelian gauge theory. Due to the duality, following the Lagrangian density L_e of Electrodynamics, $L_e = -\frac{1}{4}F_e^{\mu\nu}F_{e\mu\nu} - A_{e\mu}J_e^\mu$, we suggest the gravitational Lagrangian density L_g as,

$$L_g = -\frac{1}{4}F_g^{\mu\nu}F_{g\mu\nu} + A_{g\mu}J_g^\mu. \tag{3.1}$$

The "$-$" sign in the front of the term "$A_{e\mu}J_e^\mu$" changes to "+" in the front of the term "$A_{g\mu}J_g^\mu$", because of that the Newton's law has a negative sign in the front of the source term. Where gravitational field strength is

$$F_g^{\mu\nu} = \partial^\mu A_g^\nu - \partial^\nu A_g^\mu, \quad (3.2)$$

the A_g^μ $(\phi_g, \mathbf{A}_g)$ is the gravitational 4-potential, $J_g^\mu\left(\rho_g, \mathbf{J}_g\right)$, is the gravitational 4-current. Note the gravitational 4-current represents either 4-current of positive g-changer, $J_{g+}^\mu\left(\rho_{g+}, \mathbf{J}_{g+}\right)$, or 4-current of negative g-charge, $J_{g-}^\mu\left(\rho_{g-}, \mathbf{J}_{g-}\right)$, or net 4-current of the combination of positive and negative g-charge, $J_{g/net}^\mu = J_{g+}^\mu + J_{g-}^\mu$.

Substituting Eq. (3.1) into the Euler-Lagrange equation, $\partial_\mu \frac{\partial L_g}{\partial\left(\partial_\mu A_{g\nu}\right)} = \frac{\partial L_g}{\partial A_{g\nu}}$, and the Bianchi-type identities of $F_g^{\mu\nu}$, we obtain Maxwell-type gravitational gauge field equation,

$$\left.\begin{aligned} \frac{\partial F_g^{\mu\nu}}{\partial x^\mu} &= -\frac{1}{C} J_g^\nu, \\ \frac{\partial F_g^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_g^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_g^{\beta\mu}}{\partial x^\alpha} &= 0. \end{aligned}\right\} \quad (3.3)$$

Note Eq. (3.3) complies with SR. On the contrary, previous vector theories of gravity adopting S-WEP do not comply with SR.

Under the gauge transformation,

$$\mathbf{A}_g' = \mathbf{A}_g - \nabla f(r,t)$$
$$\phi_g' = \phi_g + \frac{\partial f(r,t)}{\partial t}, \quad (3.4)$$

the gravitational field strength $F_g^{\mu\nu}$ is invariant.

Let's define the gravitational conjugate momentum, π_g^μ, conjugating to the gravitational potential field $\mathbf{A}_g$ as, $\pi_g^\mu = \frac{\partial L_g}{\partial \dot{A}_{g\mu}}$. Then we have $\pi_g^0 = 0$, $\pi_g^i = g^i$. The A_g^0 $(= \phi_g)$ is not a dynamical potential field. The Hamiltonian is,

$$H_g = \int d^3r \frac{1}{2}\left(\mathbf{g}^2 + \mathbf{B}_g^2\right) \quad (3.5)$$

where

$$\mathbf{g} \equiv -\frac{1}{C}\frac{\partial \mathbf{A}_g}{\partial t}, \quad (3.6)$$

$$\mathbf{B}_g \equiv \nabla \times \mathbf{A}_g. \quad (3.7)$$

The Hamiltonian implies that the energy of gravitational field is positive, which resolve the negative energy issue of previous vector theories of gravity.

Eq. (3.3) shows the existence of gravitational wave (G-Wave). In vacuum, $\rho_g = 0$ and $\mathbf{v} = 0$, Eq. (3.3) give G-Wave equations:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{B_g} = 0, \quad (3.8)$$

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{g} = 0, \quad (3.9)$$

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{A}_g = 0. \quad (3.10)$$

Let's compare the G-Wave of the Gravito-dynamics and GR (Table 3.1).

Line 1 shows that G-Wave equation in both theories have the same form. The difference is that G-Wave is spin 1 and spin 2 wave in Gravito-dynamics and GR, respectively.

Line 2 shows that G-Wave potential in both theories has the same quadrupole moment.

Line 3 shows that there are field strengths in Gravito-dynamics, but in GR. This difference plays crucial role in quantizing G-Wave and in quantizing gravity.

Line 4 shows that the intensity of G-Wave in Gravito-dynamics is defined as the square of field strength, $\mathbf{B}_g$, and, for plane wave, as the square of time derivative of potentials, $\frac{\partial \mathbf{A}_g}{\partial t}$; in GR, however, the intensity of G-Wave is directly expressed as the square of time derivative of "potentials", $\frac{\partial \bar{h}^{Ttij}}{\partial t}$, without introducing the concept of the field strength; the intensity in both theories has the same form, except a factor of 4 that can be absorbed.

Table 3.1

	Gravito-dynamics	Linearized GR
Wave equation	$\frac{\partial^2 \mathbf{A_g}}{\partial t^2} - \nabla^2 \mathbf{A_g} = 0$ Spin 1	$\frac{\partial^2 \bar{h}^{\mu\nu}}{\partial t^2} - \nabla^2 \bar{h}^{\mu\nu} = 0$ Spin 2
Quadrupole Solution	$\mathbf{A}_g \sim -\frac{1}{R}\ddot{\mathbf{D}}$	$\bar{h}^{ij} \sim \frac{1}{R}\ddot{\mathbf{D}}$
Field strength	$\mathbf{B}_g \sim -\frac{\partial \mathbf{A}_g}{\partial t} \sim -\frac{1}{R}\dddot{\mathbf{D}}$	None
Radiation Intensity	$\mathbf{S_g} \sim B_g^2 \sim \left(\frac{\partial A_g^i}{\partial t}\right)^2 \sim \dddot{\mathbf{D}}^2$ $I = \int \mathbf{S_g} \cdot d\mathbf{a} = \frac{G}{20C^5}\dddot{D}_{m\alpha\beta}^2$	$T_{G-wave}^{00} \sim \left(\frac{\partial \bar{h}^{ij}}{\partial t}\right)^2 \sim \dddot{\mathbf{D}}^2$ $I = \frac{G}{5C^5}\dddot{D}_{m\alpha\beta}^2$

Thus, the detection of g-wave supports Gravito-dynamics as well.

Analogous to electrodynamics, introducing the Lagrangian of a test body in a gravitational field as,

$$L_g = -m_0 c^2 \sqrt{1-\frac{\mathbf{v}^2}{c^2}} + Q_g \mathbf{A}_g \cdot \mathbf{v} - Q_g \phi_g. \quad (3.11)$$

Expanding the term $m_i c^2 \sqrt{1-\beta_i^2}$ in power of v_i/c and keeping the terms of second order, and substituting the retarded potentials, Darwin-type gravitational Lagrangian for the whole system is

$$\begin{aligned} L_g &= \sum_i \left(\frac{1}{2} m_i v_i^2 + \frac{1}{8c^2} m_i v_i^4 \right) \\ &- \sum_{i>j} \left(\frac{Q_{gi} Q_{gj}}{R_{ij}} - \frac{Q_{gi} Q_{gj}}{2c^2 R_{ij}} \left[\begin{matrix} \mathbf{v_i} \cdot \mathbf{v_j} \\ + (\mathbf{v_i} \cdot \mathbf{n_{ij}})(\mathbf{v_j} \cdot \mathbf{n_{ij}}) \end{matrix} \right] \right), \end{aligned} \quad (3.12)$$

which can explain the perihelion precession of Mercury [11].

Substituting Eq. (3.11) into the Lagrangian equation, $\frac{d}{dt}\left(\frac{\partial L}{\partial \mathbf{V}}\right) = \frac{\partial L}{\partial \mathbf{r}}$, we obtain the equation of motion of the test body,

$$\frac{d\mathbf{P}}{dt} = Q_g \mathbf{g} + Q_g \mathbf{V} \times \mathbf{B}_g. \quad (3.13)$$

Eq. (3.3) and Eq. (3.13), derived based on the internal symmetry $U(1)_g$, form a complete set of equations for describing the gravitational field and the movement of a test body in it. We call this set of equations Gravito-dynamics that contains Newton's law of gravity, complies with SR and resolves the negative energy issue and is complete.

This again justifies our adoption of the Original-WEP.

The $U(1)_g$ gauge theory of gravity, Gravito-dynamics, and $U(1)_e$ Electrodynamics are perfect dual to each other at the classical level.

4. Quantum Gravito-dynamics (QGD)

Before we quantize Gravito-dynamics, let's review the development of the quantum mechanics. Historically, the particle nature of light wave was discovered first, which then led to the concept of the wave-particle duality of light and to establish quantum mechanics. Einstein first noted the wave-particle duality of light.

