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The Relativity Equations of Uniformly Accelerating Frames of Reference

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Abstract The work presented here is an extension of the work done in the article titled "The Conjugate Frame Method Relativity" [1]. In that work, we considered a moving reference frame that traveled at a constant speed v in a flat or curved space-time manifold. The moving reference frame traveled along a geodesic trajectory in a direction that was either directly towards or directly away from the stationary reference frame. The scalar acceleration a of the moving reference frame was equal to zero. In this work, we consider a moving reference frame that travels with a uniformly changing speed v in a flat or curved space-time manifold. The moving reference frame travels along a geodesic trajectory in a direction that is either directly towards or directly away from the stationary reference frame. The scalar acceleration a of the moving reference frame is a constant that is greater than or equal to zero. The Augmented Conjugate Frame Method is utilized in this work to derive the relativity equations of uniformly accelerating reference frames. These equations can apply to objects that are uniformly accelerated by gravitational, electric, or magnetic fields; as well as by other means, such as rocket propulsion. The relativity equations derived in this work reduce to the equations of Special Relativity when the moving reference frame has a zero scalar acceleration. The Augmented Conjugate Frame Method uses only scalar quantities in the derivation of the relativity equations of uniformly accelerating frames of reference.

Keywords: *the Augmented Conjugate Frame Method, uniform acceleration, geodesic, Planck acceleration, Planck time, Planck length, acceleration ratio, speed ratio, Planck epoch, black hole*

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1. Introduction

In this work, the Augmented Conjugate Frame Method is utilized to derive the relativity equations of an object that moves with a constant, i.e., uniform, scalar acceleration. Only scalar quantities are used in the derivation of these relativity equations. Accelerating objects are not inertial reference frames, and the equations of Special Relativity do not apply to their motion. The Augmented Conjugate Frame Method incorporates elements from classical and quantum physics into the derivation of the relativity equations of uniformly accelerating objects. The classical physics element enters the derivation by using the equation for the position of a uniformly accelerating object to determine the distance separating the stationary and moving reference frames. The uniform acceleration of the moving reference frame, i.e., object, can result from traveling in a space craft along the geodesic trajectory that connects the space craft with the stationary reference frame. The uniform acceleration can also result from the gravitational, electric, and magnetic forces that act on the object in the direction directly towards, or directly away from, the stationary reference frame. The quantum physics element enters the derivation by using the Planck acceleration as the largest possible acceleration that can be discerned in the universe [2].

For the acceleration of the moving object to be considered a constant during its motion over the distance s, we want the distance s from the moving object to an arbitrary point on it's geodesic trajectory to be much less than the distance x from the moving object to the stationary reference frame. This is analogous to the motion of an object that is freely falling near the surface of the earth. If the distance from the falling object to the surface of the earth is much less than the distance from the falling object to the center of the earth, then the acceleration of the freely falling object can be considered a constant during its motion. Throughout this work, the speed of light is a finite positive constant that has the same value c in all frames of reference. The terms reference frame and frame of reference are used interchangeably in this article, as are the terms constant acceleration and uniform acceleration. At certain places in this work, the moving and stationary reference frames are referred to as objects.

We know from classical physics that the distance x_v traveled by an object moving at a constant speed v for a time t is given by the equation $x_v = vt$, or $x_v = ct$ when the speed is equal to c. Solving this equation for the time t gives the equation $t = \frac{x_v}{c}$. Next, define the distance x to be the shortest distance separating the moving and stationary reference frames at any given time t. The moving reference frame travels along this shortest distance path, i.e., geodesic, of length x in a direction that is either directly towards, or directly away from, the stationary reference frame. The geodesic path of the moving reference frame is referred to as it's geodesic trajectory. The distance x is given by the equation $x = x_v + x_a$. The quantity x_a represents the contribution to the distance x that comes from the uniform scalar acceleration of the moving object. If the object moves with a constant scalar acceleration a, then from classical physics we have $x_a = \frac{1}{2}at^2$.

