

# No-Core Shell Model Calculations for ${}^{6,8}\text{He}$ , ${}^{8,10,12}\text{B}$

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**Abstract** The excitation energies of  ${}^{6,8}\text{He}$  and  ${}^{8,10,12}\text{B}$  isotopes in the p-shell region have been calculated by using large-basis no-core (i.e. considering all nucleons active with partial restrictions imposed on some nucleons). The shell model calculations have been performed using *spstdpf* model space with *wbt* effective residual interaction fitted for the p-shell nuclei. Three set of restrictions have been imposed named  $(0 + 1)\hbar\omega$ ,  $(0 + 2)\hbar\omega$ ,  $(0 + 3)\hbar\omega$  in our calculations have been used. The comparison of our theoretical work with the recent available experimental data shows that the restriction  $(0 + 3)\hbar\omega$  gives best results for Helium isotopes while  $(0 + 1)\hbar\omega$  is in better agreement with the experiment for Boron isotopes.

**Keywords:** excitation energies, no-core shell model, light nuclei

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## 1. Introduction

The understanding of the nuclear structure is one of the central challenges in nuclear physics. By choosing the proper effective interaction and the model space can lead to predict a wide range of observables by the nuclear shell model systematically and correctly [1]. Wide range of standard effective interaction has been studied such as the Cohen-Kurath interaction for light nuclei [2]. This effective interaction forms an essential input to all shell-model studies. Equipped with modern sophisticated effective interactions, the shell model has successfully described many properties of nuclei [3]. Large-scale shell model calculations have been successfully conducted to investigate the low-lying energy levels, binding energies and the reduced transition probabilities for even-even  ${}^{52,54,56}\text{Cr}$  and  ${}^{54,66}\text{Fe}$  isotopes by employing *gxpfl*, *gxpfla*, *fpd6* and *kb3g* effective interactions by Majeed and Ejam [4,5] where their results are in good global agreement with the experimental data. Barrett, *et al.*, [6] discussed the standard formulation and the transnational invariant of the no-core shell-model approach. They presented their results for three and four nucleon systems interacting by the nucleon-nucleon (NN) potentials or other CD-Bonn in spaces of model included up to  $18\hbar\Omega$  and  $50\hbar\Omega$  HO excitations. They apply their *ab initio* no-core shell model approach to the lightest nuclei,  ${}^3\text{H}$ ,  ${}^3\text{He}$ ,  ${}^4\text{He}$ . *Ab initio* no-core shell model (NCSM) calculations have been performed by Vary *et al.*, [7] where they started with an intrinsic Hamiltonian for all nucleons in the nucleus. They have used a realistic two-nucleon and tri-nucleon interactions recently developed from effective field theory and chiral perturbation theory. They derived a finite basis-space dependent Hermitical effective Hamiltonian. The resulting finite Hamiltonian matrix Problem is solved by

diagonalization on parallel computers. Applications range from light nuclei to multi quark systems and, recently, to quantum field theory including systems with bosons.

The aim of the present work is to study excitation energies for light isotopes  ${}^{6,8}\text{He}$  and  ${}^{8,10,12}\text{B}$  by means of large-scale shell model calculations by employing model space *spstdpf* with *wbt* effective interaction using the recent shell model code Nushellx@msu [7], then compare these theoretical attempts with the most recent experimental data.

## 2. Shell Model

The basic assumption of the nuclear shell model is that to a first approximation each nucleon moves independently in a potential that represents the average interaction with the other nucleons in a nucleus. This independent motion can be understood qualitatively from a combination of the weakness of the long-range nuclear attraction and the Pauli Exclusion Principle [9]. In a non-relativistic approximation, nuclear properties are described by the Schrödinger equation for A nucleons [9]

$$\hat{H}\psi(1,2,\dots,A) = E\psi(1,2,\dots,A) \quad (1)$$

where  $\hat{H}$  contains nucleon kinetic energy operators and interactions between nucleons of a two-body and, eventually, of three-body character, i.e.

$$H = \sum \left( -\frac{\hbar^2}{2m} \Delta_i \right) + \sum_{i < j=1}^A W(i,j) + \sum W(i,j) \quad (2)$$

