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Magnetic Force Calculation between Magnets and Coils

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Abstract Applied magnetism has a wide range of applications in technology and industry. A significant magnetic force can be applied between two parts without any contact using coils and creating a magnetic field in the environment. It is also possible to strengthen the created magnetic force by placing different cores in the coil. The purpose of this research was to calculate the force between the coil and the coaxial magnet. In this system, a core with high permeability was considered for the coil. On the other hand, the distance between the coil and the magnet is such that when the coil is off, the effect between the coil and the magnet can be considered zero. The magnetic field produced by the magnet was also determined. Lorentz's force and potential theory was used to calculate the magnetic field and force. Note that the magnetic force between the coil and the magnet was only in the direction of the coil axis.

Keywords: *magnetic field, magnetic force, coil magnet, coaxial magnet, MATLAB*

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1. Introduction

Magnetic devices used in electrical engineering require magnets or coils. However, some devices, such as magnetic couplings, are made of permanent cylindrical magnets, while others, such as transformers, are made of coils. Some devices, such as sensors, actuators, and speakers, are built with magnets, coils, or both [1].

Electromagnetic coils are very important in modern medical and electrical sensors and systems. Generation of controlled magnetic fields has many applications, including spacecraft, magnetic resonance imaging, bioelectromagnetic research, and nondestructive testing (NDT). Derived directly from the well-known Maxwell's equations [2], the equivalent magnetic charge model and equivalent magnetizing current are widely used to obtain, either analytically or numerically, the magnetic field and magnetic force of permanent magnets [3]. For instance, S. Babic et al. calculated the magnetic force between a thin cylindrical coil and a coaxial cylindrical magnet by deriving the mutual inductance between them [4], while A. Shiri et al. calculated the force between two coaxial coils using the filament method as an efficient numerical method [5]. V. Suresh et al. calculated the magnetic field from a bobbin and coil using the magnetic field vector potential method. The space of the coiled coil was considered to be completely homogeneous in these calculations, a constant permeability was considered for the whole space [2]. I.R. Ciric calculated the magnetic field within a transformer using the magnetic scalar potential method. The calculations in this study were performed by equating its electric current by fictitious magnetization [6]. N. Ebrahimi et al. designed and optimized an electromagnetic soft actuator based on a coil with a permanent magnet core. In this design, they calculated the total magnetic field using the Biot-Savart law. In design optimization, the effect of geometric parameters such as length and radius of coils and magnet was investigated [7]. G. Lemarquand et al. calculated the force between two permanent coaxial magnets. They used the equivalent of a magnet with a current-carrying coil to calculate the magnetic force. Then, they calculated the magnetic field using the potential theory. They used mutual inductance between two equivalent coils to calculate the force between the two magnets [8]. W. Robertson et al. calculated the force between a coil with an air core and a coaxial permanent magnet with four different methods (shell method, finite element method, and two integral methods) and compared them. They have shown that the shell method is an efficient and faster way to design many magnetic stimuli [9].

In this study, we calculated the magnetic force between a coil with high permeability core and a coaxial permanent magnet using the Lorentz's theory of potential and force.

2. Magnetostatic Fundamentals

Let $\overline{\mathrm{H}}$, $\overline{\mathrm{B}}$, $\overline{\mathrm{J}}$, and μ be the respective magnetic field intensity vector (A/m), magnetic flux density vector (Wb/m), electric current density vector (A/m^2), and permeability

(H/m) in a region Ω. They are related through the well-known magnetostatic following equations [10,11,12]:

$$\bar{B} = \mu\bar{H} \tag{1}$$

$$\nabla.\bar{B} = 0 \tag{2}$$

$$\nabla \times \bar{H} = \bar{J} \tag{3}$$

$$\nabla.\bar{J} = 0 \tag{4}$$

2.1. Boundary Conditions

At the interface between two media (Figure 1), the respective vector quantities $\bar{H}_i$, $\bar{B}_i$, $\bar{J}_i$, and μ_i (i = 1, 2) should satisfy the boundary conditions [11]:

$$\left(\bar{H}_2 - \bar{H}_1\right) \times \bar{n} = \bar{H}_{t2} - \bar{H}_{t1} = \bar{J}_s \tag{5}$$

$$\left(\bar{B}_2 - \bar{B}_1\right).\bar{n} = \bar{B}_{n2} - \bar{B}_{n1} = 0 \tag{6}$$

with $\hat{n}$ the normal surface vector and $\bar{J}_s$ the surface current density vector at the interface.

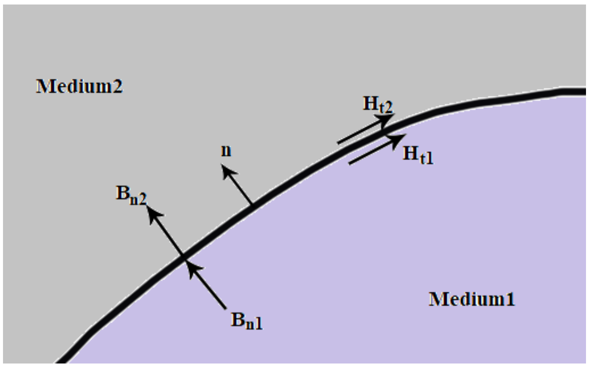

Figure 1. Fields and surface current in a general relationship at the boundary of two regions [11]

2.2. Helmholtz Decomposition

The Helmholtz theorem suggests that, via the ∇ operator, any vector F can be expressed as a sum of the gradient of a scalar function V and the curl of a vector function A. Hence [13]:

$$\bar{F} = -\nabla V + \nabla \times \bar{A} \tag{7}$$

At this step, two identities can be useful to further derive the equations [13]:

- The Curl of the gradient of a numerical field is zero (the existence of V and its derivatives is implicitly assumed in all points):

$$\nabla \times \left(\nabla V\right) = 0 \tag{8}$$

- The divergence of the curl of each vector is null:

$$\nabla.\left(\nabla \times \bar{A}\right) = 0. \tag{9}$$

2.3. Vector Potential

According to (4) and (9), the vector B can be expended as the curl of a vector potential [10].

$$\bar{B} = \nabla \times \bar{A}. \tag{10}$$

The gauge condition can be used to assume that the vector potential is unique, with [10]:

$$\nabla.\bar{A} = 0 \tag{11}$$

So, by replacing the vector potential in (3) we have:

$$-\nabla^2\bar{A} = \mu\left(\bar{r}\right).\bar{J} + \nabla\mu\left(\bar{r}\right) \times \bar{H}. \tag{12}$$