The Planck acceleration a_p is the largest possible discernible acceleration that a physical object can have in the universe [2]. This occurs when an object accelerates from a state of rest, i.e., $v = 0$, to the highest speed that is physically possible, i.e., the speed of light c, in the shortest period of time that can be discerned, i.e., the Planck time t_p. Hence, $a_p = \frac{c}{t_p}$ [2]. Solving the distance equation $x_a = \frac{1}{2}at^2$, when $a = a_p$, for the positive time t gives $t = \sqrt{\frac{2x_a}{a_p}}$. In general, the time $t = \frac{x_v}{v} = \sqrt{\frac{2x_a}{a}}$. The expressions given above will be utilized in the work that follows.

2. The Time in a Uniformly Accelerating Reference Frame

Let us use the Augmented Conjugate Frame Method to derive the relativity equation for the time in a uniformly accelerating reference frame. We postulate the stationary reference frame time equation to be

$$t_0 + \delta t = t, \tag{1}$$

where t_0 represents the time in the stationary reference frame, t represents the time in the moving reference frame, and δt represents the change in time due to the motion of the moving reference frame relative to the stationary reference frame. We can define the change in time by the equation $\delta t = \frac{x_v}{c} + \sqrt{\frac{2x_a}{a_p}}$. Substituting this expression for δt into the time equation yields

$$t_0 + \frac{x_v}{c} + \sqrt{\frac{2x_a}{a_p}} = t. \tag{2}$$

It now follows that the moving reference frame conjugate time equation, i.e., the time conjugate equation, is given by

$$\overline{t_0} - \frac{\overline{x_v}}{c} - \sqrt{\frac{2\overline{x_a}}{a_p}} = \overline{t} \tag{3}$$

[1]. Substituting the expressions $x_v = vt$ and $x_a = \frac{1}{2}at^2$ into the time equation yields

$$t_0 + \frac{vt}{c} + \sqrt{\frac{2\frac{at^2}{2}}{a_p}} = t, \tag{4}$$

$$t_0 + \frac{vt}{c} + \sqrt{\frac{a}{a_p}}t = t. \tag{5}$$

Making analogous substitutions for the corresponding barred expressions in the conjugate frame time equation gives

$$\overline{t_0} - \frac{v\overline{t}}{c} - \sqrt{\frac{a}{a_p}}\overline{t} = \overline{t}. \tag{6}$$

Solving equation (5) for t gives

$$t = \frac{t_0}{1 - (\frac{v}{c} + \sqrt{\frac{a}{a_p}})}. \tag{7}$$

Solving equation (6) for $\overline{t}$ yields

$$\overline{t} = \frac{\overline{t_0}}{1 + (\frac{v}{c} + \sqrt{\frac{a}{a_p}})}. \tag{8}$$

Now, multiplying equation (7) by its conjugate equation, i.e., equation (8), gives

$$t\overline{t} = \frac{t_0\overline{t_0}}{1 - (\frac{v}{c} + \sqrt{\frac{a}{a_p}})^2}. \tag{9}$$

Making the definitions $T^2 = t\overline{t}$ and $T_0^2 = t_0\overline{t_0}$, and substituting them into equation (9) yields

$$T^2 = \frac{T_0^2}{1 - (\frac{v}{c} + \sqrt{\frac{a}{a_p}})^2}. \tag{10}$$

Finally, taking the positive square root of both sides of equation (10) gives the result

$$T = \frac{T_0}{\sqrt{1 - (\frac{v}{c} + \sqrt{\frac{a}{a_p}})^2}}. \tag{11}$$

This is the relativity equation for the time in a uniformly accelerating reference frame. If the moving reference frame is not accelerating, then $a = 0$, and the equation reduces to the Special Relativity time equation [1,3,4].

3. The Mass, Momentum, and Energy of a Uniformly Accelerating Object

We can define the total energy ϵ of a uniformly accelerating object as the sum of its rest energy ϵ_0, kinetic energy k, and potential energy u. Therefore, in the unbarred stationary reference frame, the energy equation is given by

$$\epsilon_0 + k + u = \epsilon. \tag{12}$$

In the corresponding barred moving reference frame, the conjugate energy equation is given by

$$\overline{\epsilon_0} - \overline{k} - \overline{u} = \overline{\epsilon} \tag{13}$$

