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The Conjugate Frame Method Relativity

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Abstract Researchers typically use four dimensional vectors in Minkowski space-time to derive Albert Einstein's Special Theory of Relativity equations. In this work, I have created an original method that uses only scalar quantities in the derivation of relativity equations. The Conjugate Frame Method reproduces the time, mass, and length equations of Special Relativity. Equations for the relativistic electric charge and relativistic temperature are also derived using this method. Unlike the equations of Special Relativity, the Conjugate Frame Method Relativity equations are applicable to both inertial and non-inertial reference frames. The creation of the Conjugate Frame Method was motivated by work done in real quaternion relativity [1,2].

Keywords: *special relativity, real quaternion relativity, reference frame, scalar, geodesic*

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1. Introduction

The Conjugate Frame Method considers scalar variables from the perspectives of both rest and moving reference frames. The moving frame is referred to as being conjugate to the rest frame. Actual measurable physical quantities are determined by taking the positive square root of the product of the rest and moving reference frame variables. The Conjugate Frame Method assumes that the value of a variable in the rest frame plus any change to this value due to the relative motion of the moving frame gives the value of the variable in the moving frame as determined by an observer in the rest frame. Upon making this assumption, equations describing the relativistic behavior of various physical quantities are derived. The conjugate frame moves at a constant speed v relative to the rest frame, and v may equal zero if both frames are at rest. The moving frame travels either directly towards or directly away from the rest frame along the geodesic connecting the two reference frames. This geodesic trajectory will be a straight line in a flat space-time manifold, or a curve in a curved space-time manifold. Unbarred letters are used to represent variables relative to the stationary frame, while barred letters are used to represent variables relative to the moving frame. A zero subscript on any variable signifies that it is determined in the same reference frame as the local observer, where there is no relative motion. The moving frame barred variables are referred to as the conjugate frame variables. The term "moving frame" is used inter-changeably with the term "conjugate frame". Likewise, the terms "stationary frame" and "rest frame" are inter-changeable. The speed of light c is assumed to have the same finite constant value in all reference frames.

2. Time

Let us use the Conjugate Frame Method to derive the relativistic time equation of the Special Theory of Relativity. We begin by stating that the time in a moving reference frame can be determined from the perspective of an observer in a stationary reference frame. For an observer in the rest frame, we postulate:

$$\left\{\begin{array}{l}\text{The Present Time In The Rest Frame}\\ +\text{The Time It Takes For Light To Travel}\\ \text{From The Rest Frame To The Moving Frame}\\ =\text{The Time In The Moving Frame As}\\ \text{Determined By An Observer In The Rest Frame.}\end{array}\right\}$$

Mathematically, we can express this as

$$t_0+\frac{x}{c}=t. \qquad (1)$$

In other words, relative to the stationary frame, a photon emitted at time t_0 will travel a distance x in a time $\frac{x}{c}$ and arrive at the moving frame at time t. Conversely, we can consider the problem from the perspective of the moving frame. For someone in the moving frame, we have

$$\left\{\begin{array}{l}\text{The Present Time In The Moving Frame -}\\ \text{The Time It Takes For Light To Travel}\\ \text{From The Rest Frame To The Moving Frame}\\ =\text{The Time In The Rest Frame As}\\ \text{Determined By An Observer In The Moving Frame.}\end{array}\right\}$$

We express this mathematically as

$$\bar{t}_0 - \frac{\bar{x}}{c} = \bar{t}. \tag{2}$$

If we had chosen the opposite signs for the photon travel time terms in the two reference frame equations above, the final measurable results obtained will not be changed by having made this alternative choice. We will arrive at the same physical results for both choices of signs. This will become clear as we proceed. Relative to the unbarred frame, an object moving at a constant speed v travels a distance x in a time t. Mathematically, we write this as

$$v = \frac{x}{t}. \tag{3}$$

This equation can be rearranged as $x = vt$. The analogous equation for the conjugate reference frame is $\bar{x} = v\bar{t}$. Since the constant speed v is the magnitude of the velocity, it is independent of the direction in which the conjugate frame is moving. Observers in both reference frames use the same value for the speed v of the moving reference frame. Substituting the expression for x into equation (1) gives