If the region Ω is simply connected area of (12) then:

$$-\nabla^2\bar{A} = \mu\left(\bar{r}\right).\bar{J} \tag{13}$$

leading to

$$\begin{cases} -\nabla^2 A_x = \mu\left(\bar{r}\right).J_x \\ -\nabla^2 A_y = \mu\left(\bar{r}\right).J_y \, . \\ -\nabla^2 A_z = \mu\left(\bar{r}\right).J_z \end{cases} \tag{14}$$

In this system, each equation is a Poisson equation. If the boundary of the region is assumed infinite and A (∞) ⟶ 0, the solution is as follows [12]:

$$\bar{A} = \frac{\mu}{4\pi}\int_V \frac{\bar{J}\left(\mathbf{r}'\right)dv'}{\left|\xi\right|} \tag{15}$$

where $\bar{\mathbf{r}}$ is the vector position of the point where we calculate the potential A and $\bar{\mathbf{r}}'$ is the vector position at the current position.
with

$$\bar{\xi} = \bar{\mathbf{r}} - \bar{\mathbf{r}}' = \left(r, \theta, z\right) - \left(r', \theta', z'\right) \tag{16}$$

$$\left|\bar{\xi}\right| = \left|\bar{\mathbf{r}} - \bar{\mathbf{r}}'\right| = \sqrt{r'^2 + r^2 + \left(z - z'\right)^2 - 2rr'\cos\left(\theta' - \theta\right)} \tag{17}$$

The solution of (14) in a cylindrical coordinate system can be determined using the method of separating variables. Thus, the Laplacian of a function ψ is written in the cylindrical coordinate system as:

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\psi\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\psi + \frac{\partial^2}{\partial z^2}\psi \tag{18}$$

then we can formulate it as [14]

$$\psi = R\left(r\right)Q\left(\theta\right)Z\left(z\right) \tag{19}$$

$$\begin{aligned} \rightarrow \nabla^2\psi = &\left(\frac{1}{R}\frac{\partial^2 R}{\partial r^2} + \frac{1}{rR}\frac{\partial R}{\partial r}\right) \\ &+ \left(\frac{1}{r^2 Q}\frac{\partial^2 Q}{\partial\theta^2}\right) + \left(\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}\right) \end{aligned} \tag{20}$$

So by substituting (20) into (13), we get:

$$\begin{aligned} \nabla^2 A = &\left(\frac{1}{R}\frac{\partial^2 R}{\partial r^2} + \frac{1}{rR}\frac{\partial R}{\partial r}\right) \\ &+ \left(\frac{1}{r^2 Q}\frac{\partial^2 Q}{\partial\theta^2}\right) + \left(\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}\right) = -\mu J \end{aligned} \tag{21}$$

In order for (21) to cover all the defined points (region Ω), each statement in parentheses in (21) must have a constant value. So:

$$\frac{1}{R}\frac{\partial^2 R}{\partial r^2}+\frac{1}{rR}\frac{\partial R}{\partial r}+\left(\frac{1}{r^2 Q}\frac{\partial^2 Q}{\partial \theta^2}\right)=-\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}=k^2 \quad (22)$$

$$\rightarrow Z(z)=\sum A e^{+ikz}+A' e^{-ikz}. \quad (23)$$

With the initial condition, we obtain:

$$A(z=0,r)=0 \rightarrow Z(z)=\sum A_n \sin(k_n z) \quad (24)$$

$$\rightarrow \frac{r^2}{R}\frac{\partial^2 R}{\partial r^2}+\frac{r}{R}\frac{\partial R}{\partial r}-k^2 r^2=-\frac{1}{Q}\frac{\partial^2 Q}{\partial \theta^2}=m^2 \quad (25)$$

$$\rightarrow r^2\frac{\partial^2 R}{\partial r^2}+r\frac{\partial R}{\partial r}-\left(k^2 r^2+m^2\right)R=0 \quad (26)$$

The above equation is a Modified Bessel equation and its response is also a Bessel function of second kind. Then:

$$R(r)=\sum_m a_m I_m(k_n r)+b_m K_m(k_n r)-C_n L_m(k_n r) \quad (27)$$

which leads to:

$$J(z)=\sum_{n=0}^{\infty} J_n \sin(k_n z) \quad (28)$$

$$J_n=\frac{2J}{n\pi}\left[\cos(k_n z_1)-\cos(k_n z_2)\right] \quad (29)$$

$$C_n=\mu_0\left(\frac{\pi}{2}\right)J_n k_n^{-2}. \quad (30)$$

Here, I_m and K_m are Bessel functions of first and second kind, respectively, and L is the modified Struve function of first kind [15]. Due to the symmetry of the system, the potential does not depend on the angle θ. If there is no J current, the Struve function coefficient (C_n) will be zero. The general solution of Equation (13) is as follows:

$$A=\sum_{m,n}\begin{pmatrix} a_{mn} I_m(k_n r)+b_{mn} K_m(k_n r) \\ -C_n L_m(k_n r) \end{pmatrix}\sin(k_n z). \quad (31)$$

On the other hand, the two-dimensional format of the Poisson equation (21) is as follows [16]:

$$\frac{1}{p}\frac{\partial}{\partial p}\left(p\frac{\partial A_l}{\partial p}\right)+\frac{\partial^2 A_l}{\partial q^2}-\frac{1}{p^2}A_l=-\mu J_l \quad (32)$$

where p, q, and l refer to the normal, tangential, and longitudinal direction (direction of r, z, θ, respectively). According to (32) and (26), m = 1. Therefore, (31) can be written as

$$A=\sum_{n}\begin{pmatrix} a_n I_1(k_n r)+b_n K_1(k_n r) \\ -C_n L_1(k_n r) \end{pmatrix}\sin(k_n z). \quad (33)$$

3. Magnetic Field from a Coil with High Permeability Core

To calculate the field in (13), the environment is divided into different regions (Figure 2) and the equation solved in each separate region. The whole region is placed between z = 0 and z = z5 and it is assumed that A = 0 in these boundaries. The boundaries of z = 0 and z = z5 should be far enough away from the coil so that the boundary values do not affect the solution of the problem. The coefficients are then calculated using boundary conditions.