[1]. The potential energy is given by the equation $u = mas$, where m represents the mass of the moving object, a represents the uniform scalar acceleration of the moving object, and s represents the distance from the moving object to an arbitrary point on its geodesic trajectory. We want the distance s to be much less than the distance x from the moving object to the stationary reference frame. This ensures that the scalar acceleration of the moving object is uniform, i.e., constant, as it travels the distance s along its geodesic trajectory. We can represent the distance s by the equation $s = \frac{c^2}{a}$. If $a = a_p$, then $s = \frac{c^2}{a_p}$ [5]. This is the minimum value of s, and we show in the appendix that it is equal to the Planck length l_p. Substituting the representation $s = \frac{c^2}{a_p}$ into the potential energy equation gives $u = \frac{mac^2}{a_p}$. Now, substitute the Einstein mass-energy relations $\epsilon_0 = m_0c^2$ and $\epsilon = mc^2$, the kinetic energy relation $k = pc$; where p is the momentum of the moving object, and the potential energy relation $u = \frac{mac^2}{a_p}$ into the stationary reference frame energy equation. The energy equation then becomes

$$m_0c^2 + pc + mc^2 \frac{a}{a_p} = mc^2. \tag{14}$$

Making the analogous substitutions in equation (13) gives the corresponding conjugate frame energy equation

$$\overline{m_0}c^2 - \overline{p}c - \overline{m}c^2 \frac{a}{a_p} = \overline{m}c^2. \tag{15}$$

Now, we know from classical physics that the momentum p is given by the equation $p = mv$. Using this relation in equation (14), and it's conjugate expression $\overline{p} = \overline{m}v$ in equation (15) gives

$$m_0c^2 + mvc + mc^2 \frac{a}{a_p} = mc^2 \tag{16}$$

and

$$\overline{m_0}c^2 - \overline{m}vc - \overline{m}c^2 \frac{a}{a_p} = \overline{m}c^2, \tag{17}$$

respectively.

Dividing both sides of equations (16) and (17) by c^2, and solving for the mass m in equation (16) and the mass $\overline{m}$ in equation (17) yields

$$m = \frac{m_0}{1 - \left(\frac{v}{c} + \frac{a}{a_p}\right)} \tag{18}$$

and

$$\overline{m} = \frac{\overline{m_0}}{1 + \left(\frac{v}{c} + \frac{a}{a_p}\right)}, \tag{19}$$

respectively.

Then, multiplying equation (18) by equation (19) gives

$$m\overline{m} = \frac{m_0\overline{m_0}}{1 - \left(\frac{v}{c} + \frac{a}{a_p}\right)^2}. \tag{20}$$

Let us define $M^2 = m\overline{m}$ and $M_0^2 = m_0\overline{m_0}$. Making these substitutions in equation (20) gives

$$M^2 = \frac{M_0^2}{1 - \left(\frac{v}{c} + \frac{a}{a_p}\right)^2}. \tag{21}$$

Finally, taking the positive square root of both sides of equation (21) yields

$$M = \frac{M_0}{\sqrt{1 - \left(\frac{v}{c} + \frac{a}{a_p}\right)^2}}. \tag{22}$$

This is the relativity equation for the mass of a uniformly accelerating object. If the moving object is not accelerating, then $a = 0$, and the equation reduces to the Special Relativity mass equation [1,3,4]. Multiplying both sides of the relativity equation for the mass of a uniformly accelerating object by the speed v gives the relativity equation for the momentum of a uniformly accelerating object. Hence,

$$P = \frac{P_0}{\sqrt{1 - \left(\frac{v}{c} + \frac{a}{a_p}\right)^2}}. \tag{23}$$

Similarly, multiplying both sides of the relativity equation for the mass of a uniformly accelerating object by c^2 gives the relativity equation for the energy of a uniformly accelerating object. Therefore,

$$E = \frac{E_0}{\sqrt{1-(\frac{v}{c}+\frac{a}{a_p})^2}}. \tag{24}$$

Let us derive the energy-momentum equation, i.e., the relativistic dispersion relation, of a uniformly accelerating object. Recalling the relativity equation for the mass of a uniformly accelerating object, we have