$$t_0 + \frac{vt}{c} = t. \tag{4}$$

Making an analogous substitution for the barred expression produces the conjugate equation

$$\bar{t}_0 - \frac{v\bar{t}}{c} = \bar{t}. \tag{5}$$

Solving equation (4) for t gives

$$t = \frac{t_0}{1 - \frac{v}{c}}. \tag{6}$$

Solving equation (5) for $\bar{t}$ yields

$$\bar{t} = \frac{\bar{t}_0}{1 + \frac{v}{c}}. \tag{7}$$

Now, multiplying equation (6) by its conjugate equation (7) produces

$$t\bar{t} = \frac{t_0\bar{t}_0}{1 - \frac{v^2}{c^2}}. \tag{8}$$

Making the definitions $T^2 = t\bar{t}$ and $T_0^2 = t_0\bar{t}_0$, and substituting them into equation (8) yields

$$T^2 = \frac{T_0^2}{1 - \frac{v^2}{c^2}}. \tag{9}$$

Finally, taking the positive square root of both sides of equation (9) gives the result

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{10}$$

This is the Special Relativity equation for the time dilation of a moving frame of reference [3,4]. Taking the positive square root of the product of a variable with its conjugate frame counterpart is analogous to computing the scalar norm of vectors [5]. The variable T_0 represents the actual time that is measured in experiments in the stationary frame. The variable T represents the corresponding actual time in the moving frame as determined by an observer in the stationary frame. Note that in the work above, the lower case time variables are mathematical constructs and not the actual quantities measured in experiments.

3. Mass, Energy, and Momentum

The total energy ϵ of a body can be defined as the sum of its rest and kinetic energies. Let us represent the rest energy by the variable ϵ_0, and the kinetic energy by the variable k. In the unbarred stationary frame we have

$$\epsilon_0 + k = \epsilon. \tag{11}$$

The corresponding conjugate moving frame expression is given by

$$\bar{\epsilon}_0 - \bar{k} = \bar{\epsilon}. \tag{12}$$

The choice of signs of the kinetic energy terms in the two reference frame equations above can be reversed, and this will not change the final measurable physical results obtained. Substituting Einstein's mass-energy relation $\epsilon = mc^2$ and the kinetic energy relation $k = pc$, where p is the moving frame's momentum, in equation (11) yields

$$m_0c^2 + pc = mc^2. \tag{13}$$

Making the analogous substitutions in equation (12) gives the corresponding conjugate frame equation

$$\overline{m_0}c^2 - \bar{p}c = \bar{m}c^2. \tag{14}$$

Now, we know from classical physics that $p = mv$. This relation can be utilized in equation (13), and it's conjugate expression $\bar{p} = \bar{m}v$ in equation (14), to give

$$m_0c^2 + mvc = mc^2 \tag{15}$$

and

$$\overline{m_0}c^2 - \bar{m}vc = \bar{m}c^2, \tag{16}$$

respectively.

Solving for m in equation (15) and $\bar{m}$ in equation (16) yields

$$m = \frac{m_0}{1 - \frac{v}{c}} \tag{17}$$

and

$$\bar{m} = \frac{\overline{m_0}}{1 + \frac{v}{c}}, \tag{18}$$

respectively.

Then, multiplying equation (17) by equation (18) gives

$$m\bar{m} = \frac{m_0\bar{m}_0}{1-\frac{v^2}{c^2}}. \tag{19}$$

Let us define $M^2 = m\bar{m}$ and $M_0^2 = m_0\bar{m}_0$. Substituting these expressions into equation (19) gives

$$M^2 = \frac{M_0^2}{1-\frac{v^2}{c^2}}. \tag{20}$$

Finally, taking the positive square root of both sides of equation (20) yields

$$M = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{21}$$

This is the equation for the relativistic mass of Special Relativity [3]. Here, the variable M_0 represents the measurable physical mass of the object in the rest frame, and the variable M represents the measurable physical mass of the moving object as determined by an observer in the rest frame. Analogous to the lower case time variables in the previous section, the lower case mass variables in this section are mathematical constructs, and do not represent actual measurable quantities. It follows that multiplying the relativistic mass equation by the speed v gives the relativistic momentum equation

$$P = \frac{P_0}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{22}$$

Likewise, we see that multiplying the relativistic mass equation by c^2 gives the relativistic energy equation