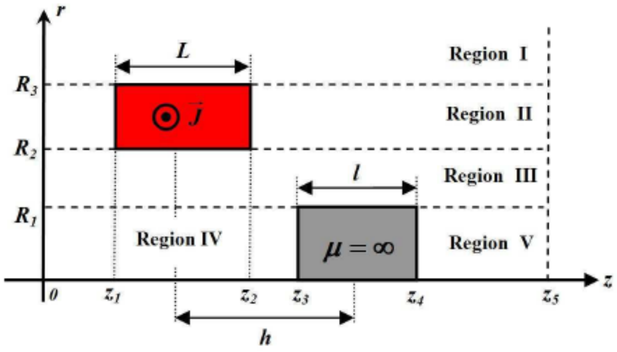

Figure 2. Division of regions to calculate the vector potential of a coil with high permeability core. The core is shown as a gray part and coil is shown as a red part. Region II contains coil with current density J. the space between coil and core is shown as Region III. The core does not belong to any region. In this figure, L, l, and h represent the respective length of the coil, length of the core and the distance between centroids of coil and core. [17]

Let us consider a coil and its core of same length and centroid and the space between the coil and the core is very small so:

$$z_1=z_3,\ z_2=z_4,\ R_2=R_1+eps \quad (34)$$

z_1 and z_2 are the axial position of the coil and z_3 and z_4 are the axial position of the core. They are shown in Figure 2. R_1 is the radius of the core and R_2 and R_3 are inner and outer radius of the coil respectively.

Because the permeability (μ) of the core is much larger than the permeability of surrounding environment, it is assumed that in the core $\mu \rightarrow \infty$. For this reason, the field's tangential component is zero at the core boundaries. Let us use the cylindrical coordinates. The current density J is in the direction of φvalue and the problem is symmetrical, so the potential for A vector is only in the direction of φ value, but its value is a function of r and z. Therefore, the magnetic field also depends only on r and z and is not in the direction of φ Since the drawn regions are hypothetical and the environment is homogeneous, then A is continuous in the boundary between these regions [8]. The current density of the coil is considered uniform and steady. Hence, for the coils with current I, length L, thickness R, and number of loops N:

$$J=\frac{IN}{LR} \quad (35)$$

Then, the Poisson's equation (13) is solved in each region separately by the method of dividing variables leading to a solution in the form expressed in (33). By influencing the boundary conditions, the unknown coefficients are calculated. Then, by using (10), the magnetic flux density is obtained. According to (33) [17]:

$$J=0 \rightarrow C_n=0. \quad (36)$$

If

$$\lim_{r\to\infty} A(r,z)=0 \to a_n=0 \quad (37)$$

If

$$A(z=0,r)=0,A(z=z_5,r)=0 \to k_n=\frac{n\pi}{(z_5-0)} \quad (38)$$

If

$$A(z=0,r)=0, \frac{\partial A}{\partial z}|_{z=z_3}=0$$
$$\to k_{2n}=0,\ k_{2n+1}=\frac{(2n+1)\pi}{2z_3}. \quad (39)$$

If

$$A(z=z_5,r)=0,\ \frac{\partial A}{\partial z}|_{z=z_4}=0 \to k_{2n}=0,$$
$$k_{2n+1}=\frac{(2n+1)\pi}{2(z_5-z_4)} \quad (40)$$

If

$$A(z,r=0)=M,|M|<\infty \to b_m^V=b_m^{IV}=0 \quad (41)$$

In region I:

$$(36),(37),(38) \to A^I=\sum_{n=1}^{\infty} b_n^I K_1(k_n r)\sin(k_n z) \quad (42)$$

In region II:

$$(38) \to A^{II}=\sum_{n=1}^{\infty}\begin{pmatrix} a_n^{II} I_1(k_n r) \\ +b_n^{II} K_1(k_n r) \\ -C_n L_1(k_n r)\end{pmatrix}\sin(k_n z) \quad (43)$$

In region III:

$$(36),(38) \to A^{III}=\sum_{n=1}^{\infty}\begin{pmatrix} a_n^{III} I_1(k_n r) \\ +b_n^{III} K_1(k_n r)\end{pmatrix}\sin(k_n z) \quad (44)$$

In region IV:

$$(36),(39),(41) \to A^{IV}=\sum_{m=1}^{\infty}\left(a_m^{IV} I_1(\beta_m r)\right)\sin(\beta_m z)$$

$$\beta_m=\frac{m\pi}{2z_3},\ m=odd\ integar \quad (45)$$

In region V:

$$(36),(40),(41) \to A^V=\sum_{m=1}^{\infty}\left(a_m^V I_1(\lambda_m r)\right)\cos(\lambda_m(z-z_4))$$

$$\lambda_m=\frac{m\pi}{2(z_5-z_4)},\ m=odd\ integar. \quad (46)$$

Now, using the boundary conditions, the value of the coefficients can be determined. The continuity of A and the tangential component of the magnetic flux density (Equation (5)) exist between the two regions, and the tangential component of the magnetic flux density at the core boundary is zero [17,18]. Thus,

$$B_z^i=\frac{1}{r}\frac{\partial rA^i}{\partial r},\ B_r^i=-\frac{\partial A^i}{\partial z},\ i=I,II,III,IV,V \quad (47)$$

$$B_{\text{tangential}}^i=\frac{1}{r}\frac{\partial rA^i}{\partial r}|_{r=R_1,R_2,R_3} \quad (48)$$

Therefore, the Boundary Conditions at r=R_3 give

$$A^I=A^{II} \quad (49)$$

$$\sum_{n=1}^{\infty} b_n^I K_1(k_n R_3)\sin(k_n z) -\sum_{n=1}^{\infty}\begin{pmatrix} a_n^{II} I_1(k_n R_3) \\ +b_n^{II} K_1(k_n R_3) \\ -C_n L_1(k_n R_3)\end{pmatrix}\sin(k_n z)=0 \quad (50)$$

$$b_n^I K_1(k_n R_3)-a_n^{II} I_1(k_n R_3) -b_n^{II} K_1(k_n R_3)+C_n L_1(k_n R_3)=0 \quad (51)$$

$$\frac{\partial rA^I}{\partial r}=\frac{\partial rA^{II}}{\partial r} \quad (52)$$