The wave nature of the propagation of gravitational field has been detected experimentally, although it has been theoretically proposed that the property of spin 2 of G-Wave (graviton) predicted by GR is not detectable by LIGO [12], even if it exists.

We have shown the duality between Electrodynamics and Gravito-dynamics at classical level. Let's extend this duality to quantum level.

4.1. Quantizing Gravitational Wave (g-Wave): Gravito-photon

We ask: does G-wave exhibit particle nature? As the first step of quantizing gravity, this question needs to be answered theoretically (Section 4.1) and empirically (Thought-experiment in Section 7.2). In quantizing light Wave, the strengths of electrodynamics field are utilized.

To study the particle nature of G-Wave, we begin with standing G-Wave. The standing G-Wave is confined in a rectangular volume V with sides A, B, C. The vector potential can be expanded in Fourier series,

$$\mathbf{A}_g = \sum_{\mathbf{k_g}} \mathbf{A}_{g\mathbf{k_g}}(t) e^{i\mathbf{k_g}\cdot\mathbf{r}} \quad (4.1)$$

The G-Wave vector $\mathbf{k_g}$ has components,

$$k_{gx} = \frac{2\pi n_x}{A}, k_{gy} = \frac{2\pi n_y}{B}, k_{gz} = \frac{2\pi n_z}{C},$$

where n_x, n_y, and n_z are integers.

Introducing the gravito-Coulomb-type gauge,

$$\nabla \cdot \mathbf{A}_g = 0, \quad (4.2)$$

$$\mathbf{k_g} \cdot \mathbf{A}_{g\mathbf{k_g}} = 0, \quad (4.3)$$

where $\mathbf{A}_{g\mathbf{k_g}}$ is time dependent, perpendicular to $\mathbf{k_g}$, and satisfies the equation,

$$\ddot{\mathbf{A}}_{g\mathbf{k_g}} + \omega_{g\mathbf{k_g}}^2 \mathbf{A}_{g\mathbf{k_g}} = 0, \quad (4.4)$$

$$\omega_{gk_g} = ck_g. \quad (4.5)$$

From Eq. (3.6) and Eq. (3.7), the field strengths in the terms of $\mathbf{A}_g$ are

$$\mathbf{g} = -\frac{\partial \mathbf{A}_g}{\partial t} = -\frac{1}{C}\sum_{k_g} \dot{\mathbf{A}}_{gk_g} e^{ik_g\cdot r}, \quad (4.6)$$

$$\mathbf{B}_g = \nabla \times \mathbf{A}_g = i\sum_{k_g} (\mathbf{k}_g \times \mathbf{A}_{gk_g}) e^{ik_g\cdot r}. \quad (4.7)$$

Eq. (4.6) and Eq. (4.7) give the total energy, W_{gT}, of G-wave,

$$\begin{aligned} W_{gT} &= \frac{1}{8\pi}\int \left(g^2 + B_g^2\right) dV \\ &= \frac{V}{8\pi c^2}\sum_{\mathbf{k}_g} (\dot{\mathbf{A}}_{gk_g} \cdot \dot{\mathbf{A}}_{gk_g}^* + \omega_{gk_g}^2 \mathbf{A}_{gk_g} \cdot \mathbf{A}_{gk_g}^*). \end{aligned} \quad (4.8)$$

For standing wave, Eq. (4.1) can be written as,

$$\mathbf{A}_g = \sum_{k_g} (\mathbf{a}_{gk_g} e^{i\mathbf{k_g}\cdot\mathbf{r}} + \mathbf{a}_{gk_g}^* e^{-i\mathbf{k_g}\cdot\mathbf{r}}), \quad (4.9)$$

where $\mathbf{a}_{gk_g} \sim e^{-i\omega_{gk_g}\mathbf{t}}$. Then we have

$$\mathbf{A}_{gk_g} = \mathbf{a}_{gk_g} + \mathbf{a}_{g,-k_g}^*. \quad (4.10)$$

Substituting Eq. (4.10) into Eq. (4.8), we obtain the total energy,

$$W_{gT} = \sum_{k_g} W_{g,k_g} = \sum_{k_g} \frac{k_g^2 V}{2\pi} \mathbf{a}_{gk_g} \cdot \mathbf{a}_{gk_g}^*. \quad (4.11)$$

Eq. (4.11) shows that the total energy of G-Wave, is the summation of the energy, W_{g,k_g}, of each plane G-Wave, and that G-Wave is expressed in the terms of a series of discrete parameters, $\mathbf{a}_{gk_g}$.

To study the Hamiltonian of G-Wave, let's introduce gravitation canonical variables,

$$q_{gk_g} \equiv \sqrt{\frac{V}{4\pi c^2}} \frac{1}{\sqrt{2\omega_{gk_g}}} \left(\mathbf{a}_{gk_g} + \mathbf{a}^*_{gk_g} \right), \quad (4.12)$$

$$p_{g\mathbf{k_g}} \equiv -i\sqrt{\frac{V}{4\pi c^2}} \sqrt{\frac{\omega_{gk_g}}{2}} \left(\mathbf{a}_{gk_g} - \mathbf{a}^*_{gk_g} \right). \quad (4.13)$$

Eq. (4.12) and Eq. (4.13) give the Hamiltonian of G-Wave,

$$H_{G-Wave} = \sum_{k_g} \frac{1}{2} \left(p^2_{gk_g} + \omega^2_{gk_g} q^2_{gk_g} \right). \quad (4.14)$$

This Hamiltonian, Eq. (4.14), has the form of the "harmonic oscillator". Now to quantize G-Wave becomes to quantize "harmonic oscillator" of G-Wave. According to quantum mechanics, we introduce the gravitational ladder operators, which satisfy commutation relation,

$$\left[a_{g\sigma,k_g}, a^*_{g\sigma,k_g} \right] = 1. \quad (4.15)$$

Then we have,

$$H_{G-Wave} = \sum_{k_g} \sum_{\sigma=1}^{2} \hbar\omega_{gk_g} \left(N_{\sigma,k_g} + \frac{1}{2} \right), \quad (4.16)$$

where

$$N_{\sigma,k_g} \equiv a_{g\sigma,k_g} a^*_{g\sigma,k_g}, \quad (4.17)$$

$$\left[a_{g\sigma,k_g}, H \right] = \hbar\omega_{gk_g} a_{g\sigma,k_g}, \quad (4.18)$$

$$\left[a^*_{g\sigma,k_g}, H \right] = -\hbar\omega_{gk_g} a^*_{g\sigma,k_g}. \quad (4.19)$$

Eq. (4.16) implies that the energy levels are quantized, i.e., G-Wave is quantized, and that the ground state energy is $\hbar\omega_{gk_g}/2 > 0$. Zero energy of quanta of G-Wave is not allowed. Eq. (4.18) and Eq. (4.19) show that $\mathbf{a}_{gk_g}$ and $\mathbf{a}^*_{gk_g}$ form a set of creation and annihilation operators.

The quanta of G-Wave are spin 1 boson; we denote the boson as "Gravito-photon", which does not carry g-charge, i.e., no rest mass; so, gravity is a long-range force.

We also obtain the total momentum of G-Wave,

$$P_{G-Wave} = \sum_{k_g} \sum_{\sigma=1}^{2} \mathbf{k}_g \, N_{\sigma,k_g}. \quad (4.20)$$

In this section, we have shown that G-Wave does have particle nature and that Gravito-dynamics is compatible with quantum mechanics. It will be crucial to detect the particle nature of G-Wave empirically (Thought-experiment in Section 7.2) [13]. In Gravito-dynamics, the energy of G-Wave is localizable. In GR, the energy-momentum of G-Wave is not localizable.

We have established the wave-particle duality of G-Wave. We suggested that the energy and momentum of Gravito-photon can be expressed as, respectively,

$$E_{g-photon} \equiv h\nu_g, \quad (4.21)$$

$$P_{g-photon} \equiv \frac{h}{\lambda_g}, \quad (4.22)$$

where h is Planck constant, ν_g and λ_g are frequency and wavelength.

4.2. Quantizing Gravito-dynamics (QGD)

The difficulties in quantizing geometric theories of gravity have been roughly classified into two categories. One is the "conceptual difficulties", which is a lack of clarity of the nature of gravity. Another is the "technical difficulties", e.g., cannot renormalize quantum theories of gravity. The quantization of geometric theories of gravity is the quantization of either geometrical quantities or space-time itself. To this aim, various approaches have been proposed that have drawn great attentions [14,15,16]. The "technical difficulties" and the "conceptual difficulties" are closely related to each other.