$\psi(1,2,3,\dots,A)$  is an A-body wave function, while  $i$  denotes all relevant coordinates  $\vec{r}_i, \vec{s}_i, \vec{t}_i$  of a given particle ( $i=1,2,\dots,A$ ). Although the three-body forces are proved to be important, in the present work we will consider only the two-body interaction. We can re-write

the Hamiltonian (2), adding and subtracting a one-body potential of the form  $\sum_{i=1}^A U(i)$  [9]

$$\hat{H} = \sum_{i=1}^A \left[ -\frac{\hbar^2}{2m} \Delta_i + U(i) \right] + \sum_{i < j=1}^A W(i, j) - \sum_{i=1}^A U(i) = \hat{H}^{(0)} + \hat{V} \quad (3)$$

where we denoted a sum of single-particle Hamiltonians as  $\hat{H}^0$

$$\hat{H}^0 = \sum_{i=1}^A \left[ -\frac{\hbar^2}{2m} \Delta_i + U_i \right] = \sum_{i=1}^A \hat{h}(i) \quad (4)$$

and  $\hat{V}$  is called a residual interaction. Existence of a nuclear average potential allows to assume that we can find such a potential  $\sum_{i=1}^A U_i$ , that the residual interaction  $V$  is small.

### 3. Results and Discussions

The calculations are performed using the shell model wrapper code Nushellx@msu for windows. The model space used is *spsdpf* with *wbt* effective residual effective interaction. The low-lying energy levels for light  ${}^6,8\text{He}$  and  ${}^{8,10,12}\text{B}$  isotopes have been calculated by using large-scale no-core shell model approach with partial restrictions on some nuclei with three set of restrictions namely  $(0 + 1)\hbar\omega$ ,  $(0 + 2)\hbar\omega$ ,  $(0 + 3)\hbar\omega$ .

Figure 1, displays the comparison of the calculated low-lying energy levels of the  ${}^6\text{He}$  isotope. The first  ${}^+2$  state are predicated at 4.598 MeV, 3.190 MeV and 1.769 MeV by using  $(0 + 1)\hbar\omega$ ,  $(0 + 2)\hbar\omega$ ,  $(0 + 3)\hbar\omega$  respectively, it is noticed from the figure that the best results achieved by using  $(0 + 3)\hbar\omega$  restrictions in comparison with the experimental value at 1.797 MeV is in better agreement with the experiment by using *ckpot* [10] effective interaction with  ${}^4\text{He}$  taken as core, therefore the no-core shell model calculations with some restrictions imposed on some nucleons proves to be good approach in shell model calculations for light nuclei.

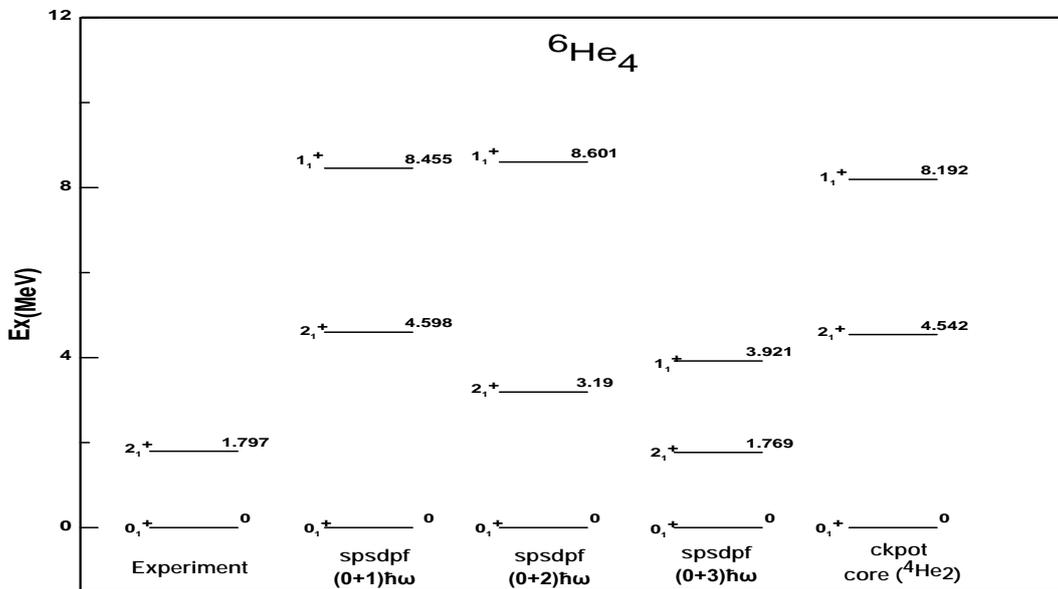


Figure 1. Shows the comparison of calculated energy levels for  ${}^6\text{He}$  isotope with the experimental data [11]