$$\sum_{n=1}^{\infty} b_n^I\left(-k_n K_0(k_n R_3)-\frac{K_1(k_n R_3)}{R_3}\right)\sin(k_n z) =\sum_{n=1}^{\infty}\begin{pmatrix} a_n^{II}\left(k_n I_0(k_n R_3)-\frac{I_1(k_n R_3)}{R_3}\right) \\ +b_n^{II}\left(-k_n K_0(k_n R_3)-\frac{K_1(k_n R_3)}{R_3}\right) \\ -C_n\left(k_n L_0(k_n R_3)-\frac{L_1(k_n R_3)}{R_3}\right)\end{pmatrix}\sin(k_n z) \quad (53)$$

$$k_n\begin{pmatrix} -b_n^I K_0(k_n R_3)+b_n^{II} K_0(k_n R_3) \\ -a_n^{II} I_0(k_n R_3)+C_n L_0(k_n R_3)\end{pmatrix} =\frac{1}{R_3}\begin{pmatrix} b_n^I K_1(k_n R_3)-b_n^{II} K_1(k_n R_3) \\ -a_n^{II} I_1(k_n R_3)+C_n L_1(k_n R_3)\end{pmatrix}=0 \quad (54)$$

$$-b_n^I K_0(k_n R_3)-a_n^{II} I_0(k_n R_3) +b_n^{II} K_0(k_n R_3)+C_n L_0(k_n R_3)=0 \quad (55)$$

$$(51)+(55) \to \begin{cases} b_n^{II}-b_n^I=C_n\dfrac{\begin{bmatrix} L_0(k_n R_3)I_1(k_n R_3) \\ -L_1(k_n R_3)I_0(k_n R_3)\end{bmatrix}}{\begin{bmatrix} K_0(k_n R_3)I_1(k_n R_3) \\ +K_1(k_n R_3)I_0(k_n R_3)\end{bmatrix}} \\ a_n^{II}=C_n\dfrac{\begin{bmatrix} L_0(k_n R_3)K_1(k_n R_3) \\ +L_1(k_n R_3)K_0(k_n R_3)\end{bmatrix}}{\begin{bmatrix} K_0(k_n R_3)I_1(k_n R_3) \\ +K_1(k_n R_3)I_0(k_n R_3)\end{bmatrix}} \end{cases} \quad (56)$$

$$\rightarrow\begin{cases} b_n^{II}-b_n^{I}=C_n k_n R_3\begin{pmatrix} L_0(k_nR_3)I_1(k_nR_3) \\ -L_1(k_nR_3)I_0(k_nR_3)\end{pmatrix} \\ a_n^{II}=C_n k_n R_3\begin{pmatrix} L_0(k_nR_3)K_1(k_nR_3) \\ +L_1(k_nR_3)K_0(k_nR_3)\end{pmatrix}\end{cases} \quad (57)$$

while at r=R2 we have (Boundary Conditions)

$$A^{III}=A^{II} \quad (58)$$

$$\begin{aligned}&\sum_{n=1}^{\infty}\left(a_n^{III}I_1(k_nR_2)+b_n^{III}K_1(k_nR_2)\right)\sin(k_nz)\\&-\sum_{n=1}^{\infty}\begin{pmatrix} a_n^{II}I_1(k_nR_2)+b_n^{II}K_1(k_nR_2) \\ -C_nL_1(k_nR_2)\end{pmatrix}\sin(k_nz)=0\end{aligned} \quad (59)$$

$$\begin{aligned}&a_n^{III}I_1(k_nR_2)-a_n^{II}I_1(k_nR_2)+b_n^{III}K_1(k_nR_2)\\&-b_n^{II}K_1(k_nR_2)+C_nL_1(k_nR_2)=0\end{aligned} \quad (60)$$

$$\frac{\partial rA^{III}}{\partial r}=\frac{\partial rA^{II}}{\partial r} \quad (61)$$

$$\begin{aligned}&\sum_{n=1}^{\infty}\begin{pmatrix} a_n^{III}\left(k_nI_0(k_nR_2)-\frac{I_1(k_nR_2)}{R_2}\right) \\ +b_n^{III}\left(-k_nK_0(k_nR_2)-\frac{K_1(k_nR_2)}{R_2}\right)\end{pmatrix}\sin(k_nz)\\&=\sum_{n=1}^{\infty}\begin{pmatrix} a_n^{II}\left(k_nI_0(k_nR_2)-\frac{I_1(k_nR_2)}{R_2}\right) \\ +b_n^{II}\left(-k_nK_0(k_nR_2)-\frac{K_1(k_nR_2)}{R_2}\right) \\ -C_n\left(k_nL_0(k_nR_2)-\frac{L_1(k_nR_2)}{R_2}\right)\end{pmatrix}\sin(k_nz)\end{aligned} \quad (62)$$

$$\begin{aligned}&k_n\begin{pmatrix} a_n^{III}I_0(k_nR_2)-a_n^{II}I_0(k_nR_2)-b_n^{III}K_0(k_nR_2) \\ +b_n^{II}K_0(k_nR_2)+C_nL_0(k_nR_2)\end{pmatrix}\\&=\frac{1}{R_2}\begin{pmatrix} a_n^{III}I_1(k_nR_2)-a_n^{II}I_1(k_nR_2)+b_n^{III}K_1(k_nR_2) \\ -b_n^{II}K_1(k_nR_2)+C_nL_1(k_nR_2)\end{pmatrix}\\&=0\end{aligned} \quad (63)$$

$$\begin{aligned}&a_n^{III}I_0(k_nR_2)-b_n^{III}K_0(k_nR_2)-a_n^{II}I_0(k_nR_2)\\&+b_n^{II}K_0(k_nR_2)+C_nL_0(k_nR_2)=0\end{aligned} \quad (64)$$

$$(60)+(64)\rightarrow\begin{cases} b_n^{III}-b_n^{II}=C_n\dfrac{\begin{bmatrix} L_0(k_nR_2)I_1(k_nR_2) \\ -L_1(k_nR_2)I_0(k_nR_2)\end{bmatrix}}{\begin{bmatrix} K_0(k_nR_2)I_1(k_nR_2) \\ +K_1(k_nR_2)I_0(k_nR_2)\end{bmatrix}} \\ a_n^{II}-a_n^{III}=C_n\dfrac{\begin{bmatrix} L_0(k_nR_2)K_1(k_nR_2) \\ +L_1(k_nR_2)K_0(k_nR_2)\end{bmatrix}}{\begin{bmatrix} K_0(k_nR_2)I_1(k_nR_2) \\ +K_1(k_nR_2)I_0(k_nR_2)\end{bmatrix}}\end{cases} \quad (65)$$

$$\rightarrow\begin{cases} b_n^{III}-b_n^{II}=C_n k_n R_2\begin{pmatrix} L_0(k_nR_2)I_1(k_nR_2) \\ -L_1(k_nR_2)I_0(k_nR_2)\end{pmatrix} \\ a_n^{II}-a_n^{III}=C_n k_n R_2\begin{pmatrix} L_0(k_nR_2)K_1(k_nR_2) \\ +L_1(k_nR_2)K_0(k_nR_2)\end{pmatrix}\end{cases} \quad (66)$$

