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On the Test of Time Dilation Using the Relativistic Doppler Shift Equation

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Abstract In a recent research study entitled "Test of Time Dilation Using Stored Li+ Ions as Clocks at Relativistic Speed" (*Phys. Rev. Lett*. 113, 120405 – Published 16 September 2014), an Ives–Stilwell type experiment,it was claimed that a conducted time dilation experiment using the relativistic Doppler effect on the Li+ ions resonance frequencies had verified, with a greatly increased precision, the relativistic frequency shift formula, derived in the Special Relativity from the Lorentz Transformation, thus indirectly proving the time dilation predicted by the Special Relativity. The test was based on the validation of an algebraic equality relating a set of measured frequencies, and deduced from the relativistic Doppler equations. In this study, it was shown that this algebraic equality, used as a validation criterion, did not uniquely imply the validity of the relativistic Doppler equations. In fact, using an approach in line with the referenced study, it was revealed that an infinite number of frequency shift equations would satisfy the employed validation criterion. Nonetheless, it was shown that even if that claim was hypothetically accepted, then the experiment would prove nothing but a contradiction in the Special Relativity prediction. In fact, it was clearly demonstrated that the relativistic blue shift was the consequence of a time contraction, determined via the light speed postulate, leading to the relativistic Doppler formula in the case of an approaching light source. The experiment would then be confirming a relativistic time contraction. It was also shown that the classical relativity resulted in perceived time alterations leading to the classical Doppler Effect equations. The "referenced study" result could be attributed to the classical Doppler shift within 10 % difference.

Keywords: *Special Relativity, relativistic Doppler Effect, Ives–Stilwell, time dilation test, time contraction*

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1. Introduction

Considering two inertial reference frames in relative motion, and assuming that event information is ultimately communicated between the frames through light—or electromagnetic—signals, proper time interval measured in the "traveling" frame between two co-local events occurring at the frame origin would be perceived altered in the "stationary" frame. This apparent time interval alteration is due to the time delay in the communication signals when transferring the information, and to the frames motion. Hence, the time alteration factor depends on the speed of light with respect to the frames, and the relative motion velocity.

An observer in the "stationary" frame should then correct the perceived time interval, so as the absolute time can be calculated, using corrected equations relating the frames space coordinates.

If the apparent time variations between the reference frames were to be taken into consideration and incorporated in the classical Galilean transformation converting space coordinates from one reference frame to another, this transformation shall take different corrected forms, depending on the assumptions made on the speed of conveyance of events from one frame to another (i.e., on the light signal speed).

In classical physics, there are two principal theories governing the nature and behavior of light; the Emission Theory and the Ether Theory.

In the light Emission Theory, also known as the ballistic theory, often attributed to Isaac Newton for his Corpuscular theory, light is assumed to exhibit a nature incorporating a corpuscular behavior. Under this conjecture, light is emitted at a constant speed c relative to its emission source. So, if the source (emitter) is moving at a speed v relative to an observer, light will travel towards the observer at a speed of $c \pm v$. Hence, there is no preferred reference frame for light propagation.

On the other hand, the Ether Theory was prevailed in the 19th century when the electromagnetic wave nature of light had been established and described by the Maxwell's equations. The corpuscular nature of light conjecture had been dropped. Since waves must propagate through a medium, then the ether, an assumed medium required to carry light waves, was supposed to fill the entire interstitial space. Therefore, light must propagate at a constant speed c relative to the ether rest frame. Hence, the speed of light with respect to a certain reference frame would depend on the speed of that frame with respect to the ether rest frame. However, doubts about the Ether Theory had been raised following the negative results of the famous Michelson-Morley experiment [[1]] designed to verify the Ether Theory by attempting to detect directional variations in the light speed relative to the earth, supposedly moving with respect to the ether. The Special Relativity theory came later on to replace the Ether Theory, introducing new concepts of space and time.

In the theory of Special Relativity, [2] Einstein postulated that there was no preferred reference frame for light propagation (first postulate: physics laws are the same in all inertial reference frames), and that the speed of light was independent of the source state of motion. Hence, the speed of light would always be the same and equal to a universal constant c in all inertial reference frames (second postulate). Consequently, since an observer measures the same speed of light in his rest frame and in another traveling inertial frame, space and time in the latter must be deformed with respect to the observer in order for this speed invariance to be maintained.