$$E = \frac{E_0}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{23}$$

Continuing with the analysis of the relativistic mass equation, we have

$$M\sqrt{1-\frac{v^2}{c^2}} = M_0, \tag{24}$$

$$M^2\left(1-\frac{v^2}{c^2}\right) = M_0^2, \tag{25}$$

$$M^2 - M^2\frac{v^2}{c^2} = M_0^2, \tag{26}$$

$$M^2c^2 - M^2v^2 = M_0^2c^2, \tag{27}$$

$$ME - P^2 = M_0E_0, \tag{28}$$

$$ME = M_0E_0 + P^2. \tag{29}$$

Finally, multiplying both sides of equation (29) by c^2 gives the relativistic energy-momentum equation

$$E^2 = E_0^2 + P^2c^2. \tag{30}$$

Then,

$$E^2 - E_0^2 = P^2c^2, \tag{31}$$

$$\left(E+E_0\right)\left(E-E_0\right) = P^2c^2. \tag{32}$$

Let us define δE to be the difference between the total energy E and the rest energy E_0 of an object. Hence, $\delta E = E - E_0$. Then,

$$\left(E+E_0\right)\delta E = P^2c^2, \tag{33}$$

$$\delta E = \frac{P^2c^2}{E+E_0}, \tag{34}$$

$$\delta E = \frac{P^2c^2}{Mc^2+M_0c^2}, \tag{35}$$

$$\delta E = \frac{P^2}{M+M_0}. \tag{36}$$

If $v << c$, then from equation (21) and equation (22) we see that $M \simeq M_0$ and $P \simeq P_0$, respectively. Hence,

$$\delta E \simeq \frac{P_0^2}{M_0+M_0}, \tag{37}$$

$$\delta E \simeq \frac{P_0^2}{2M_0}. \tag{38}$$

This shows that the difference δE between the total and rest energies of a moving object is approximately equal to the object's classical kinetic energy when $v << c$ [1].

4. Length

Let us now employ the Conjugate Frame Method to derive the length x of a moving object as determined by an observer in a stationary frame. We begin by considering the change in length of an object that is moving at a constant speed v along the geodesic path connecting the rest and moving frames. The moving object travels either directly towards or directly away from the stationary frame. Let x_0 represent the length of an object at rest. This length is defined as the distance light travels when going from one end of the object to the other end. The variable t_0 represents the time it takes for light to travel this distance. Mathematically, we express this as

$$x_0 = ct_0. \tag{39}$$

The change in length δx_0 of the moving object is given by the distance the object travels in the time t_0. Namely,

$$\delta x_0 = vt_0. \tag{40}$$

We now postulate the length equation to be

$$x_0 + \delta x_0 = x. \tag{41}$$

Then,

$$x_0 + vt_0 = x. \tag{42}$$

From the perspective of the moving frame, the conjugate frame equation is

$$\overline{x_0} - v\overline{t_0} = \overline{x}. \tag{43}$$

As in all of the Conjugate Frame Method derivations, the signs of the speed terms can be reversed without affecting the final results. This freedom in the choice of signs is a symmetry of the equations.

Using the relation $t_0 = \frac{x_0}{c}$ from equation (39) above, we obtain for the stationary frame

$$x_0 + v\frac{x_0}{c} = x. \tag{44}$$

Simplifying this expression then gives

$$x = x_0\left(1 + \frac{v}{c}\right). \tag{45}$$

Substituting the corresponding $\overline{t_0}$ variable expression into equation (43) and following the same procedure used above for the rest frame, but now for the moving frame, yields the conjugate equation

$$\overline{x} = \overline{x_0}\left(1 - \frac{v}{c}\right). \tag{46}$$

Multiplying equation (45) by equation (46) then gives

$$x\overline{x} = x_0\overline{x_0}\left(1 - \frac{v^2}{c^2}\right). \tag{47}$$

Let us define $X_0^2 = x_0\overline{x_0}$ and $X^2 = x\overline{x}$. Substituting these into equation (47) yields

$$X^2 = X_0^2\left(1 - \frac{v^2}{c^2}\right). \tag{48}$$

Taking the positive square root of both sides of the equation, we obtain

$$X = X_0\sqrt{1 - \frac{v^2}{c^2}}. \tag{49}$$

This is the Special Relativity equation for the length contraction of a moving object [4].