We argue that it is the "conceptual difficulties" that causes the "technical difficulties". Namely the origin of the difficulty is to apply the concepts and methods used in quantizing those physical quantities to quantize the geometric quantities of gravity. It has been pointed out that the 'conceptual difficulties' requires a conceptual resolution [17,18]. Now we examine the "conceptual difficulties", i.e. the physical nature of gravity first.

Instead of considering gravity as a consequence of the geometry of space-time, we treat gravity as physical fields acting in space-time. This allows successful application of the procedures used to quantize Electrodynamics to quantize gravity. It also provides an alternative approach to unify gravity with electrodynamics fields in the same conceptual/mathematical framework. This conceptual resolution simultaneously resolves the "technical difficulties", whereas resolving the "technical difficulties" would not necessarily solve the "conceptual difficulties".

The duality between Gravito-dynamics and Electrodynamics suggests that the Gravito-dynamics can be quantized and renormalized [19] by following the same approach of quantizing Electrodynamics.

We start with the gravitational Lagrangian for free fields, $\rho_g = 0$ and $\mathbf{J}_g = 0$,

$$L_g = -\frac{1}{4} F_g^{\mu\nu} F_{g\mu\nu} = \frac{1}{2} \left(\mathbf{g}^2 - \mathbf{B}_g^2 \right). \quad (4.23)$$

Adopting the Gravito-Coulomb-type gauge condition,

$$\nabla \cdot \mathbf{A}_g = 0, \quad (4.24)$$

Eq. (4.23) gives the massless d'Alembert equation,

$$\partial_\nu \partial^\nu A_g^\mu = 0. \quad (4.25)$$

Its solution is

$$\left. \begin{array}{l} A_{gi}(x) = \int \frac{d^3k}{\sqrt{2\boldsymbol{\omega}}} \sum_s \varepsilon_{gi}(k,s) \begin{bmatrix} a_g(k,s) e^{-i\mathbf{k}\cdot\mathbf{x}} \\ + a_g^\dagger(k,s) e^{i\mathbf{k}\cdot\mathbf{x}} \end{bmatrix}, \\ \mathbf{k}\cdot\boldsymbol{\varepsilon}_g(k,s) = 0. \end{array} \right\}. \quad (4.26)$$

The $\boldsymbol{\varepsilon}_g(k,s)$ is the polarization vector of gravito-photon. s = 1, 2. $\omega = |\mathbf{k}| = k_0$.

The gravitational commutation rule is

$$\left.\begin{aligned}\left[a_g(k,s),a_g(k',s')\right]&=0,\\ \left[a_g^\dagger(k,s),a_g^\dagger(k',s')\right]&=0,\\ \left[a_g(k,s),a_g^\dagger(k',s')\right]&=\delta_{ss'}\delta(\mathbf{k}-\mathbf{k}')\end{aligned}\right\}. \tag{4.27}$$

Then the Hamiltonian H_g and momentum become respectively,

$$H_g = \int d^3k\,\frac{\omega}{2}\sum_{s=1,2}\begin{bmatrix}a_g(k,s)a_g^\dagger(k,s)\\ +a_g^\dagger(k,s)a_g(k,s)\end{bmatrix}, \tag{4.28}$$

$$p_g = \int d^3k\,\frac{\mathbf{k}}{2}\sum_{s=1,2}\begin{bmatrix}a_g(k,s)a_g^\dagger(k,s)\\ +a_g^\dagger(k,s)a_g(k,s)\end{bmatrix}. \tag{4.29}$$

The gravitational vacuum is defined as $a_g(k,s)|0\rangle = 0$.

The gravitational field is quantized, denoted as *QGD*. Where $a_g(k,s)$ and $a_g^\dagger(k,s)$ form a set of creation and annihilation operators. The quanta of the gravitational fields, which is the gauge boson and we denoted it as the gravito-photon, have the following properties: (1) zero rest mass and spin 1; (2) carries energy ω and momentum $\mathbf{k}$.

Note, according to QGD, although the gravito-boson carries energy, it does not carry g-charge, which implies that the gravito-photon is not the source of gravitational field. On the contrary, in GR, the energy of gravitational field is the source of gravitational field and thus, the energy of the gravitational field generates secondary gravitational field, and so on, no end.

Note, once we change the conceptual perspective of gravity from geometrics to physical dynamics, we no longer have "technical difficulties" in quantizing vector Gravito-dynamics of gravity. This is another justification for the Gravito-dynamics, a gauge theory.

4.3. Renormalizing QGD

We renormalize QGD by applying the same procedure used to renormalize QED. Since gravito-bosons are massless, it is straightforward to normalize QGD by introducing the Largangian,

$$\begin{aligned}L_{QGD} &= -\frac{1}{4}F_g^{\mu\nu}F_{g\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu - iQ_gA_{g\mu})\psi\\ &\quad - m\bar{\psi}\psi - \frac{\lambda_g}{2}\left(\partial_\mu A_g^\mu\right)^2.\end{aligned} \tag{4.30}$$

To renormalize QGD, let's introduce renormalization constants in front of each term of the Lagrangian L_{QGD}. The full Lagrangian is then,

$$\begin{aligned}L_{QGD} &= -\frac{1}{4}Z_{g3}F_g^{\mu\nu}F_{g\mu\nu} + Z_{g2}i\bar{\psi}\gamma^\mu\partial_\mu\psi\\ &\quad + Z_{g1}Q_g\bar{\psi}\gamma^\mu A_{g\mu}\psi - Z_{gm}m\bar{\psi}\psi\\ &\quad - \frac{\lambda_{gB}}{2}\left(\partial_\mu A_g^\mu\right)^2\end{aligned} \tag{4.31}$$

Where the term $(\partial_\mu A_g^\mu)^2$ is the gauge fixing term. The normalization constants are,

$$Z_{g3} = 1 + \delta Z_{g3},\ Z_{g2} = 1 + \delta Z_{g2},$$

$$Z_{g1} = 1 + \frac{\delta Q_g}{Q_g},\ Z_{gm} = 1 + \frac{\delta m}{m}, \tag{4.32}$$

and define

$$\begin{aligned}\Psi_B &= \sqrt{Z_{g2}}\,\psi,\ A_{gB}^\mu = \sqrt{Z_{g3}}\,A_g^\mu,\\ m_B &= \frac{Z_m}{Z_{g2}}m,\ Q_{gB} = \frac{Z_{g1}}{Z_2\sqrt{Z_{g3}}}Q_g,\\ \lambda_{gB} &= \frac{\lambda_g}{Z_{g3}},\ F_{gB}^{\mu\nu} = \partial^\mu A_{gB}^\nu - \partial^\nu A_{gB}^\mu.\end{aligned} \tag{4.33}$$

Substituting Eq. (4.32) and Eq. (4.33) into Eq. (4.31), we obtain the renormalized Lagrangian of QGD that has the form identical to that of Lagrangian, Eq. (4.30),

$$\begin{aligned}L_{QGD} &= -\frac{1}{4}F_{gB}^{\mu\nu}F_{gB\mu\nu} + i\bar{\psi}_B\gamma^\mu\partial_\mu\psi_B\\ &\quad + Q_{gB}\bar{\psi}_B\gamma^\mu A_{gB\mu}\psi_B\\ &\quad - m_B\bar{\psi}_B\psi_B - \frac{\lambda_{gB}}{2}\left(\partial_\mu A_{gB}^\mu\right)^2.\end{aligned} \tag{4.34}$$

We conclude that the local $U(1)_g$ gauge theory of gravity based on the Original-WEP does not encounter the non-renormalizable issue. It has been proved that any gauge theory can be renormalized [20].

This is another justification for Original-WEP and Gravito-dynamics.