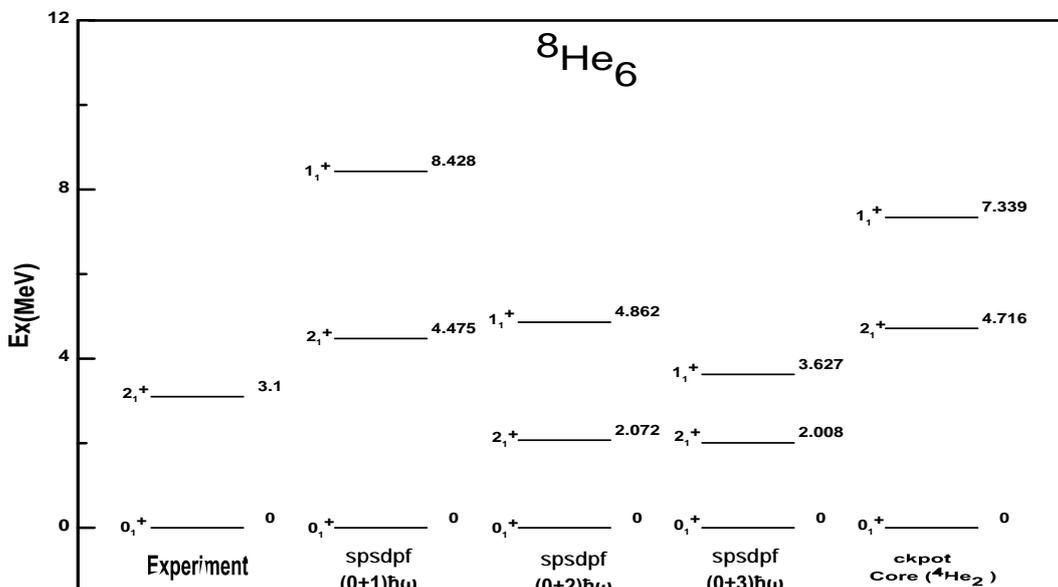
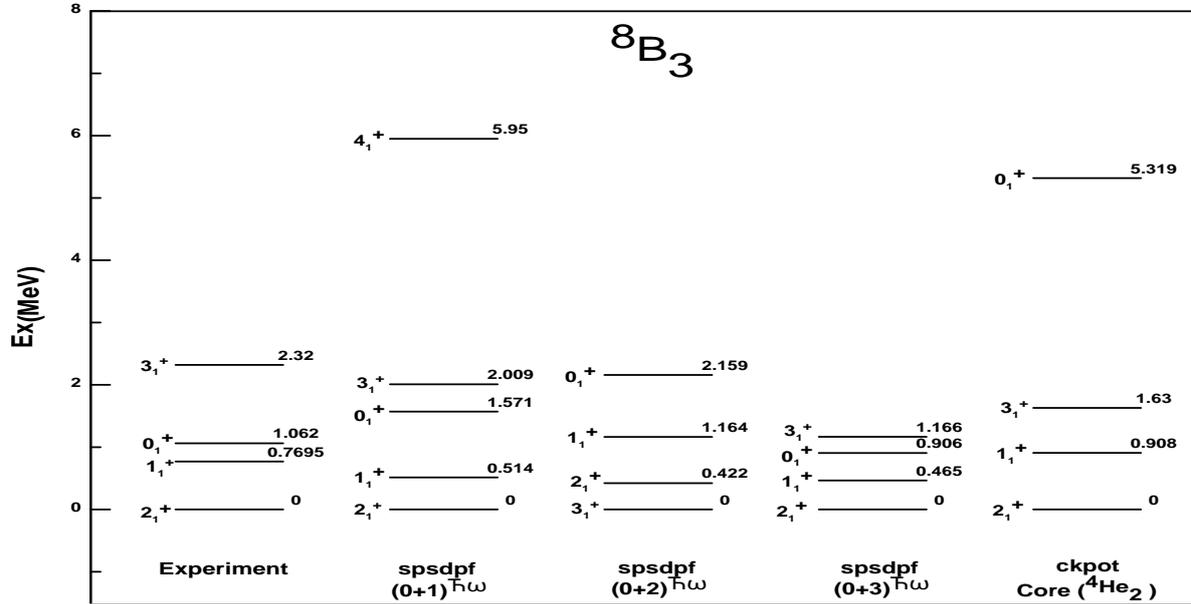