$$M\sqrt{1-(\frac{v}{c}+\frac{a}{a_p})^2} = M_0, \tag{25}$$

$$M^2[1-(\frac{v}{c}+\frac{a}{a_p})^2] = M_0^2, \tag{26}$$

$$M^2 - M^2(\frac{v}{c}+\frac{a}{a_p})^2 = M_0^2, \tag{27}$$

$$M^2c^2 - \left(Mv + Mc\frac{a}{a_p}\right)^2 = M_0^2c^2, \tag{28}$$

$$ME - \left(P + Mc\frac{a}{a_p}\right)^2 = M_0E_0, \tag{29}$$

$$ME = M_0E_0 + \left(P + Mc\frac{a}{a_p}\right)^2. \tag{30}$$

Finally, multiplying both sides of equation (30) by c^2 gives

$$E^2 = E_0^2 + \left(Pc + E\frac{a}{a_p}\right)^2. \tag{31}$$

This is the energy-momentum equation, i.e., the relativistic dispersion relation, of a uniformly accelerating object.

4. The Length of a Uniformly Accelerating Object

Let us now derive the relativity equation for the length of a uniformly accelerating object. We begin by considering the change in length of an object that is moving with a constant scalar acceleration a, and a uniformly changing speed v, along the geodesic path connecting the rest and moving reference frames. Let l represent the length of the moving object, and let l_0 represent the length of the object at rest. The rest length l_0 is defined as the distance light travels when going from one end of the stationary object to the other end. Let t_0 represent the time it takes for light to travel the distance l_0. Since the speed of light c is a constant, we know that the distance l_0 is given by the equation

$$l_0 = ct_0. \tag{32}$$

The change in length δl_0 of the uniformly accelerating object is given by the distance the object travels in the time t_0. Therefore, from classical physics, we have

$$\delta l_0 = vt_0 + \frac{1}{2}at_0^2. \tag{33}$$

We postulate the length equation to be

$$l_0 + \delta l_0 = l. \tag{34}$$

Substituting the expression for δl_0 into the length equation gives

$$l_0 + vt_0 + \frac{1}{2}at_0^2 = l. \tag{35}$$

From the perspective of the moving reference frame, the conjugate frame length equation is

$$\overline{l_0} - v\overline{t_0} - \frac{1}{2}a\overline{t_0}^2 = \overline{l} \tag{36}$$

[1]. Substituting the representation $t_0 = \frac{l_0}{c}$ into the speed v term of the length equation, and substituting the representation $t_0^2 = \frac{2l_0}{a_p}$ into the scalar acceleration a term of the length equation yields

$$l_0 + v\frac{l_0}{c} + \frac{2l_0a}{2a_p} = l. \tag{37}$$

We can see that $t_0^2 = \frac{l_0^2}{c^2} = \frac{2l_0}{a_p}$. Solving for l_0, we obtain $l_0 = \frac{2c^2}{a_p} = \frac{2c^2}{\frac{c}{t_p}} = 2ct_p = 2l_p$, where t_p is the Planck time and l_p is the Planck length [2]. See the Appendix below for information regarding the Planck time and the Planck length. Simplifying equation (37), we obtain

$$l = l_0[1+(\frac{v}{c}+\frac{a}{a_p})]. \tag{38}$$

Similarly, substituting the representation $\overline{t_0} = \frac{\overline{l_0}}{c}$ into the speed v term of the conjugate frame length equation, and substituting the representation $\overline{t_0}^2 = \frac{2\overline{l_0}}{a_p}$ into the scalar acceleration a term of the conjugate frame length equation yields

$$\overline{l} = \overline{l_0}[1-(\frac{v}{c}+\frac{a}{a_p})]. \tag{39}$$

Analogous to l_0, we can show that $\overline{l_0} = 2l_p$. Multiplying equation (38) by equation (39) then gives

$$l\overline{l} = l_0\overline{l_0}[1-(\frac{v}{c}+\frac{a}{a_p})^2]. \tag{40}$$

Let us define the variables $L_0^2 = l_0\overline{l_0} = 4l_p^2$ and $L^2 = l\overline{l}$. Substituting these expressions into equation (40) yields