At r=R$_1$, we have

$$A^{III}=A^{IV} \quad (67)$$

$$\begin{aligned}&\sum_{n=1}^{\infty}\left(a_n^{III}I_1(k_nR_1)+b_n^{III}K_1(k_nR_1)\right)\sin(k_nz)\\&=\sum_{m=1}^{\infty}a_m^{IV}I_1(\beta_mR_1)\sin(\beta_mz)\end{aligned} \quad (68)$$

$$\sum_{n=1}^{\infty}A_n\sin(k_nz)=\sum_{m=1}^{\infty}B_m\sin(\beta_mz) \quad (69)$$

$$\begin{aligned}B_m&=\frac{2}{z_3}\int_0^{z_3}A^{III}\sin(\beta_mz)dz\\&=\frac{2}{z_3}\sum_{n=1}^{\infty}A_n\left(\int_0^{z_3}\sin(k_nz)\sin(\beta_mz)dz\right)\\&=\frac{2}{z_3}\sum_{n=1}^{\infty}A_nf(n,m)\end{aligned} \quad (70)$$

$$f(n,m)=\begin{cases} -\dfrac{k_n\sin\left(\frac{m\pi}{2}\right)\cos(k_nz_3)}{\left(k_n^2-\beta_m^2\right)} & k_n\neq\beta_m \\ 0.5*z_3 & k_n=\beta_m\end{cases} \quad (71)$$

$$a_m^{IV}=\frac{2}{z_3}\sum_{n=1}^{\infty}\begin{pmatrix} a_n^{III}\dfrac{I_1(k_nR_1)}{I_1(\beta_mR_1)} \\ +b_n^{III}\dfrac{K_1(k_nR_1)}{I_1(\beta_mR_1)}g(n,m)\end{pmatrix}f(n,m) \quad (72)$$

$$A^{III}=A^{V} \quad (73)$$

$$\begin{aligned}&\sum_{n=1}^{\infty}\left(a_n^{III}I_1(k_nR_1)+b_n^{III}K_1(k_nR_1)\right)\sin(k_nz)\\&=\sum_{m=1}^{\infty}a_m^{V}I_1(\lambda_mr)\cos\left(\lambda_m(z-z_4)\right)\end{aligned} \quad (74)$$

$$\sum_{n=1}^{\infty}A_n\sin(k_nz)=\sum_{m=1}^{\infty}B_m\cos\left(\lambda_m(z-z_4)\right) \quad (75)$$

$$\begin{aligned}B_m&=\frac{2}{z_5-z_4}\int_0^{z_3}A^{III}\cos(\lambda_mz)dz\\&=\frac{2}{z_5-z_4}\sum_{n=1}^{\infty}A_n\left(\int_0^{z_3}\sin(k_nz)\cos(\lambda_mz)dz\right)\\&=\frac{2}{z_5-z_4}\sum_{n=1}^{\infty}A_ng(n,m)\end{aligned} \quad (76)$$

$$g(n,m)=\begin{cases} +\dfrac{k_n\cos(\lambda_m z_4)\cos(k_n z)}{\left(k_n^2-\lambda_m^2\right)} \\ -\dfrac{\lambda_m\sin(\lambda_m z_4)\sin(k_n z)}{\left(k_n^2-\lambda_m^2\right)}\Big|_{z_4}^{z_5} & k_n\neq\lambda_m \\ -\dfrac{\cos(2k_n z_4)}{4k_n} & k_n=\lambda_m \end{cases} \quad (77)$$

$$a_m^V=\frac{2}{z_5-z_4}\sum_{n=1}^{\infty}\left(\begin{array}{l} a_n^{III}\dfrac{I_1(k_n R_1)}{I_1(\lambda_m R_1)} \\ +b_n^{III}\dfrac{K_1(k_n R_1)}{I_1(\lambda_m R_1)}\end{array}\right)g(n,m) \quad (78)$$

$$\begin{cases} \dfrac{\partial rA^{III}}{\partial r}=\dfrac{\partial rA^{IV}}{\partial r} & z\in[0,z_3] \\ \dfrac{\partial rA^{III}}{\partial r}=0 & z\in[z_3,z_4] \\ \dfrac{\partial rA^{III}}{\partial r}=\dfrac{\partial rA^{V}}{\partial r} & z\in[z_4,z_5] \end{cases} \quad (79)$$

$$\frac{\partial rA^{III}(r,z)}{\partial r}=\sum_{n=1}^{\infty}A_n\sin(k_n z) \quad (80)$$

$$\begin{aligned} A_n&=\frac{2}{z_5}\int_0^{z_5}\frac{\partial rA^{III}(r,z)}{\partial r}\sin(k_n z)dz \\ &=\frac{2}{z_5}\left(\begin{array}{l}\displaystyle\int_0^{z_3}\frac{\partial rA^{IV}(r,z)}{\partial r}\sin(k_n z)dz \\ \displaystyle+\int_{z_4}^{z_5}\frac{\partial rA^{V}(r,z)}{\partial r}\sin(k_n z)dz\end{array}\right) \end{aligned} \quad (81)$$

$$\begin{aligned} &a_n^{III}-b_n^{III}\left(\frac{K_0(k_n R_1)}{I_0(k_n R_1)}\right) \\ &=\sum_{m=1}^{\infty}\left(\begin{array}{l} a_m^{IV}\dfrac{m}{nz_3}\dfrac{I_0(\beta_m R_1)}{I_0(k_n R_1)}f(n,m) \\ +a_m^{V}\dfrac{m}{n(z_5-z_4)}\dfrac{I_0(\lambda_m R_1)}{I_0(k_n R_1)}g(n,m)\end{array}\right). \end{aligned} \quad (82)$$

By solving the equations for the coefficients b_n^I ،a_n^{II} ، b_n^{II} ،a_n^{III} ،b_n^{III} ،a_m^{IV} و a_m^V the potential A will be obtained at all points in the Ω region (except for the inside of the core). Then, the magnetic flux density is obtained at all points using (47).