4.4. Coupling QGD to Dirac Particles

We have shown in Section 2.2 that the internal space θ_g is the function of spacetime x_μ, and the internal group $U(1)_g$ is the local symmetry group. When the invariance of the Lagrangian L_{QGD} under the gravitation local $U(1)_g$ transformation is required, gravitational vector potential field $A_{g\mu}$ must exist. Let's introduce the Lagrangian density of the QGD for a spin-1/2 field interacting with gravitational field, in natural unit,

$$\begin{aligned}L_{QGD} &= -\frac{1}{4}F_g^{\mu\nu}F_{g\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu - iQ_gA_{g\mu})\psi\\ &\quad - m_0\bar{\psi}\psi.\end{aligned} \tag{4.35}$$

The Lagrangian L_{QGD} and the $|\psi|^2$ are invariant under the gravitational local $U(1)_g$ gauge transformation,

$$\left.\begin{aligned}\psi &\to \psi' = e^{-iQ_g\alpha_g(x)}\psi,\\ A'_{g\mu} &\to A_{g\mu} + \partial_\mu\alpha_g(x).\end{aligned}\right\} \tag{4.36}$$

As shown by Eq. (2.21), $U(1)_g$ is local symmetry. Let's introduce the covariant derivative,

$$D_{g\mu} = \partial_\mu - iQ_gA_{g\mu}, \tag{4.37}$$

which transforms in the same way as ψ,

$$D'_{g\mu}\psi' = e^{-iQ_g\alpha_g(x)}D_{g\mu}\psi, \tag{4.38}$$

$$D'_{g\mu} = \partial_\mu - iQ_gA'_{g\mu}. \tag{4.39}$$

The Lagrangian L_{QGD} yields the following equations:

$$i\gamma^{\mu}(\partial_{\mu} - iQ_g A_{g\mu})\psi - m_0\psi = 0. \quad (4.40)$$

$$\frac{\partial F_g^{\mu\nu}}{\partial x^{\mu}} = -Q_g \bar{\psi}\gamma^{\nu}\psi. \quad (4.41)$$

Let's define the effective rest mass m_{eff} as

$$m_{eff} \equiv (1 - \sqrt{G}\gamma^{\mu}A_{g\mu})m_0. \quad (4.42)$$

Eq. (4.40) can be written as

$$i\gamma^{\mu}\partial_{\mu}\psi - m_{eff}\psi = 0. \quad (4.43)$$

Eq. (4.43) implies that to describe the effect of gravitational fields on a Dirac particle is equivalent to describe a Dirac particle with effective mass and without gravitational interaction. The appearance mass of a spin ½ particle is its effective mass and contains two parts, one is its rest mass, another is due to the gravitational interaction.

In a strong external gravitational field, $A^E_{g\mu}$, the term $A_{g\mu}$ in Eq. (4.40) should be replaced by the total gravitational fields $A^T_{g\mu}$,

$$A^T_{g\mu} = A_{g\mu} + A^E_{g\mu}. \quad (4.44)$$

Then Eq. (4.40) becomes,

$$i\gamma^{\mu}(\partial_{\mu} - iQ_g A^T_{g\mu})\psi - m_0\psi = 0. \quad (4.45)$$

Where the total effective rest masse is defined as

$$m^T_{eff} \equiv (1 - \sqrt{G}\gamma^{\mu}A^T_{g\mu})m_0. \quad (4.46)$$

Eq. (4.45) becomes,

$$i\gamma^{\mu}\partial_{\mu}\psi - m^T_{eff}\psi = 0. \quad (4.47)$$

For a situation of the strong external gravitational fields, the term, $\sqrt{G}\gamma^{\mu}A_{g\mu}$, can be ignored and the term, $\sqrt{G}\gamma^{\mu}A^E_{g\mu}$, becomes dominate then we have

$$m^T_{eff} \approx (1 - \sqrt{G}\gamma^{\mu}A^E_{g\mu})m_0. \quad (4.48)$$

At the microscopic scale, studying the behaviors of Dirac particles in strong external gravitational fields will test the validity of QGD. Recently the effects of Newtonian gravity on the quantum mechanics have been investigated [21,22].

4.5. Einstein Hypothesis

In 1912, in exploring the nature and effect of gravity, Einstein assumed the existence of gravito-magnetic field. Then he adopted a model that consists of the ball shell K with mass M over the sphere surface and a material point P arranged in the center of this ball shell with the rest mass "m". He then analyzed the model and proposed an effect that the inertias of objects are changed due to the gravitational interaction between the objects [1]. He shows that the presence of the shell K increases the mass of the mass P in it, at the classical level,

$$m' = m + \frac{GmM}{Rc^2} \quad (4.49)$$

Now let's study what we have in quantum level. Eq. (4.48) gives,

$$m^T_{eff} \approx \begin{pmatrix} 1 - \sqrt{G}\gamma^0 A^E_{g0} - \gamma^1 A^E_{g1} \\ -\gamma^2 A^E_{g2} - \gamma^3 A^E_{g3} \end{pmatrix} m_0 \quad (4.50)$$

The scalar potential of the external gravitational field is,

$$A^E_{g0} = -\frac{\sqrt{G}}{r}M_0 \quad (4.51)$$

Where M_0 is the rest mass of the source of the external gravitational field. Substituting it into Eq. (4.50), we, have

$$m^T_{eff} \approx \begin{pmatrix} 1 + \frac{GM_0}{r}\gamma^0 - \gamma^1 A^E_{g1} \\ -\gamma^2 A^E_{g2} - \gamma^3 A^E_{g3} \end{pmatrix} m_0. \quad (4.52)$$

The first two terms on the right-hand side agree with the Einstein hypothesis Eq. (4.49), but it is at quantum level and derived from QGD.

The other three terms on the right-hand side represent the interaction between spin of particle and external gravito-magnetic potential, which also contribute to the effective mass and will be considerable in a strong gravito-magnetic field of rotating dense star.

Now let's summarize the effects of gravity on a test object at both classical level and quantum level (Table 4.1).

We come to interesting conclusions.

In Einstein point of view, the effects of gravity are due to both force and potential. The former is described by S-WEP. Einstein (1907) "assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system", which leads to GR. The latter was described by Einstein Hypothesis.

Table 4.1.

	Gauge theory of gravity	Einstein
Classical level: Field Strength: $\mathbf{g}$	Original-WEP $m_i\mathbf{a} = Q_g\mathbf{g}$ $Q_g = \sqrt{G}m_0$	S-WEP: $m_i\mathbf{a} = m_g\mathbf{g}$ $m_i = m_g$
Classical level: Scalar potential		$m' = m + \frac{GmM}{Rc^2}$
Quantum level: Field Potential: $A^T_{g\mu}$	QGD: $m^T_{eff} \equiv (1 - \sqrt{G}\gamma^{\mu}A^T_{g\mu})m_0$	Non

In Gravito-dynamics, the effect of Newton's gravitational force is represented by Original-WEP. In QGD, the effect of gravity is due to potential, which affects the rest mass of particle and contains classical Einstein Hypothesis.

4.6. Negative Gravitational Charge

The total g-charge Q_{gT} can be obtained from Eq. (4.41),

$$Q_{gT} = Q_g \int d^3x\, \bar{\psi}\gamma^0\psi. \quad (4.53)$$

Treating this as a quantum equation, we then have

$$Q_{gT} = Q_g \int d^3p \sum_{s=1,2}(b^{s\dagger}_p b^s_p - c^{s\dagger}_p c^s_p). \quad (4.54)$$

Eq. (4.54) indicates that the total g-charge is equal to the number of "gravitational particles", $b^{s\dagger}_p b^s_p$, minus the number of "gravitational antiparticles", $c^{s\dagger}_p c^s_p$. The "gravitational particles" carry "positive g-charge" and the "gravitational antiparticles" carry "negative g-charges".

On the other hand, to satisfy CPT symmetry requires opposite g-charges. Analogous to the positron in the Dirac Hole theory, Particle/Anti-particle Conjugation would transform the "gravitational particle" to the "gravitational antiparticles", and vice versa. Thus, the definitions of the positive and negative g-charges,

$$Q_{g+} = +\sqrt{G}\, m_g = +\sqrt{G}\, m_0$$

$$Q_{g-} = -\sqrt{G}\, m_g = -\sqrt{G}\, m_0,$$

in Section 2, are justified. To describe the gravitational fields of "negative g-charges", we assume that the negative g-charges generate the gravitational fields in the same way as that of the positive g-charges do.

To exam its existence, negative g-charge has been introduced into scalar Newton, vector Gravito-EM, and tensor Einstein, and explains equally well the acceleration of the expansion of the universe. The property of *theory-independent* of this mechanism provides a theoretical support. Most significantly, the mechanism predicts that the acceleration is *time-dependent*, e.g., is accelerating [23]. Recently a most significant observation is reported [24]: the acceleration of the universe is accelerating, although not confirmed yet. On the contrary, the dark energy model with cosmological constant predicts a constant acceleration. Therefore, we argue that these observations are strong indications of the existence of negative g-charge, although there is no direct evidence yet.

An interesting possibility is that one sub-universe may be full of "positive g-charges" that attract each other, while a different sub-universe may be full of "negative g-charges" that attract each other, but these two sub-universes repel each other. Note those two sub-universes may be overlapped.

5. Unifying $U(1)_g$ Gravity with Electromagnetic Force and Electroweak Force

For unification, based on the duality between gravity and electrodynamics, we introduce a novel symmetry to bridge both interactions.

5.1. Ultra-Symmetry: e-Charge/g-Charge Conversion

By analogy to Boson/Fermion Supersymmetry, we define a novel symmetry, *Ultra-symmetry*, which states that under conversion between g-charge and e-charge,

$$Q_e \leftrightarrow -Q_g, \tag{5.1}$$

Electrodynamics/QED converts to Gravito-dynamics/QGD, respectively, and vice versa.