$$L^2 = L_0^2[1-(\frac{v}{c}+\frac{a}{a_p})^2]. \quad (41)$$

By taking the positive square root of both sides of this equation, we obtain

$$L = L_0\sqrt{1-(\frac{v}{c}+\frac{a}{a_p})^2}. \quad (42)$$

This is the relativity equation for the length of a uniformly accelerating object that has a rest length $L_0 = 2l_p$. Let us define the arbitrary length X_0 of an object at rest by the equation $X_0 = rL_0$, where the scale factor r is a non-negative real number. Let us define the length X of a uniformly accelerating object that has a rest length X_0 by the equation $X = rL$. Multiplying both sides of equation (42) by the scale factor r, and substituting the length variables X_0 and X into the resulting equation yields

$$X = X_0\sqrt{1-(\frac{v}{c}+\frac{a}{a_p})^2}. \quad (43)$$

This is the relativity equation for the length of a uniformly accelerating object that has an arbitrary rest length. If the moving object is not accelerating, then $a = 0$, and the equation reduces to the Special Relativity length equation [1,3,4].

5. The Electric Charge and the Electric Current in a Uniformly Accelerating Reference Frame

Consider the electric charge q_0 at rest. When the charge moves with a uniformly changing speed v and a constant scalar acceleration a, it becomes the electric current I. We postulate that the value q_0 of the electric charge at rest plus the value It of the electric charge in motion equals the total value q of the electric charge. The time can be defined by the equation $t = \frac{x_v}{c}+\sqrt{\frac{2x_a}{a_p}}$. The stationary reference frame electric charge equation is given by

$$q_0 + It = q. \quad (44)$$

Substituting the expression for the time t into the electric charge equation gives

$$q_0 + I(\frac{x_v}{c}+\sqrt{\frac{2x_a}{a_p}}) = q. \quad (45)$$

Let us define the electric current by $I = \frac{q_0}{t}$. Substituting the expressions $I = \frac{q_0}{t}$, $x_v = vt$, and $x_a = \frac{1}{2}at^2$ into the electric charge equation yields

$$q_0 + \frac{q_0 vt}{ct} + \frac{q_0}{t}\sqrt{\frac{2\frac{at^2}{2}}{a_p}} = q, \quad (46)$$

$$q_0 + q_0\frac{v}{c} + q_0\sqrt{\frac{a}{a_p}} = q. \quad (47)$$

We can write this equation for the electric charge q as

$$q = q_0[1+(\frac{v}{c}+\sqrt{\frac{a}{a_p}})]. \quad (48)$$

Following this same procedure, we can derive the electric charge equation relative to the moving reference frame. Upon doing this, we arrive at the conjugate frame electric charge equation, which is given by

$$\overline{q} = \overline{q_0}[1-(\frac{v}{c}+\sqrt{\frac{a}{a_p}})] \quad (49)$$

[1]. Multiplying equations (48) and (49) together, defining the physical charges Q_0 and Q, substituting Q_0 and Q into the product of equations (48) and (49), and taking the positive square root of both sides of the resulting equation yields

$$Q = Q_0\sqrt{1-(\frac{v}{c}+\sqrt{\frac{a}{a_p}})^2}. \quad (50)$$

This is the relativity equation of a uniformly accelerating electric charge.

The uniformly accelerating electric current I is given by the equation $I = \frac{Q}{T}$. Substituting the relativity expressions for the electric charge Q and time T into the equation for I, and setting $I_0 = \frac{Q_0}{T_0}$, we obtain

$$I = I_0[1-(\frac{v}{c}+\sqrt{\frac{a}{a_p}})^2]. \quad (51)$$

This is the relativity equation of a uniformly accelerating electric current.

6. The Temperature of a Uniformly Accelerating Object

We first define the variables that we will use to derive the relativity equation for the temperature of a uniformly accelerating object. Let θ_0 represent the temperature of an object at rest. Let θ represent the temperature of an identical object that moves with the uniformly changing speed v and constant scalar acceleration a. The moving object travels either directly towards or directly away from the rest object along the geodesic path that connects them. Let x represent the shortest distance separating the moving object and the stationary object at any given time t. The distance $x = x_v + x_a$, where $x_v = vt$ and $x_a = \frac{1}{2}at^2$. Let D_v represent the speed pseudo temperature gradient at the location of the moving object, and let D_a represent the scalar acceleration pseudo temperature gradient at the location of the moving object [1]. Next, we postulate the temperature equation to be

$$\theta_0 + \delta\theta = \theta, \quad (52)$$

where $\delta\theta$ represents the change in temperature of the moving object due to it's motion relative to the stationary object, and is defined by the equation