4. Lorentz Force

In the presence of an electric or magnetic field, an electric charge is applied to a force known as the Lorentz force [19].

$$\overline{F}=q\left(\overline{E}+\overline{v}\times\overline{B}\right) \quad (83)$$

with $\overline{E}$ the electrical field intensity vector and $\overline{v}$ the velocity vector of the moving charge.

Since the electric current in a volume V is related to the moving loads, the magnetic force applied to the object is equal to [12]:

$$\overline{F}=\int_V\overline{J}\times\overline{B}dv=I\int_V\overline{dv}\times\overline{B}. \quad (84)$$

5. The Magnetic Force Applied to the Cylindrical Magnet

Indeed, the magnetic field produced by a cylindrical permanent magnet can be determined with the same analytical formulation as the one used for a cylindrical thin coil. However, the magnetic field produced by a permanent magnet is not of the same order of magnitude as the one produced by a thin coil [1]. With a coil of same radius and length as the magnet, its uniform current density is [1]:

$$\overline{J}_{mag}=\frac{\overline{P}\times\overline{n}}{\mu_0} \quad (85)$$

where P is the polarization vector (Tesla), n the normal vector, and μ0 the permeability of free space. If the magnet is coaxial, the current density of the coil will be in the θ direction [12]. Using (84), the force applied to the magnet can be calculated. This force is only in the direction of the axis. It is important to note that in this case, the distance between the coil and the magnet is such that the magnet has a very small effect on the magnetization of the core of the coil, which is considered zero. This allows the magnetic field around the coil to be calculated independently of the magnet effect.

6. Generated Code

A Matlab code was generated. In this code, the potential resulting from a cylindrical coil with a high permeability core (assuming μ →∞) is initially calculated. The magnetic flux density is then calculated using a curl operator (∇×). Also, the cylindrical magnet calculations are equivalent to a thin-walled coil of the same length as the magnet. In this case, the coil and the magnet are coaxial, and their axis is the z-axis of the cylindrical coordinate system. The current of coil is uniform and only in the direction of θ. The polarization of the magnet is in the direction of the magnet axis, which is the z-axis. Thus, the current of the coil is only in the direction of θ. To calculate the force applied to the magnet, the effect of the magnetic field on the current of the equivalent coil is calculated using the Lorentz Equation.

The code inputs include coil length, internal and external coil radius, number of coil loops, magnet length, magnet radius, magnet polarization, and distance between magnet and coil centroids.

The coefficients b_n^I ،a_n^{II} ،b_n^{II} ،a_n^{III} ،b_n^{III} ،a_m^{IV} and a_m^V, which are the coefficients of the potential Bessel functions in the five defined regions, are calculated.

Initially, a_n^{II} and a_n^{III} ،are first calculated then, the coefficients b_n^{III} ،a_m^{IV} and a_m^V are obtained by solving the system of linear equations Ax = b (In principle, each unknown coefficient is an unknown vector) with A the matrix of the coefficients of the unknown variables x and b the constant vector of each equation.

Next, the coefficients b_n^I and b_n^{II} are obtained and the potential and magnetic flux and force derived in each of the regions considered.

Then, the modified Struve function of first kind is set to calculate the potential and density of the magnetic flux.

7. Problem Description

In this work, we considered a cylindrical coil with a very high permeability core and a coaxial magnet. The current density flowing through the coil is uniform. The magnet can be converted into a thin-walled coil of same size as the magnet by evaluating its polarization (the flux density in magnet). Moreover, the magnetic force applied to it can be calculated using the Lorentz Equation. We considered 9 inputs namely, the coil length, the inner radius of the coil (core radius), the outer radius of the coil, the number of coil loops, the magnet polarization, the distance between the centers of the coil and the magnet, the magnet radius, the length of the magnet, and the coil current. In nine different tests, we calculated the effect of these elements on the force between the coil and the magnet.

7.1. First Test: Effect of Coil Length

The coil has an inner radius of 0.03m, an outer radius of 0.05m, a current of 1A, and a number of loops of 1000. The magnet has a length of 0.02m, a radius of 0.03m, and a polarization of 1T. The distance between the coil and the magnet is 0.10m. By changing the length of the coil, we calculated its effect on the force (Figure 3).

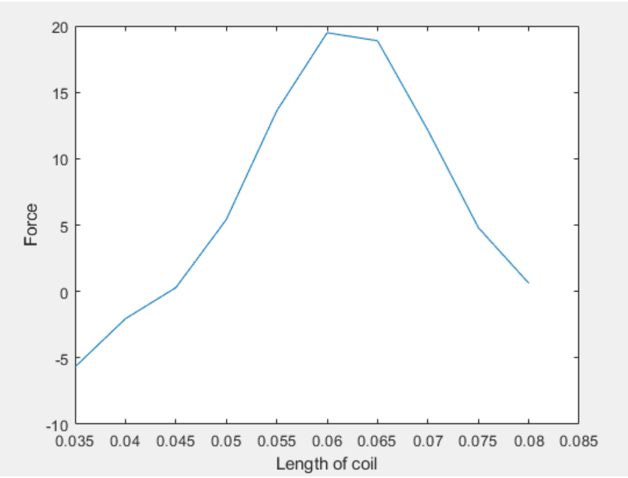

Figure 3. The effect of coil length on the magnetic force between the coil and the magnet

As shown in Figure 3, if the length of the coil is twice its inner radius, the force between the coil and the magnet increases.

7.2 Second Test: Effect on the Inner Radius of the Coil

The coil has an outer radius of 0.08m, and a length of 0.05m, a current of 1A, and a number of loops of 1000. The magnet has a length of 0.02m, a radius of 0.04m, and a polarization of 1T. The distance between the coil and the magnet is 0.10m. The inner radius of the coil was varied and we obtained its effect on the magnetic force (Figure 4)

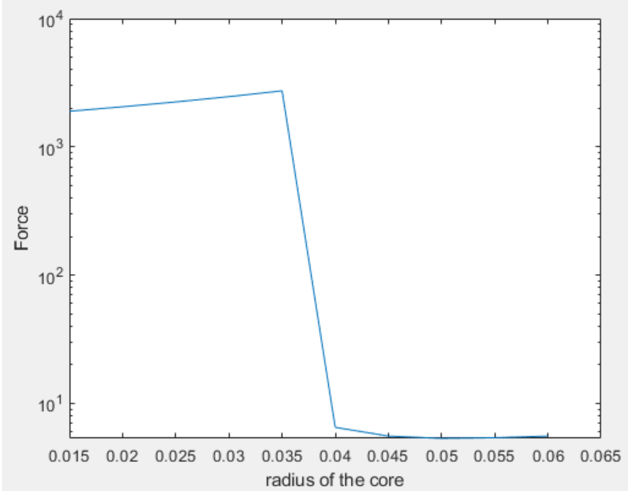

Figure 4. The effect of the inner radius of the coil on the magnetic force between the coil and the magnet

As shown in Figure 4, when the core radius of the coil is smaller than the radius of the magnet, the force is much higher, and the highest value occurs when there is a small difference between the radius of the core and the magnet.