Ultra-symmetry is a type of symmetry for theoretical unification of Electrodynamics and Gravity. Ultra-symmetry is fruitful theoretically as shown in this article. By converting e-charge to g-charge, and vice versa, Ultra-symmetry implies the following conversions (Table 5.1):

Table 5.1. Ultra-Symmetries

Charge	$Q_e \leftrightarrow -Q_g$
Symmetry	$U(1)_e \leftrightarrow U(1)_g$
Classical theory	Electro-dynamics ↔ Gravito-dynamics
Quantum theory	QED ↔ QGD

Ultra-symmetry establishes fundamental conjectured relationships between two charges, e-charge and g-charge, and between two long-range interactions, electrodynamics and gravity, in Nature.

5.2. Unifying Gravito-dynamics with Electrodynamics

It is difficult to unify General Relativity, a geometric theory of gravity, with Electrodynamics, a physical gauge theory. The duality between two interactions is the basis of the unification. The important and significant benefits of the duality is to helps us understanding Gravity/QGD by comparing with Electrodynamics/QED. Namely, effect/phenomena of gravity may be predicted and directly obtained from well-established Electrodynamics/QED. We can now unify Electrodynamics and Gravito-dynamics readily.

Let's consider a simple case where a moving object carries both the g-charge and e-charge. The Maxwell's equations and Maxwell-type equations of gravity of the moving charge are, respectively,

$$\left.\begin{aligned} \frac{\partial F_e^{\mu\nu}}{\partial x^\nu} = \frac{1}{c} J_e^\mu \\ \frac{\partial F_e^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_e^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_e^{\beta\mu}}{\partial x^\alpha} = 0 \end{aligned}\right\}, \tag{5.2}$$

$$\left.\begin{aligned} \frac{\partial F_g^{\mu\nu}}{\partial x^\nu} = -\frac{1}{c} J_g^\mu \\ \frac{\partial F_g^{\mu\alpha}}{\partial x^\beta} + \frac{\partial F_g^{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_g^{\beta\mu}}{\partial x^\alpha} = 0 \end{aligned}\right\}, \tag{5.3}$$

where $F_e^{\mu\alpha} (= \partial^\mu A_e^\alpha - \partial^\alpha A_e^\mu)$ and $F_g^{\mu\alpha} (= \partial^\mu A_g^\alpha - \partial^\alpha A_g^\mu)$ are the field tensors, A_e^μ and A_g^μ are the four-potentials, J_e^μ and J_g^μ are the four-currents of Electrodynamics and Gravito-dynamics, respectively

Combining Eq. (5.2) and Eq. (5.3), we obtain the following universal equations,

$$\left.\begin{aligned} \frac{\partial U_a^{\mu\nu}}{\partial x^\nu} = \frac{1}{C} J_a^\mu, \\ \frac{\partial U_a^{\mu\alpha}}{\partial x^\beta} + \frac{\partial U_a^{\alpha\beta}}{\partial x^\mu} + \frac{\partial U_a^{\beta\mu}}{\partial x^\alpha} = 0. \end{aligned}\right\} \tag{5.4}$$

Where $U_a^{\mu\alpha}$ represents either $F_g^{\mu\alpha}$ or $F_e^{\mu\alpha}$; J_a^μ represents either J_g^μ or J_e^μ:

$$U_1^{\mu\alpha} \equiv F_g^{\mu\alpha} = \partial^\mu A_g^\alpha - \partial^\alpha A_g^\mu, \tag{5.5}$$

$$U_2^{\mu\alpha} \equiv F_e^{\mu\alpha} = \partial^\mu A_e^\alpha - \partial^\alpha A_e^\mu, \tag{5.6}$$

$$J_1^\mu = -J_g^\mu, J_2^\mu = J_e^\mu. \tag{5.7}$$

$$A_1^\beta \equiv A_g^\beta, A_2^\beta \equiv A_e^\beta, \tag{5.8}$$

Eq. (5.4) is the unified field equation of Electrodynamics and Gravito-dynamics at classical level.

Under gauge transformation,

$$A_a^{\mu'} \to A_a^\mu + \frac{\partial f_a}{\partial x_\mu}, \tag{5.9}$$

the field strength $U_a^{\mu\nu}$ is invariant.

For a system containing both e-charges and g-charges, the total Lagrangian density, L_{field}, includes two parts, Gravito-dynamics Lagrangian, L_g, and Electrodynamics Lagrangian, L_e,

$$L_{field} = L_g + L_e = -\sum_{a=1,2}\left(\frac{1}{4}U_a^{\mu\nu}U_{\mu\nu}^a\right) - \sum_{a=1,2}\left(A_a^{\mu}J_{\mu}^a\right). \quad (5.10)$$

Now let's unify the equations of motion of a particle carrying both e-charge and g-charge in the presence of both gravity and electromagnetic field. The duality between force laws of Electrodynamics and Gravito-dynamics are the following: converting e-charge q_e to g-charge q_g, and the field strengths $\mathbf{E}$ and $\mathbf{B}$ to the field strengths $\mathbf{g}$ and $\mathbf{B}_g$, respectively, then the Lorentz force law of Electrodynamics converts to Lorentz-type force law of Gravito-dynamics, and vice versa.

Note when converting e-charge to g-charge, a "minus" sign must be placed only once either in front of g-charge of the source term in field equation, or in front of g-charge in equation of motion. If place "minus" sign in front of g-charge in both the field equation and equation of motion, then the effect of "minus" sign will be canceled out. Following Newton, we choose to place the "minus" sign in front of source term of field equation.

The unified Lagrangian of the equation of motions of e-charge/g-charge in the presence of both gravity and electrodynamics field is,

$$L_{motion} = -m_0c^2\sqrt{1-\frac{v^2}{c^2}} + \sum_{a=1,2}\left(Q_a\mathbf{v}\cdot\mathbf{A}_a - Q_a\phi_a\right), \quad (5.11)$$

where $Q_1 = Q_g$ (g-charge), $Q_2 = Q_e$ (e-charge), $\mathbf{A}_1 = \mathbf{A}_g$ (gravito-magnetic vector potential), $\mathbf{A}_2 = \mathbf{A}_e$ (electromagnetic vector potential), $\phi_1 = \phi_g$ (gravitational scalar potential), $\phi_2 = \phi_e$ (electrodynamics scalar potential).

Eq. (5.11) gives the unified equation of motion,

$$\frac{dP}{dt} = \sum_{a=1,2}\left(Q_a\mathbf{E}_a + Q_a\mathbf{v}\times\mathbf{B}_a\right), \quad (5.12)$$

where $\mathbf{E}_1 = \mathbf{g}$ (gravitational field), $\mathbf{E}_2 = \mathbf{E}$ (electric field), $\mathbf{B}_1 = \mathbf{B}_g$ (Gravito-magnetic field), $\mathbf{B}_2 = \mathbf{B}$ (magnetic field).

We denote Eq. (5.4) and Eq. (5.12) as *Electro-Gravito-Dynamics*.

5.3. Unifying QGD and QED: Electro-gravity Interaction

Duality between QED and QGD implies that the physical laws are expressed by the equations of exactly same form, and their physical quantities are correspondence. In this type of unification, the equation is considered as the fundamental entity.

The Lagrangian of the unified QED and QGD with internal symmetry group $U(1)_g \times U(1)_e$ is

$$L_{QED-QGD} = L_g + L_e = -\sum_{a=1,2}\left(\frac{1}{4}U_a^{\mu\nu}U_{\mu\nu}^a\right) = -\frac{1}{4}F_g^{\mu\nu}F_{g\mu\nu} - \frac{1}{4}F_e^{\mu\nu}F_{e\mu\nu}. \quad (5.13)$$

Following the conventional rule of naming, we referred this unified interaction of QED and QGD as "*Electro-gravity Interaction*".

Next, let's study a situation of coupling Electro-gravity Interaction to a Dirac particle that carries both e-charge and g-charge. The Lagrangian and covariant derivative in QGD are, respectively,

$$L_{QGD} = -\frac{1}{4}F_g^{\mu\nu}F_{g\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} - iQ_gA_{g\mu})\psi - m_0\bar{\psi}\psi, \quad (5.14)$$

$$D_{g\mu} = \partial_{\mu} - iQ_gA_{g\mu}, \quad (5.15)$$

which have the form same as that of QED,

$$L_{QED} = -\frac{1}{4}F_e^{\mu\nu}F_{e\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + iQ_eA_{e\mu})\psi - m_0\bar{\psi}\psi, \quad (5.16)$$

$$D_{e\mu} = \partial_{\mu} + iQ_eA_{e\mu}. \quad (5.17)$$

Which shows the Ultra-symmetry between QED and QGD.