5. Electric Charge and Electric Current

Consider the electric charge q_0 at rest. When the charge moves with a constant speed v it becomes the electric current I. We postulate that the value of the charge q_0 at rest + the value of the charge It in motion = the total value of the charge q. Here, t is the time variable. Mathematically, we have

$$q_0 + It = q, \tag{50}$$

$$q_0 + I\frac{x}{c} = q. \tag{51}$$

Defining the electric current as $I = \frac{q_0}{t}$, we obtain

$$q_0 + \frac{q_0 x}{ct} = q, \tag{52}$$

$$q_0 + \frac{q_0 vt}{ct} = q. \tag{53}$$

Solving for q, we get

$$q = q_0\left(1 + \frac{v}{c}\right). \tag{54}$$

Following the same procedure for the moving frame yields the corresponding conjugate frame equation

$$\overline{q} = \overline{q_0}\left(1 - \frac{v}{c}\right). \tag{55}$$

Multiplying these two reference frame equations together, defining the physical charges Q_0 and Q, and taking the positive square root of both sides of the resulting equation yields the relativistic electric charge equation

$$Q = Q_0\sqrt{1 - \frac{v^2}{c^2}}. \tag{56}$$

This equation, in combination with the relativistic mass equation (21) given above, indicates that electrically charged particles cannot be accelerated to speeds greater than the speed of light by electromagnetic fields. This is attributed to the vanishing of a charged particle's acceleration while under the influence of an electromagnetic field, as its speed approaches the value c. The electric charge is an invariant quantity in the Special Theory of Relativity, hence $Q = Q_0$. Using the relativistic time equation of Special Relativity, we can show that the Special Relativity electric current is given by

$$I = I_0\sqrt{1 - \frac{v^2}{c^2}}. \tag{57}$$

Here, I_0 represents the electric current in an electric circuit that is at rest, and I represents the electric current in an identical electric circuit that is moving at a constant speed v relative to the rest frame circuit. For the Conjugate Frame Method, we impose the additional requirement that the moving electric circuit travels either directly towards or directly away from the stationary circuit along a geodesic path. From equation (57), we see that the electric current of Special Relativity is not an invariant quantity. In the Conjugate Frame Method Relativity, we can see from equation (56) that the electric charge is not an invariant quantity. Using the Conjugate Frame Method Relativity equations to derive the relativistic electric current equation yields

$$I = I_0\left(1 - \frac{v^2}{c^2}\right). \tag{58}$$

From this equation, we see that the electric current of the Conjugate Frame Method Relativity is not an invariant quantity. Experiments can be performed to determine the validity of these equations.

6. Temperature

We begin by making the following variable assignments. Let θ_0 represent the temperature of an object at rest. Let θ represent the temperature of an identical object that is moving at a constant speed v along the geodesic connecting the two objects. The object in motion moves in a direction that is either directly towards or directly away from the rest object. Let x represent the shortest distance separating the two objects at any given time, and let G represent the temperature per unit length at the location of the moving object (I call this quantity the pseudo temperature gradient). Next, we postulate the temperature equation to be

$$\theta_0 + Gx = \theta. \tag{59}$$

Making the usual substitutions gives

$$\theta_0 + Gvt = \theta, \tag{60}$$

$$\theta_0 + Gv\frac{x}{c} = \theta. \tag{61}$$

Now, defining the pseudo temperature gradient as

$$G = \frac{\theta}{x}, \tag{62}$$

we obtain

$$\theta_0 + \frac{vx\theta}{cx} = \theta. \tag{63}$$

Then,

$$\theta = \frac{\theta_0}{1 - \frac{v}{c}}. \tag{64}$$

Continuing with the Conjugate Frame Method, we ultimately arrive at the result

$$\Theta = \frac{\Theta_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{65}$$

This is the relativistic temperature equation. The variable Θ represents the measurable temperature of the moving object as determined by an observer in the rest frame, and the variable Θ_0 represents the measurable temperature of the object at rest. This equation shows that the temperature of a moving object is greater than its rest temperature, as determined by an observer in the rest frame.