7.3. Third Test: Effect of the Outer Radius of the Coil

The coil has an inner radius of 0.025m, an outer radius of 0.05m, and a current of 1A, and a number of loops of 1000. The magnet has a length of 0.02m, a radius of 0.04m, and a polarization of 1T. The distance between the coil and the magnet is 0.10m. The outer radius was varied and we obtained its effect on the magnetic force (Figure 5).

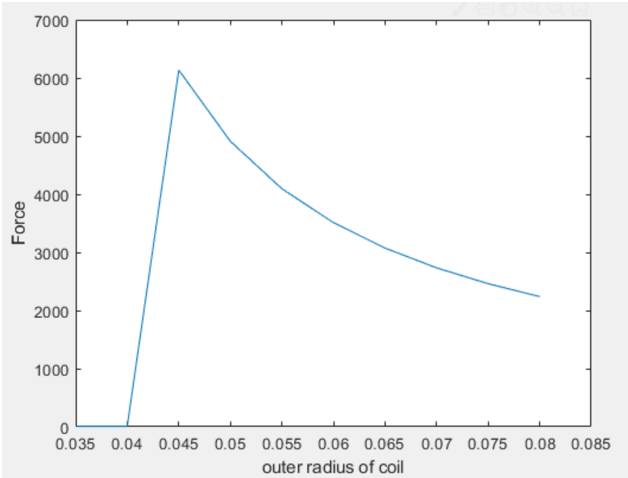

Figure 5. The effect of the outer radius of the coil on the magnetic force between the coil and the magnet

As shown in Figure 5, when outer radius of the coil is bigger than the radius of the magnet, the force is much higher. The highest value of the force occurs when there is small difference between outer radius of coil and radius of the magnet.

7.4. Fourth Test: Effect of the Number of Coil Loops

The coil has an inner radius of 0.03m, an outer radius of 0.05m, a length of 0.05m, and current of 1A. The magnet has a length of 0.02m, a radius of 0.04m, and a polarization of 1T. The distance between the coil and the magnet is 0.1m. We changed the number of loops in coil and calculated its effect (Figure 6).

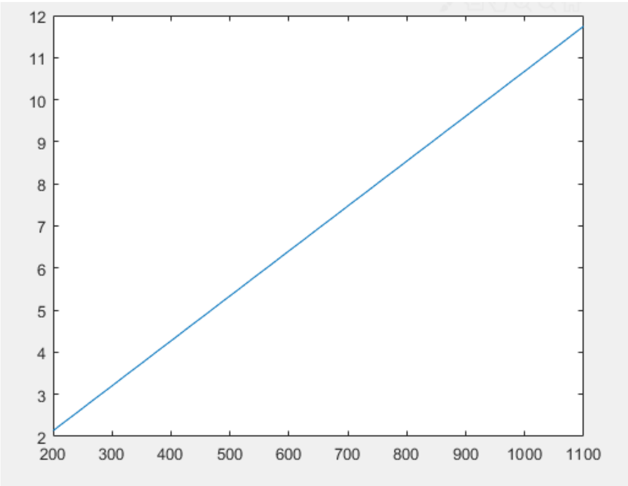

Figure 6. The effect of the number of coil loops on the magnetic force between the coil and the magnet

As shown in Figure 6, the effect of the number of coil loops on the magnetic force is linear.

7.5. Fifth Test: Effect of Magnet Polarization

The coil has an inner radius of 0.03m, an outer radius of 0.05m, a length of 0.05m, and current of 1A, and a number of loops of 1000. The magnet has a length of 0.02m, a radius of 0.04m. The distance between the coil and the magnet is 0.10m. We changed the polarization of the magnet and calculated its effect on the magnetic force (Figure 7).

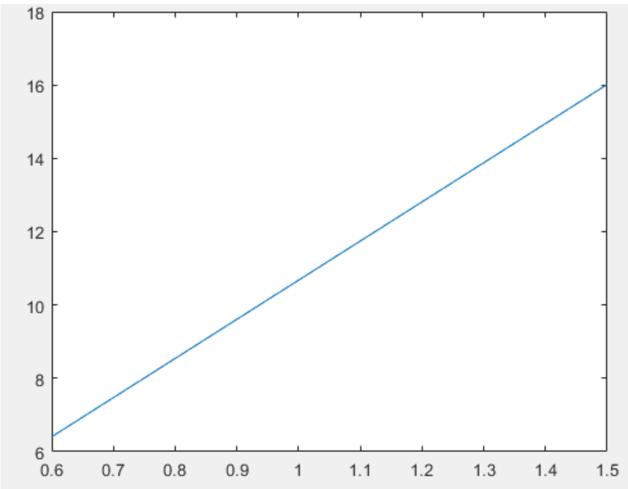

Figure 7. The effect of magnet polarization on the magnetic force between the coil and the magnet

For calculation the magnetic force between coil and magnet, the polarization of the magnet has linear relationship with its fictitious current density. The current density is uniform and has no effect on the final value of the integral of the force. The effect of the fictitious current density is linear, so as shown in Figure 7, the relationship between polarization and the magnetic force is also linear.

7.6. Sixth Test: Effect of the Distance between the Coil and the Magnet

The coil has an inner radius of 0.03m, an outer radius of 0.05m, a length of 0.05m, and current of 1A, and a number of loops of 1000. The magnet has a length of 0.02m, a radius of 0.04m, and a polarization of 1T. The distance between the coils and the magnet was varied and we calculated the magnetic force between them (Figure 8).