Based on the Special Relativity postulates, the Lorentz transformation, a set of space-time equations to convert coordinates between two inertial frames of reference in relative motion, predicting time dilation and length contraction under particular interpretations, was derived. [[2]] Relativistic Doppler Effect equations were deduced from the Lorentz transformation.

Many experimental studies attempting to verify the Special Relativity prediction of the time dilation have been conducted. The principle of the empirical verification of the time dilation factor via the relativistic Doppler Effect had been proposed by Ives& Stilwell [3]. Later variations of the Ives–Stilwell type experiment have been conducted. In a recent research, [4] referred to in the paper as the "referenced study", a modern version of this type, the Special Relativity time dilation was claimed to be tested implicitly via the experimental verification of the relativistic Doppler effect formula through the validation of an algebraic equality (deduced from the relativistic Doppler equations)relating a set of measured frequencies. In this study, it will be shown that the verified equality does not uniquely imply the validity of the relativistic Doppler equations. Even if it was the case, the verification of the relativistic Doppler formula would actually be a validation, within the Special Relativity frame, of the contraction in the stationary frame of a proper time interval measured at the origin of the traveling frame for the approaching frame case, which contradicts the Special Relativity prediction of time dilation regardless of the relative motion orientation. The time contraction equation and the resulting Doppler shift will be derived and verified within the frame of the Special Relativity using a basic physics approach.

Each of the above light theories would give different extents of time and space alterations between inertial reference frames in relative motion. Modified versions of the Galilean transformation would be obtained. Given the objective of this paper, Special Relativity assumption will be considered to derive the respective time alteration and Doppler Effect equations. The classical Doppler equations will be derived as well, as to offer a verification means for the used method. Although the employed approach leads to the Lorentz transformation in the case of the Special Relativity assumption for the light speed, several misconceptions with the Special Relativity interpretations are revealed.

2. Doppler Shift Approach to Test Time Dilation

In the "referenced research study", [2] the test for time dilation is based on verifying the relativistic Doppler equations, for the receding and approaching source, respectively:

$$f_R = \gamma f'_R \left(1 - \frac{v}{c}\right), \tag{1}$$

$$f_A = \gamma f'_A \left(1 + \frac{v}{c}\right). \tag{2}$$

where f is the perceived frequency in the "stationary" frame, and f' the proper frequency in the emitter rest frame receding from [Eq. (1)] or approaching [Eq. (2)] the "stationary" observer. γ is the Lorentz dilation factor given by

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}.$$

Li+ ions beam moving at a relativistic speed receives two laser beams: one from the front, and one from the back, relative to the ions direction of travel. In the ions reference frame, the ion perceived (shifted) absorbed frequency f_o is equal to its rest state resonance frequency, which is the shift of the emitted frequency f' in the lab frame. The absorbed frequencies in the lab frame from the front and the back laser beams are measured and identified as f_F and f_B. Using Eqs. (1) and (2), we can write

$$f_o = \gamma f_B \left(1 - \frac{v}{c}\right), f_{o'} = \gamma f_F \left(1 + \frac{v}{c}\right),$$

leading to

$$f_B = \gamma f_o \left(1 + \frac{v}{c}\right), f_F = \gamma f_{o'} \left(1 - \frac{v}{c}\right),$$

and therefore,

$$\frac{f_B f_F}{f_o f_{o'}} = \gamma^2 \left(1 - \frac{v^2}{c^2}\right); \frac{f_B f_F}{f_o f_{o'}} = 1 \tag{3}$$

The measured resonance frequencies were claimed to obey Eq. (3) within a minimal error, thus theoretically verifying the relativistic Doppler Eqs. (1) and, (2), and hence the time dilation was assumed to be tested.