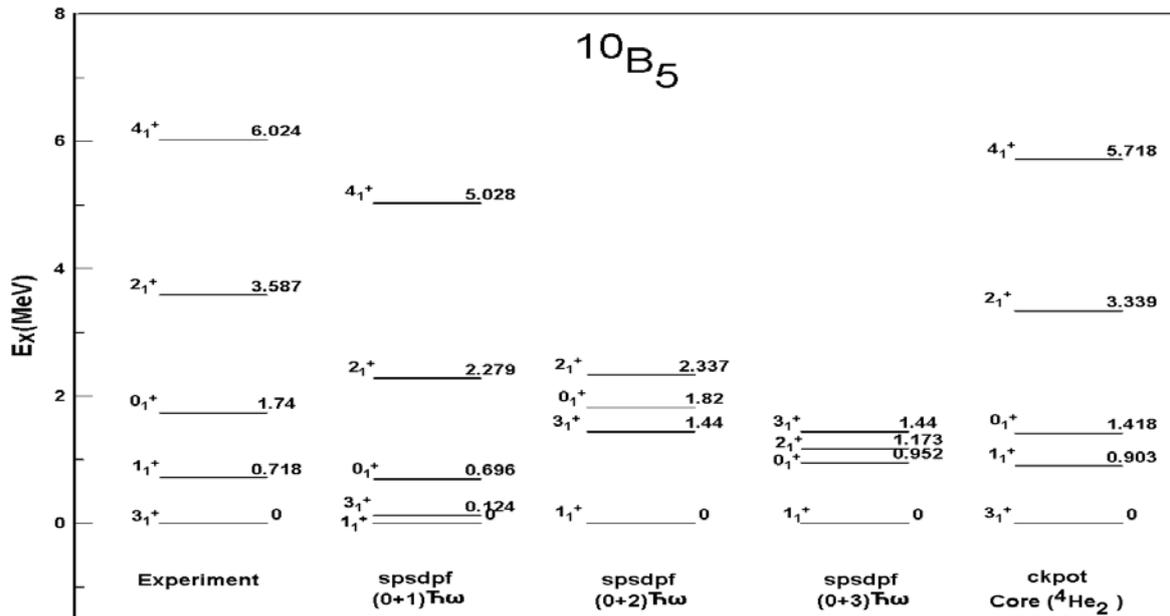
Figure 2. Shows the comparison of calculated energy levels for  ${}^8\text{He}$  isotope with the experimental data [11]

**Figure 2**, presents the comparison of the predicated values of the no-core shell model calculations for  $^8\text{He}$  there is general good global agreement for all sent of restrictions imposed on the active nucleons and the best agreement with the experimental values achieved with  $(0 + 1)\hbar\omega$  and even more closer than the core calculations using ckpot effective interaction.

**Figure 3**, shows the compassion between the theoretical work and the experimental date for  $^8\text{B}$  isotope. The ground state is correctly predicted with  $(0 + 1)\hbar\omega$  and  $(0 + 3)\hbar\omega$  while is not the case with  $(0 + 2)\hbar\omega$  and the best results achieved by using  $(0 + 1)\hbar\omega$  restriction which proved to be able to predict the correct ordering of the low-lying excitation energies.



**Figure 3.** Shows the comparison of calculated energy levels for  $^6\text{B}$  isotope with the experimental data [11]



**Figure 4.** Shows the comparison of calculated energy levels for  $^{10}\text{B}$  isotope with the experimental data [11]

**Figure 4**, shows the comparison of our no-core shell model calculations for  $^{10}\text{B}$  isotope, it is clear from the figure that none of the imposed restrictions are able to predict the ground state, this might be attributed to the fact that this nucleus is odd-odd and the ground state need full coupling to predict the ground state and the restrictions imposed might restrict the full coupling that's why the ground state could not be predicted correctly, therefore we cannot rely on the restrictions and a full coupling needed which need a considerable computer power to perform such calculation for this nucleus.