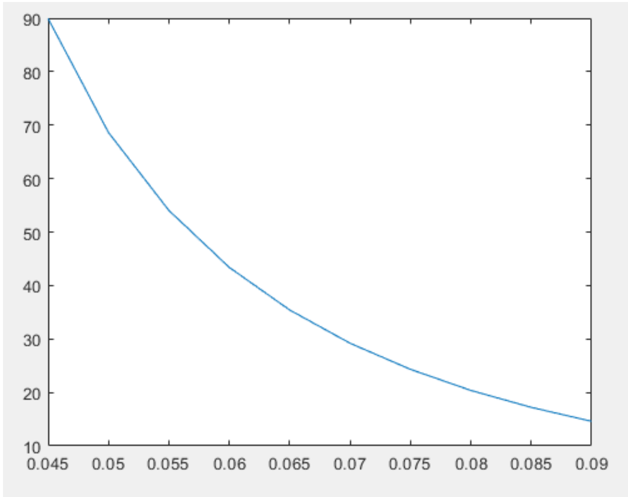

Figure 8. The effect of the distance between the coil and the magnet on the magnetic force between the coil and the magnet

The magnetic field decreases with distance. So, as we can see in Figure 8, it makes sense that when the distance between the coil and the magnet increases, the force between them decreases.

7.7. Seventh Test: Effect of the Magnetic Radius

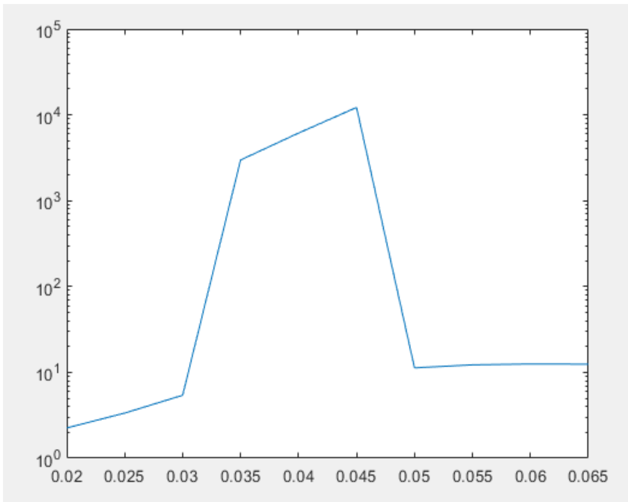

Figure 9. The effect of the magnetic radius on the magnetic force between the coil and the magnet

The coil has an inner radius of 0.03m, an outer radius of 0.05m, a length of 0.05m, and current of 1A, and a number of loops of 1000. The magnet has a length of 0.02m, and a polarization of 1T. The distance between the coil and the magnet is 0.10m. We have changed the radius of the magnet and calculated its effect on the magnetic force (Figure 9).

As we saw in Tests 2 and 3, there will be more force when the radius of the magnet has a value between the inner radius and the outer radius of the coil. As can be seen in Figure 9, the force will be greater if the radius of the magnet is closer to the outer radius of the coil.

7.8. Eighth Test: Effect of Magnet Length

The coil has an inner radius of 0.03m, an outer radius of 0.05m, a length of 0.05m, and current of 1A, and a number of loops of 1000. The magnet has a radius of 0.04m, and a polarization of 1T. The distance between the coil and the magnet is 0.10m. By changing the length of the magnet, its effect on the magnetic force was derived (Figure 10).

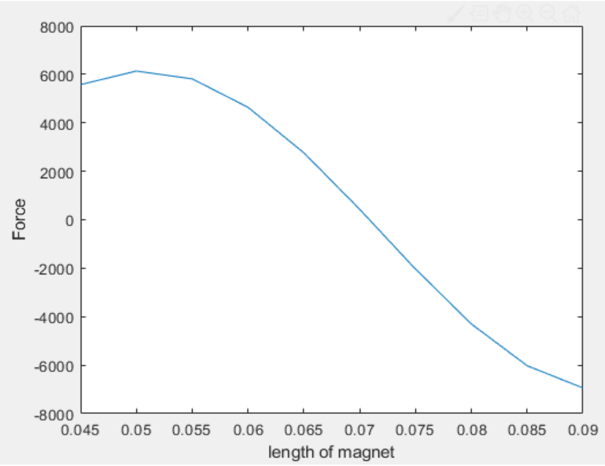

Figure 10. The effect of magnet length on magnetic force between coil and magnet

7.9. Ninth Test: Effect of Coil Current

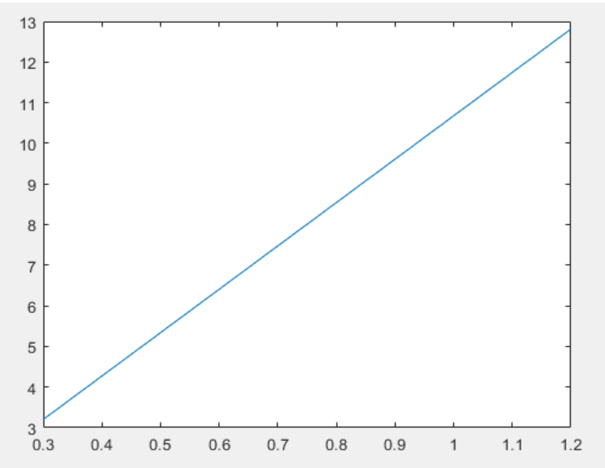

Figure 11. The effect of coil current on the magnetic force between the winding and the magnet, the effect of the coil current is linear to the magnetic force because we assume the coil current is uniform

The coil has an inner radius of 0.03m, an outer radius of 0.05m, a length of 0.05m, and a number of loops of 1000. The magnet has a length of 0.02m, a radius of 0.04m, and a polarization of 1T. The distance between the coil and the magnet is 0.10m. We changed the current and examined its effect on the magnetic force (Figure 11).

8. Conclusion

In this paper, the axial force between a cylindrical coil of high permeability and a coaxial cylindrical magnet was calculated using the Lorentz's force and the potential theory. The magnetic vector potential of the coil was initially calculated, and then, the magnetic flux density derived using a curl operator. The magnetic force exerted by the coil on the magnet was calculated using the Lorentz's force. Simulation were performed using a Matlab code, showing the effects of the size of the coil, the size of the magnet, the distance between them, the current of the coil and the polarization of the magnet on the magnetic force. The results showed that the distance and ratio of the magnetic radius to the radius of the coil core had the most significant effect on the magnitude of the force and that the effect of the number of coil loops and polarization on the force is linear.

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