We combine both Lagrangians of QGD and QED, referred as the unified Lagrangian, $L_{QED-QGD}$, to describe the effects of Electro-gravity Interaction. The Lagrangian with internal symmetry group $U(1)_g \times U(1)_e$ is

$$L_{QED-QGD} = -\frac{1}{4}F_g^{\mu\nu}F_{g\mu\nu} - \frac{1}{4}F_e^{\mu\nu}F_{e\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} - iQ_gA_{g\mu} + iQ_eA_{e\mu})\psi - m_0\bar{\psi}\psi. \quad (5.18)$$

The equations of motion of Electro-gravity interaction are,

$$i\gamma^{\mu}\left(\partial_{\mu} + iQ_eA_{e\mu} - iQ_gA_{g\mu}\right)\psi - m_0\psi = 0, \quad (5.19)$$

$$\frac{\partial F_g^{\mu\nu}}{\partial x^{\nu}} = -Q_g\bar{\psi}\gamma^{\mu}\psi, \quad (5.20)$$

$$\frac{\partial F_e^{\mu\nu}}{\partial x^{\nu}} = Q_e\bar{\psi}\gamma^{\mu}\psi. \quad (5.21)$$

Following the conventional rule of naming, we referred to this unified interaction of QED and QGD as "*Electro-gravity Interaction*".

By the definition of g-charge, Eq. (5.18) gives

$$L_{QED-QGD} = -\frac{1}{4}F_g^{\mu\nu}F_{g\mu\nu} - \frac{1}{4}F_e^{\mu\nu}F_{e\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + iQ_eA_{e\mu})\psi - M_{effective}\bar{\psi}\psi, \quad (5.22)$$

$$M_{effective} \equiv m_0(1 - \sqrt{G}\gamma^{\mu}A_{g\mu}). \quad (5.23)$$

The effects of the gravitational potential fields $A_{g\mu}$ generated by a Dirac particle is negligible, i.e., $Q_gA_{g\mu} \ll Q_eA_{e\mu}$. Thus Eq. (5.19) reduces to the Dirac equation of the QED,

$$i\gamma^{\mu}\left(\partial_{\mu}+iQ_{e}A_{e\mu}\right)\psi-m_{0}\psi=0,$$

which implies the negligibility of gravitational effect, namely explains why we do not observe the gravitational effect on quantum systems.

5.4. Renormalizing Electro-gravity Interaction

We renormalize Quantum Electro-Gravity Interaction by following the same procedure used to renormalize QED or QGD. The bare Lagrangian for Quantum Electro-Gravity (QED-QGD) with gauge fixing terms for QGD and QED respectively can be written as

$$\begin{aligned}L_{QED-QGD}&=-\frac{1}{4}F_{g}^{\mu\nu}F_{g\mu\nu}-\frac{\lambda_{g}}{2}\left(\partial_{\mu}A_{g}^{\mu}\right)^{2}\\&-\frac{1}{4}F_{e}^{\mu\nu}F_{e\mu\nu}-\frac{\lambda_{e}}{2}\left(\partial_{\mu}A_{e}^{\mu}\right)^{2}\\&+i\bar{\psi}\gamma^{\mu}\begin{pmatrix}\partial_{\mu}-iQ_{g}A_{g\mu}\\+iQ_{e}A_{e\mu}\end{pmatrix}-m\bar{\psi}\psi.\end{aligned}\tag{5.24}$$

Let's introduce the renormalization Lagrangian,

$$\begin{aligned}L_{QED-QGD}&=-\frac{1}{4}Z_{g3}F_{g}^{\mu\nu}F_{g\mu\nu}-\frac{1}{4}Z_{e3}F_{e}^{\mu\nu}F_{e\mu\nu}\\&+Z_{2}i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi+Z_{g1}Q_{g}\bar{\psi}\gamma^{\mu}A_{g\mu}\psi-Z_{e1}Q_{e}\bar{\psi}\gamma^{\mu}A_{e\mu}\psi\\&-\frac{\lambda_{gB}}{2}\left(\partial_{\mu}A_{g}^{\mu}\right)^{2}-\frac{\lambda_{eB}}{2}\left(\partial_{\mu}A_{e}^{\mu}\right)^{2}-Z_{m}m\bar{\psi}\psi\end{aligned}\tag{5.25}$$

Where

$$\left.\begin{aligned}&Z_{g3}=1+\delta Z_{g3},Z_{e3}=1+\delta Z_{e3},\\&Z_{g1}=1+\frac{\delta Q_{g}}{Q_{g}},Z_{e1}=1+\frac{\delta Q_{e}}{Q_{e}},\\&Z_{2}=1+\delta Z_{2},Z_{m}=1+\frac{\delta m}{m}.\end{aligned}\right\}\tag{5.26}$$

Let's define

$$\left.\begin{aligned}&\psi_{B}=\sqrt{Z_{2}}\psi,m_{B}=\frac{Z_{m}}{Z_{2}}m,\\&F_{gB}^{\mu\nu}=\sqrt{Z_{g3}}F_{g}^{\mu\nu},F_{eB}^{\mu\nu}=\sqrt{Z_{e3}}F_{e}^{\mu\nu},\\&Q_{gB}=\frac{Z_{g1}}{Z_{g2}\sqrt{Z_{g3}}}Q_{g},Q_{eB}=\frac{Z_{e1}}{Z_{e2}\sqrt{Z_{e3}}}Q_{e},\\&\lambda_{gB}=\frac{\lambda_{g}}{Z_{g3}},\lambda_{eB}=\frac{\lambda_{e}}{Z_{e3}}\end{aligned}\right\}\tag{5.27}$$

where $F_{gB}^{\mu\nu}=\partial^{\mu}A_{gB}^{\nu}-\partial^{\nu}A_{gB}^{\mu}$ and $F_{eB}^{\mu\nu}=\partial^{\mu}A_{eB}^{\nu}-\partial^{\nu}A_{eB}^{\mu}$.

Putting Eq. (5.26) and Eq. (5.27) into Eq. (5.25), then the renormalization Lagrangian $L_{QED-QGD}$ becomes

$$\begin{aligned}L_{QED-QGD}&=-\frac{1}{4}F_{gB}^{\mu\nu}F_{gB\mu\nu}\,\square\frac{1}{4}F_{eB}^{\mu\nu}F_{eB\mu\nu}\\&+i\bar{\psi}_{B}\gamma^{\mu}\left(\partial_{\mu}-iQ_{gB}A_{gB\mu}+iQ_{eB}A_{eB\mu}\right)\\&-\frac{\lambda_{gB}}{2}\left(\partial_{\mu}A_{gB}^{\mu}\right)^{2}-\frac{\lambda_{eB}}{2}\left(\partial_{\mu}A_{eB}^{\mu}\right)^{2}-m_{B}\bar{\psi}_{B}\psi_{B}.\end{aligned}\tag{5.28}$$

5.5. Coupling Electro-gravity Interaction to Dirac Particle

To detect the effects of gravitational fields on a Dirac particle, we need to introduce a strong external gravitational potential $A_{g\mu}^{E}$. In this case the term $A_{g\mu}$ in Eq. (5.19) should be replaced by the external gravitational potential $A_{g\mu}^{E}$,

$$i\gamma^{\mu}\left(\partial_{\mu}+iQ_{e}A_{e\mu}-iQ_{g}A_{g\mu}^{E}\right)\psi-m_{0}\psi=0,\tag{5.29}$$

which can be written as

$$i\gamma^{\mu}\left(\partial_{\mu}+iQ_{e}A_{e\mu}\right)\psi-M_{eff}^{E}\psi=0,\tag{5.30}$$

$$M_{eff}^{E}\equiv\left(1-\sqrt{G}\gamma^{\mu}A_{g\mu}^{E}\right)m_{0}.\tag{5.31}$$

In terms of Pauli matrix, the effective mass can be expressed as

$$M_{eff}^{E}=\begin{bmatrix}1-\sqrt{G}\gamma^{0}A_{g0}^{E}\\-\sqrt{G}\begin{pmatrix}0&\boldsymbol{\sigma}\cdot\mathbf{A}_{g}^{E}\\-\boldsymbol{\sigma}\cdot\mathbf{A}_{g}^{E}&0\end{pmatrix}\end{bmatrix}m_{0}\tag{5.32}$$

Where the term, ($\boldsymbol{\sigma}\cdot\mathbf{A}_{g}^{E}$), represents the coupling between the spin of the Dirac particle and the external gravito-magnetic potential $\mathbf{A}_{g}^{E}$. There are two coupling mechanisms, whereby gravitational potentials can change the effective rest mass of the Dirac particles, i.e. by coupling to both the gravito-electric potential, A_{g0}^{E}, and to the gravito-magnetic potential, $\mathbf{A}_{g}^{E}$.