$$\delta\theta = D_v x_v + D_a x_a. \quad (53)$$

Substituting this expression for $\delta\theta$ into the temperature equation gives

$$\theta_0 + D_v x_v + D_a x_a = \theta. \quad (54)$$

By substituting the representations $x_v = vt$ and $x_a = \frac{1}{2}at^2$ into the temperature equation, we obtain

$$\theta_0 + D_v vt + D_a \frac{1}{2}at^2 = \theta. \quad (55)$$

Substituting the representation $t = \frac{x_v}{c}$ into the speed v term of the temperature equation, and substituting the representation $t^2 = \frac{2x_a}{a_p}$ into the scalar acceleration a term of the temperature equation yields

$$\theta_0 + D_v v \frac{x_v}{c} + D_a \frac{2x_a a}{2a_p} = \theta. \quad (56)$$

Now, define the speed pseudo temperature gradient by the equation

$$D_v = \frac{\theta}{x_v}, \quad (57)$$

and define the scalar acceleration pseudo temperature gradient by the equation

$$D_a = \frac{\theta}{x_a}. \quad (58)$$

Substituting these expressions for D_v and D_a into the temperature equation gives

$$\theta_0 + \frac{\theta}{x_v} v \frac{x_v}{c} + \frac{\theta}{x_a} \frac{2x_a a}{2a_p} = \theta, \quad (59)$$

$$\theta_0 + \theta\left(\frac{v}{c} + \frac{a}{a_p}\right) = \theta. \quad (60)$$

Simplifying this equation, we obtain

$$\theta_0 = \theta\left[1 - \left(\frac{v}{c} + \frac{a}{a_p}\right)\right]. \quad (61)$$

Solving for θ, we have

$$\theta = \frac{\theta_0}{1 - \left(\frac{v}{c} + \frac{a}{a_p}\right)}. \quad (62)$$

To complete the derivation of the relativity equation for the temperature of a uniformly accelerating object, the following steps should be taken. First, derive the corresponding conjugate frame temperature equation [1]. Second, multiply the stationary reference frame temperature equation by the conjugate moving reference frame temperature equation. Third, define the physical temperature variables Θ and Θ_0. Fourth, substitute these physical temperature variables into the equation that was produced by multiplying the two reference frame temperature equations together. Fifth, take the positive square root of both sides of the resulting temperature equation. Upon completion of these five steps, we obtain the result

$$\Theta = \frac{\Theta_0}{\sqrt{1 - \left(\frac{v}{c} + \frac{a}{a_p}\right)^2}}. \quad (63)$$

This is the relativity equation for the temperature of a uniformly accelerating object.

7. Discussion and Conclusions

The relativity equations of uniformly accelerating objects that are derived in this work are extensions of the relativity equations of non-accelerating objects that were derived in the work titled "The Conjugate Frame Method Relativity" [1]. The relativity equations of the Augmented Conjugate Frame Method reduce to the equations of Special Relativity when the moving reference frame has a scalar acceleration that is equal to zero [1,3,4]. The values of the scalar acceleration and speed of the moving reference frame, at any given time, are needed in order to evaluate the relativity equations of the Augmented Conjugate Frame Method. The relativity equations derived in this work are independent of the directions that the moving reference frame is moving and accelerating in as it travels along its geodesic trajectory. In other words, it does not matter whether the moving reference frame is traveling towards or away from the stationary reference frame, or whether the moving reference frame is accelerating towards or away from the stationary reference frame. The relativity equations are the same in all cases. The Augmented Conjugate Frame Method generates two distinct dimensionless factors in the derivation of its relativity equations of uniformly accelerating reference frames. This is in contrast with having one Lorentz factor in the Special Relativity equations of inertial reference frames [3,4].