**Figure 5**, illustrates the calculated low-lying energy levels for the data of energy of  $^{12}\text{B}$  isotope. From figure we can see that the  $(0 + 2)\hbar\omega$  is unable to predict the ground state correctly and the best results achieved by imposing  $(0 + 1)\hbar\omega$  restriction while we could not get any results by imposing  $(0 + 3)\hbar\omega$  restriction that's why is not appearing in this case, The no-core shell model calculations are in better agreement compared with the calculations with the core to be taken at  $^4\text{He}$  with ckpot effective interaction.

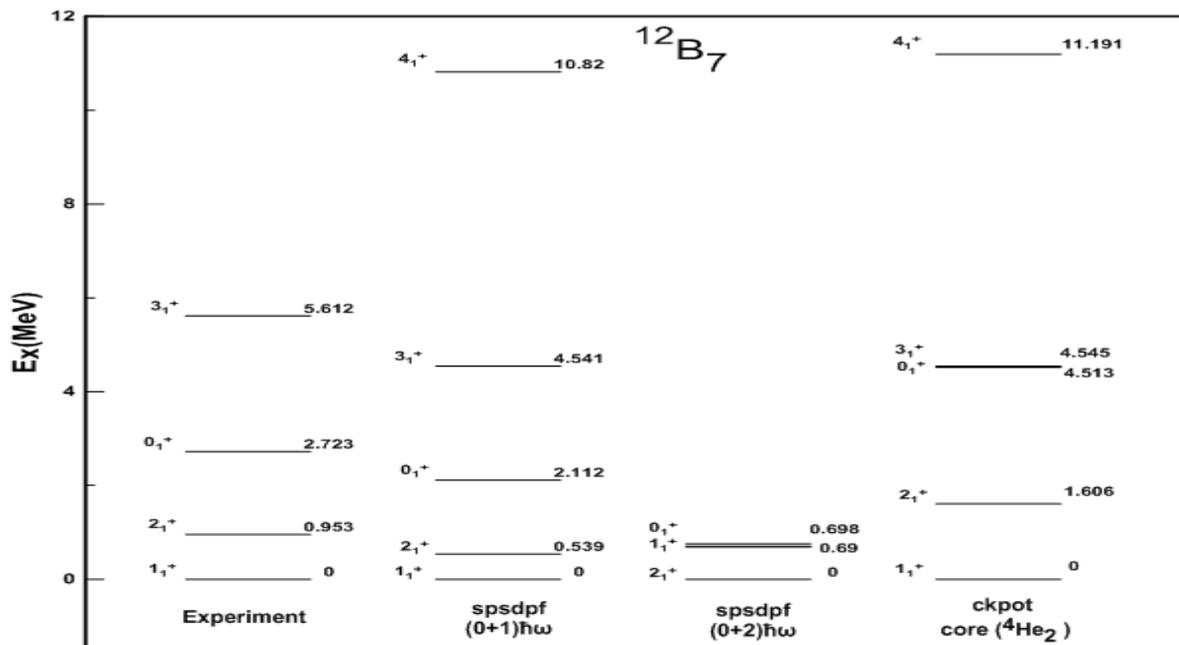


Figure 5. Shows the comparison of calculated energy levels for  $^{12}\text{B}$  isotope with the experimental data [11]

## 4. Conclusions

In the present work, large-scale shell model calculations have been performed for light nuclei  $^6,8\text{He}$  and  $^8,10,12\text{B}$  isotopes by using the shell model code Nushellx@msu by adopting the model space SPSPDF with WBT effective interactions. Three sets of restrictions namely  $(0 + 1)\hbar\omega$ ,  $(0 + 2)\hbar\omega$ ,  $(0 + 3)\hbar\omega$  have been imposed in our work. The comparison of the calculated low-lying energy levels with the most recent experimental data proved that the no-core shell model calculations with extended model space is able to predict the low-lying energy levels for light nuclei and the results might be improved if we remove this restriction's and considering full space without imposing any restrictions on nucleons, but this will need a considerable computer power with parallel processors which might we have access in the future.

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