For spin 1/2 particles carrying negative g-charge, we have, the Dirac-type equations,

$$i\gamma^{\mu}\left(\partial_{\mu}+iQ_{e}A_{e\mu}\right)\psi-M_{-eff}^{E}\psi=0,\tag{5.33}$$

where the effective mass of Dirac particles carrying negative g-charges is,

$$M_{-eff}^{E}\equiv\left(1+\sqrt{G}\gamma^{\mu}A_{g\mu}^{E}\right)m_{0}.\tag{5.34}$$

5.6. Unifying QGD with Electroweak Force: Electroweak-Gravity Interaction

The electromagnetic force and weak force have been unified with symmetry group $SU(2)\times U(1)$. Once we unified QED and QGD, it is expected to unify Electro-weak force and QGD. The Lagrangian of electroweak (L_{EW}) and QGD (L_{QGD}) with internal symmetry group $SU(2)\times U(1)\times U(1)_{g}$ is

$$\begin{aligned}L_{EW-QGD}&=L_{QGD}+L_{EW}\\&=-\frac{1}{4}F_{g}^{\mu\nu}F_{g\mu\nu}-\frac{1}{4}F_{U}^{\mu\nu}F_{U/\mu\nu}-\frac{1}{4}F_{SU}^{i\mu\nu}F_{SU/\mu\nu}^{i},\end{aligned}\tag{5.35}$$

Where

$$F_{g}^{\mu\nu}=\partial^{\mu}A_{g}^{\nu}-\partial^{\nu}A_{g}^{\mu},$$

$$F_{U}^{\mu\nu}=\partial^{\mu}A_{U}^{\nu}-\partial^{\nu}A_{U}^{\mu},$$

$$F_{SU/\mu\nu}^{i}=\partial_{\mu}A_{SU/\nu}^{i}-\partial_{\nu}A_{SU/\mu}^{i}+gf^{ijk}A_{\mu}^{j}A_{\nu}^{k},\tag{5.36}$$

$A_{SU/\mu}^{i}$ is the SU(2) weak isospin gauge field, A_{U}^{μ} is the U(1) weak hypercharge field, A_{g}^{μ} is the $U(1)_{g}$ gravitational field, f^{abc} the SU(2) structure constant, g is the coupling constant. Following the conventional rule of naming, we referred this unified interaction of electroweak and QGD as "*Electroweak-gravity Interaction*".

6. $SU(2)_g$ Short-Range Gravity and Unification

The Abelian electrodynamics is generalized to Non-Abelian gauge theory. Along this line, it is nature to generalize Abelian gravity to Non-Abelian gravity. Now we propose an approach to investigate Non-Abelian gravity and unification. The basic approach includes the followings: (1) Extend $U(1)_g$ gravity to $SU(2)_g$ gravity, and establish $U(1)_g \times SU(2)_g$ gravity; (2) Unify electroweak and $SU(2)_g$ gravity; (3) Unify electroweak and $U(1)_g \times SU(2)_g$ gravity. Section 6 forms a natural development of previous works [9,10,19,25]; it is intended to take the theory to its logical consequences.

6.1. $SU(2)_g$ Gravity

Fundamental particles have rest mass and thus, carry g-charge that generates long-range gravitational force. Taking into account two facts: (1) Abelian symmetry is generalized to non-Abelian symmetry, (2) Maxwell theory is generalized to Yang-Mills theory, we suggest to generalize gravity to contain a Non-Abelian gauge theory. We propose a non-Abelian symmetry, $SU(2)_g$, for gravity [10]. By analogy to Weak isospin, we postulate that:

"*All leptons and quarks have a property, called gravito-isospin (g-isospin), which serves as a quantum number and governs how that particle interact with a novel gravitational interaction*".

The Lagrangian of the $SU(2)_g$ Gravity is,

$$\mathcal{L}_{gSU} = -\frac{1}{4} F_{gSU}^{a\mu\nu} F_{gSU/\mu\nu}^{a}, \qquad (6.1)$$

where,

$$F_{gSU}^{a\mu\nu} \equiv A_{gSU}^{a\nu,\mu} - A_{gSU}^{a\mu,\nu} + g_{gSU} f^{abc} A_{gSU}^{b\mu} A_{gSU}^{c\nu}, \qquad (6.2)$$

$A_{gSU}^{a\mu,\nu}$ is the $SU(2)_g$ gravitational gauge field, f^{abc} is $SU(2)_g$ structure constant, g_{gSU} is the coupling constant. According to the gauge theory, the gauge bosons of the non-Abelian $SU(2)_g$ gravity carry g-charges. The simplest assumption about the physical meaning of the g_{gSU} is that the g_{gSU} is g-charge, Q_g, which is equal to $\sqrt{G}m_0$.

So, unlike the force of $U(1)_g$ Gravito-dynamics, which is mediated by massless gravito-photons, the $SU(2)_g$ gravitational interaction is mediated by gauge bosons carrying g-charge $g_{gSU\,2}$. i.e., rest mass, although its unit is not directly mass. Thus, a short range of gravity exist. For massive mediate particles of mass m_0, the potential of interaction is

$$V(r)_{SU2} = -\frac{K}{r} e^{-m_0 r}, \qquad (6.3)$$

where K is the coupling parameter. Eq. (6.3) indicates a short-range $SU(2)_g$ gravitational force. This is a Yukawa-type expression, and is a theoretical derivation of microscopic short-range gravity. The further task is to experimentally explore it [26]. On the other hand, macroscopic short-range gravity has been discussed [27].

This fact leads to a significant conclusion that, since g-charge of a particle is related to its mass, the $SU(2)_g$ gauge bosons automatically carry mass.

6.2. $U(1)_g \times SU(2)_g$ Gravity

To unify long-range and short-rang gravity, the Lagrangian of the $U(1)_g \times SU(2)_g$ gravity is,

$$\mathcal{L}_{g(U\times SU)} = -\frac{1}{4} F_g^{\mu\nu} F_{g\mu\nu} - \frac{1}{4} F_{gSU}^{a\mu\nu} F_{gSU/\mu\nu}^{a}, \qquad (6.4)$$

where,

$$F_g^{\mu\nu} \equiv A_g^{\nu,\mu} - A_g^{\mu,\nu}, \qquad (6.5)$$

is the $U(1)_g$ gravitational field. The potential of total gravitational force including $U(1)_g$ long-range and $SU(2)_g$ short-range gravitational forces between leptons/quarks is,

$$V(r)_{total} = -\frac{\sqrt{G}M_{souce}}{r} - \frac{K}{r} e^{-m_0 r}. \qquad (6.6)$$

6.3. Unifying $SU(2)_g$ Gravity with Electroweak Interaction

We have shown that, at microscopic scale, long-range gravity created by elementary particle is negligible. We now consider short-range gravity. The Lagrangian of the unified electroweak (Eq. 5.35) and $SU(2)_g$ gravity (Eq. 6.1) is,

$$\begin{aligned}\mathcal{L}_{EW\times SUg} = &-\frac{1}{4} F_U^{\mu\nu} F_{U/\mu\nu} - \frac{1}{4} F_{SU}^{\mu\nu} F_{SU/\mu\nu} \\ &-\frac{1}{4} F_{gSU}^{a\mu\nu} F_{gSU/\mu\nu}^{a},\end{aligned} \qquad (6.7)$$

Where $F_U^{\mu\nu}$, $F_{SU}^{\mu\nu}$ and $F_{gSU}^{a\mu\nu}$ are the U(1) weak hypercharge field, SU(2) weak isospin gauge field and the $SU(2)_g$ gravitational field, respectively.

6.4. Unifying $U(1)_g \times SU(2)_g$ Gravity with Electroweak Interaction

The leptons and the $W^{\pm}$ bosons carry both the electric charge and mass (g-charge), and Z boson carries mass (g-charge). This fact suggests that Gravity, Electrodynamics, and Weak interactions are related. We unify Electroweak (Eq. 5.35) and $U(1)_g \times SU(2)_g$ gravity (Eq.6.4) with the Lagrangian,

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{g(U\times SU)} + \mathcal{L}_{EW} \\ &= -\frac{1}{4} F_g^{\mu\nu} F_{g\mu\nu} - \frac{1}{4} F_{gSU}^{a\mu\nu} F_{gSU/\mu\nu}^{a} \\ &\quad -\frac{1}{4} F_U^{\mu\nu} F_{U\mu\nu} - \frac{1}{4} F_{SU}^{i\mu\nu} F_{SU/\mu\nu}^{i}.\end{aligned} \qquad (6.8)$$

We postulate that a SU(2) gauge boson and its corresponding $SU(2)_g$ gauge boson are actual bond of a gauge boson that either carrying both e-charge and g-charge or carrying g-charge only. This postulation provides a mechanism that SU(2) gauge bosons gain mass from corresponding $SU(2)_g$ gauge bosons, besides the symmetry breaking mechanism.

$U(1)_g$ gravity, $SU(2)_g$ gravity and $U(1)_g \times SU(2)_g$ gravity are unified with Electroweak interaction, respectively, within the single mathematical framework. The Unification is structural naturalness. New perspectives bring insights: (1) the underlying physics is simpler, clearer and self-consistent; (2) deepening our conceptually understanding of gravity.