7. Discussion and Conclusions

With the exceptions of the relativistic electric charge and relativistic electric current equations, the Conjugate Frame Method Relativity reproduces the equations of Special Relativity. The equations in this work, however, were derived using only scalar quantities. This is different from Special Relativity, where four vectors in Minkowski space-time are routinely used in the derivation of relativity equations. In the Conjugate Frame Method Relativity, objects moving towards the rest frame are constrained to travel along the same shortest distance trajectory, or geodesic, as light would take when traveling from the moving frame to the rest frame. Objects moving away from the rest frame travel along the same geodesic as objects moving towards the rest frame, but in the opposite direction. In a flat space-time manifold, moving objects traveling at a constant speed along straight line trajectories are inertial reference frames. If the manifold is not flat, then objects travel along curved trajectories. The speed of the moving object must remain constant as it travels along the curved geodesic for the Conjugate Frame Method to be applicable. Objects that move along curved trajectories are not inertial reference frames. Since the derivation of the equations in the Conjugate Frame Method Relativity do not depend on the geometry of the geodesic path traveled by a moving frame, the equations apply to both inertial and non-inertial reference frames, as long as the speed of the moving frame remains constant. This is unlike the equations of Special Relativity that apply only to inertial reference frames.

Table 1. The Conjugate Frame Method Relativity Equations. Here, $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ Is The Lorentz Factor [4]

Table Of Results	
Time	$T = T_0\gamma$
Length	$X = X_0\gamma^{-1}$
Mass	$M = M_0\gamma$
Energy	$E = E_0\gamma$
Momentum	$P = P_0\gamma$
Electric Charge	$Q = Q_0\gamma^{-1}$
Electric Current	$I = I_0\gamma^{-2}$
Temperature	$\Theta = \Theta_0\gamma$

In the Conjugate Frame Method, actual measurable quantities are determined by taking the positive square root of the product of a rest frame variable with its moving frame counterpart. The choice of sign used for the speed dependent term in the rest frame equation is immaterial. The two frames of reference will have opposite signs for their respective speed dependent terms. Therefore, the product of these terms will be negative regardless of the choice of sign that was made. The resulting relativity equations are the same for both choices of sign. It also does not matter whether the conjugate frame is moving directly towards or directly away from the rest frame. Moving in either direction, the resulting relativity equations are the same. The relativistic electric charge equation has applications in various fields including high energy physics, astrophysics, and relativistic space travel. Equation (45) and equation (46) above suggest that the Conjugate Frame Method can also be used to derive the classical and relativistic Doppler effect wavelength and frequency equations. The Conjugate Frame Method Relativity equations derived in this work predict that objects traveling near the speed of light undergo mass and temperature increases, and interact weakly with electromagnetic fields, as determined by observers in the rest frame. A listing of the Conjugate Frame Method Relativity equations is shown in Table 1.

Appendix

The rest frame equation can be represented in the general form $a_0 + the\ speed\ dependent\ term = a$. Similarly, the moving frame conjugate equation can be represented in the general form $\overline{a_0} - \overline{the\ speed\ dependent\ term} = \overline{a}$. The physical quantity A can be measured in an actual experiment, and it is determined mathematically by calculating the positive square root of the product of the two reference frame variables a and $\overline{a}$. Stated mathematically, $A = \sqrt{a\overline{a}}$. Calculating the value of the measurable quantity A is analogous to computing the positive norm of a mathematical object. Examples of mathematical objects having positive norms include vectors, quaternions, and complex numbers [1,5]. Using the subscripts 1 and 2 in the relativistic time equation to represent the times of two different events, the period of time δT separating these two events is given by the relation $\delta T = T_2 - T_1$. After making the relativistic time substitutions for the two events, this equation becomes

$$\delta T = \frac{\delta T_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad (66)$$

where $\delta T_0 = T_{20} - T_{10}$. Similarly, using the relativistic length equation, we can show that the distance δX separating two points in a moving object is given by the relativistic equation

$$\delta X = \delta X_0 \sqrt{1 - \frac{v^2}{c^2}}, \qquad (67)$$

where $\delta X_0 = X_{20} - X_{10}$.

Equation (67) expresses the length contraction of a moving object in the direction of it's motion. This equation is applicable when the two points in the moving object are on the same geodesic that connects the two reference frames.

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