For many purposes, the contributions from the scalar acceleration terms in the relativity equations derived in this work are negligible, and can be neglected. The acceleration ratio $\frac{a}{a_p}$ is exceedingly small for scalar accelerations encountered in classical physics. An example of a physical event where the acceleration ratio may be large enough to affect the relativity equations in a meaningful way is the Big Bang. The universe is thought to have expanded with an acceleration that was possibly approaching the value of the Planck acceleration during the Planck epoch of the early universe [2]. Another example of where the acceleration ratio may be of sufficient magnitude to affect the relativity equations is inside the horizon of a black hole [2]. More work needs to be done to determine the effects of the acceleration ratio $\frac{a}{a_p}$ and speed ratio $\frac{v}{c}$ on the relativity equations of uniformly accelerating frames of reference. The combined effects of these two ratios can be significant.

The speed v is a non-negative quantity, and the speed of light c is a positive quantity. The scalar acceleration a is a non-negative quantity, and the Planck acceleration a_p is a positive quantity. The acceleration ratio and the speed ratio are dimensionless real numbers that are greater than or equal to zero and less than or equal to one. It follows from Special Relativity that the speed of a moving object cannot exceed the speed of light c [3,4]. This requires the moving object's scalar acceleration to approach zero as its speed approaches c. The relativity equations that are derived in this work apply only to regions along the moving object's geodesic trajectory where its scalar acceleration can be considered a constant.

Table 1. The Relativity Equations of Uniformly Accelerating Objects. Here, $\zeta = \left[1-\left(\frac{v}{c}+\sqrt{\frac{a}{a_p}}\right)^2\right]^{-\frac{1}{2}}$ **and** $\eta = \left[1-\left(\frac{v}{c}+\frac{a}{a_p}\right)^2\right]^{-\frac{1}{2}}$.

Table Of Results	
Variable	Equation
Time	$T = T_0\zeta$
Length	$X = X_0\eta^{-1}$
Mass	$M = M_0\eta$
Energy	$E = E_0\eta$
Momentum	$P = P_0\eta$
Electric Charge	$Q = Q_0\zeta^{-1}$
Electric Current	$I = I_0\zeta^{-2}$
Temperature	$\Theta = \Theta_0\eta$
Relativistic Dispersion	$E^2 = E_0^2 + \left(Pc + E\frac{a}{a_p}\right)^2$

The relativity equations of the Augmented Conjugate Frame Method combine elements from classical and quantum physics. The equation that we used in this work for the distance x separating the stationary and moving reference frames comes from classical physics. In classical physics, this equation is used to determine the trajectory of an object that is uniformly accelerated by a gravitational field. This equation also applies to objects that are uniformly accelerated by electric fields, magnetic fields, space craft engines, and other means of acceleration. We can utilize this equation in the Augmented Conjugate Frame Method as long as the uniformly accelerated object travels along the geodesic trajectory that connects the object to the stationary reference frame. The relativity equations of uniformly accelerating objects do not depend on how the objects are accelerated. The Planck acceleration, Planck time, and Planck length are constants that have applications in quantum physics and quantum gravity. The Augmented Conjugate Frame Method explicitly incorporates the Planck acceleration into the relativity equations of uniformly accelerating objects. This allows these equations to apply to regions ranging in size from the macroscopic down to the quantum scale. See Table 1 for a listing of the relativity equations of uniformly accelerating objects that are predicted by the Augmented Conjugate Frame Method. These equations may be verified by conducting experiments.

Appendix

The Planck acceleration a_p is considered by researchers to be the largest possible acceleration that can be discerned in the universe [2]. The value of a_p is given by the equation

$$a_p = \frac{c}{t_p}, \tag{64}$$

where c is the speed of light and t_p is the Planck time. By substituting the values $c = 299792958\frac{m}{s}$ and $t_p = 5.39116\times10^{-44}s$ into the Planck acceleration equation, we obtain

$$a_p = \frac{299792958\frac{m}{s}}{5.39116\times10^{-44}s}, \tag{65}$$

$$a_p \approx 5.5608185\times10^{51}\frac{m}{s^2} \tag{66}$$

[6]. It is shown above that the minimum value of the distance s separating the moving object and an arbitrary point on its geodesic trajectory is given by the equation $s = \frac{c^2}{a_p}$. Let us call this quantity s_{min}. Now, $s_{min} = \frac{c^2}{a_p} = \frac{c^2}{\frac{c}{t_p}} = ct_p = l_p$, where l_p is the Planck length.

The Planck length $l_p = 1.616229\times10^{-35}m$, and is thought to be on the scale where quantum gravitational effects become relevant [2,6].

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