7. Thought-Experiments

7.1. Thought-experiment: Testing both Original-WEP and S-WEP

The S-WEP is an extrapolated hypothesis yet to be supported directly by experimental evidence. For experimentally testing Original-WEP, it is feasible to put such test particles in a longitudinal gravitational field. However, for testing S-WEP with relativistic test particles, it is feasible to utilize a transverse gravitational field.

Thus, we propose a synchrotron-type experiment of measuring the free fall of relativistic test particles, which is in the transverse gravitational field of Earth in y-axis. Test particles move a distance X in the x-axis, and free fall either a distance H_1 or a distance H_2 along the y-axis as predicted by Observers O_1 and O_2, respectively, as illustrated in Figure 7.1.

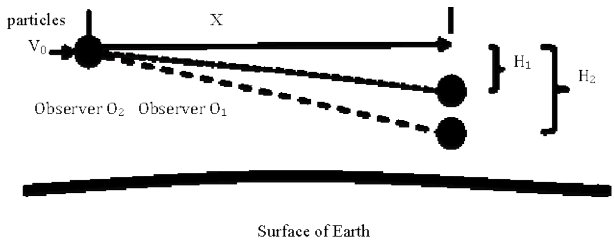

Figure 7.1. Observers O_1 and O_2 are at rest on Earth

In this experiment, relativistic test particles with high initial velocity v_0, momentum P_0, and energy E_0 move along the x-axis. Testing S-WEP becomes a measurement of the test particles' free fall distances in the y-direction. All of measurements are performed on the Earth, i.e., the observers are at rest on the Earth.

In the presence of a gravitational field, $\mathbf{g}_0$, the relativistic motion is determined by,

$$\frac{d}{dt}\frac{m_{i0}}{\sqrt{1-\frac{V^2}{C^2}}}\mathbf{v} = m_g \mathbf{g}_0. \tag{7.1}$$

Eq. 7.1 give us the travel distance X of the test particles along the x-axis, the time t_x required to travel the distance X, the angle $\theta(t_x)$ the trajectory makes with the x-axis, and the free fall distance H in the y-axis, which are correspondingly

$$X = \frac{p_0 C}{m_g g_0}\sinh^{-1}\frac{m_g g_0 t_X C}{E_0}, \tag{7.2}$$

$$t_x = \frac{E_0}{m_g g_0 C}\sinh\frac{m_g g_0 X}{P_0 C}, \tag{7.3}$$

$$\tan\theta(t_x) = \frac{m_g g_0 t_X}{P_0}, \tag{7.4}$$

$$H = \frac{E_0}{m_g g_0}\left[\sqrt{1+\left(\frac{m_g g_0 t_x C}{E_0}\right)^2} - 1\right]. \tag{7.5}$$

The theoretical predictions of the values of X, t_x, tan $\theta(t_x)$, and H provide measurable variables.

According to Original-WEP, Eq. 7.5 becomes

$$H_1 = \frac{E_0}{m_{i0} g_0}\left[\sqrt{1+\left(\frac{m_{i0} g_0 t_X C}{E_0}\right)^2} - 1\right]. \tag{7.6}$$

On the other hand, S-WEP gives,

$$H_2 = \frac{E_0}{\gamma m_{i0} g_0}\left[\sqrt{1+\left(\frac{\gamma m_{i0} g_0 t_X C}{E_0}\right)^2} - 1\right]. \tag{7.7}$$

The value of H_2 is different from that of H_1. This difference is likely detectable in a real experiment, for example, the test particles do circular motion.

7.2. Thought Experiment: Detecting Particle Nature of G-Wave

It has been argued that LIGO can't be used to detect spin-2 gravitons, the quanta of G-Wave in GR [12]. However, the particle nature of G-Wave implies that spin-1 gravito-photons will transport its energy to g-charges encountered on the way of propagation. Adopting the concept of detecting neutrinos, we propose an experiment [13], as shown below (Figure 7.2), to detect the particle nature of G-wave by detecting a beam of gravito-photons, instead of detecting a single gravito-photon.

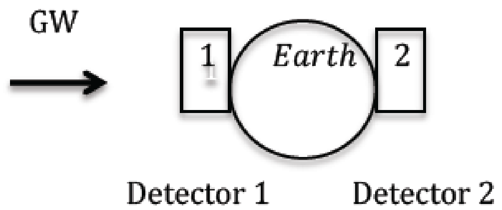

Figure 7.2. Detecting particle nature of g-wave

Experiment set up. G-Wave comes from a source and hits the detector 1, such as a LIGO detector, with the beam density $I_{g,inc}$. We want to measure the remaining beam density $I_{g,rem}$, on the detector 2, such as another LIGO detector, located on the opposite of a "gravito-conductor", which is defined as a system of g-charges under the influence of gravitation force, such as Earth. During their propagation in a uniform gravito-conductor, Gravito-photons collide with g-charges. Probability, $P_{g-photon}$, per unit length, that a single Gravito-photon interacts with g-charges in the slab, is

$$P_{g-photon} = n_{gc}\sigma_g dz, \quad (7.8)$$

where n_{gc} is the volume density of g-charges, σ_g is the scattering cross section. Then we have,

$$I_{g,rem}(z) = I_{g,inc}\, e^{-n_{gc}\sigma_g z}. \quad (7.9)$$

The results of observation allow us to estimate the cross section,

$$\sigma_{g-p} = \frac{1}{z n_{gc}} \ln\left(\frac{I_{g,inc}}{I_{g,rem}}\right). \quad (7.10)$$

For Earth, its diameter $z \approx 1.3 \times 10^9$cm, and $n_{gc} \approx 3.3 \times 10^{24}/\text{cm}^3$, we have

$$\sigma_{g-p} \approx 2.5\times10^{-34} \ln\left(\frac{I_{g,inc}}{I_{g,rem}}\right)\text{cm}^2. \quad (7.11)$$

If $I_{g,rem}$ would be detected and less than $I_{g,inc}$, the experimental result would be an evidence of the particle nature of G-Wave. Here we have ignored the cross-section of absorption of gravito-photons. Gravito-photons penetrate into a gravito-conductor with attenuation.

Once we determined σ_{g-p}, we have gravitational mean free path of gravito-photon through a gravito-conductor with known density. Eq. (7.11) will help us to calculate the intensity of G-Wave at the source, $I_{g,source}(z)$, by,

$$I_{g,source}(z) = I_{g,earth}\, e^{n_{g,universe}\ \sigma_{g-p} z_{source\ -earth}}, \quad (7.12)$$

where $I_{g,earth}$ is the intensity detected on Earth; $n_{g,universe}$ is the number density of the universe; σ_{g-p} is the gravito-photon cross-section; $z_{source\ -earth}$ is the distance between the source and Earth.

8. Discussion and Conclusion

In this article, gravity is studied as physical fields acting in space-time. Based on the experimentally confirmed Original-WEP for slow moving objects, we reveal an internal U(1) symmetry of gravity and then, established Gravito-dynamics and quantize it (QGD). The duality between electrodynamics and gravity is exposed. The underlying idea of duality is that the analogy between different interactions in Nature is not a mere coincidence. The duality between Electrodynamics and Gravito-dynamics and between QED and QGD show that the concept of duality is a powerful tool for studying the relations between theories of fundamental interactions.

One promising option to achieve the goal of unification is to establish a gauge theory of gravity at classical level and then, quantize it. We unify gravity with electrodynamics force at classical level first and then, unify QGD with QED to describe Electro-gravity interaction, which is then renormalized. We also extend gravity to contain SU(2) short-range force.

Obviously, all the theoretical results in this article depend on the basic Original-WEP being true. Although Original-WEP is consistent with existing experiments, we have proposed a synchrotron-type experiment to directly test whether the gravitational mass is equal to the inertial rest mass for relativistic objects [9,25]. We hope experimental physicists would find our theory intriguing enough to use their expertise to test both Original-WEP and S-WEP. S-WEP is foundation of GR.

At the macroscopic scale, the experiments testing the existence of gravito-magnetic fields and g-waves will test the validity of Gravito-dynamics. The GP-B experiment of Stanford University has reported positive results [28].

At the microscopic scale, studying the behaviors of Dirac particles in strong external gravitational fields will test the validity of QGD. Examining the behaviors of Dirac particles in both electromagnetic field and strong external gravitational field will test the validity of the unified Electro-gravity interaction.

We argue that Gravito-dynamics and QGD established in this article bridge the Newton's law and an ultimate theory of gravity, without difficulties in quantization, renormalization and unification with other interactions in Nature. "Breakthroughs typically arise from a change of perspective, and the ability to discover simplicity in the hitherto superficially complicated problem" [16].

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