2.1. Criticism

The mere empirical verification that the compounded frequency ratio given in Eq. (3) is equal to 1, doesn't necessarily offer a validation of the equations leading to that ratio. In fact, we notice that the relativistic red shift factor is the inverse of the blue shift's given in Eqs. (1) and (2), respectively. Indeed, simple algebraic manipulations will lead to

$$\gamma\left(1+\frac{v}{c}\right)=\left[\gamma\left(1-\frac{v}{c}\right)\right]^{-1}.$$

Hence, the relativistic Doppler shift Eqs. (1)and(2) can be written in the form

$$f_R = f'_R\gamma\left(1-\frac{v}{c}\right), \quad (4)$$

$$f_A = f'_A\left[\gamma\left(1-\frac{v}{c}\right)\right]^{-1}, \quad (5)$$

revealing that the sign change of the velocity term in Eqs. (1) and (2) is not merely the result of the reversed velocity direction, but rather the inversion of the shift factor; a hidden illusion in the math of these equations. It follows that any red and blue shift equations where the shift factor in one is the inverse of the other, will meet the criterion of Eq. (3). For instance, one can postulate that the frequency shift occurs in accordance with the following equations:

$$f_R = f'_R\left(1-\frac{v}{c}\right), \quad (6)$$

$$f_A = f'_A\left(1-\frac{v}{c}\right)^{-1}, \quad (7)$$

Satisfying the expected red and blue shift for the receding and approaching source, respectively. Therefore, the equations for the absorbed frequencies in the lab frame from the front and the back laser beams become

$$f_B = f_o\left(1-\frac{v}{c}\right)^{-1},\ f_F = f_{o'}\left(1-\frac{v}{c}\right),$$

leading to

$$\frac{f_B\, f_F}{f_o\, f_{o'}}=1,$$

satisfying Eq. (3).

It follow that the “referenced study” could be claimed to be a verification of Eqs. (6) and (7), thus contradicting the relativistic Doppler shift.

As a matter of fact, the following group of frequency shift equations,

$$f_R = f'_R\left(1-\frac{v^n}{c^n}\right)^p,\ f_A = f'_A\left(1-\frac{v^n}{c^n}\right)^{-p},$$

obtained for the different values of the positive numbers n and p, would be verified by the “referenced study”. Hence, the claim that the study offers a test of time dilation has no ground for logical substantiation.

It is worth mentioning that the classical frequency shift equations

$$f_R = f'_R\left(1-\frac{v}{c}\right), f_A = f'_A\left(1+\frac{v}{c}\right),$$

conform to reality in the sense that the source velocity direction is reflected in the equations. i.e., the red and blue shifts are correlated with the source motion direction, $-v$ or $+v$ (receding or approaching source). Whereas, in the common form of the relativistic frequency shift equations shown as Eqs. (1) and (2), the velocity sign change is only delusive, as the real implication of these equations appears when they are written in their primitive form given by Eqs. (4) and (5), with the same sign of the v term in both equations.

Nevertheless, even if the “referenced study” was hypothetically accepted, it will be clearly shown in this paper that Eq. (2) is derived from a time contraction equation for approaching frames (i.e., the contraction in the stationary frame of a proper time interval measured at the origin of the traveling frame), verified within the Special Relativity frame, as it leads to the Lorentz transformation time equation. Therefore, the “referenced study” would be verifying a time contraction equation, in contradiction with the Special Relativity predictions.

Equations (1) and (2) will be derived from scratch using basic physics concepts, showing they correspond to time dilation and contraction, respectively. The classical Doppler Effect equations will be derived as well, using the same approach, as to validate the used strategy.

On the other hand, using the classical Doppler Effect equations, the expression given by Eq. (3) will be modified to

$$\frac{f_B\, f_F}{f_o\, f_{o'}}=\left(1-\frac{v^2}{c^2}\right).$$

For the relative velocity of 0.338 used in the “referenced study”, the theoretical ratio of the product of the shifted frequencies to the product of the corresponding rest state resonance frequencies would become 0.9, 10% less than the measured ratio.

3. Time Interval Alteration

Consider two inertial frames of reference, $K(x, y, z,)$ and $K'(x', y', z')$, in relative translational motion with parallel corresponding axes, and let their origins be aligned along the overlapped x- and x'-axes. Let v be the relative motion velocity in the direction of the x-x' axis oriented in such a way that with respect to an observer in K (i.e., K is “stationary” relative to this observer), the relative travel direction of K' (“traveling” frame) is in the positive x-x' direction when the frames are receding.

3.1. General Time Alteration Factor Derivation for Receding Frames

Suppose that K and K' are overlapping at the time $t = t' = 0$. The event coordinates can then be considered as space and time intervals measured from the initial zero coordinates of the overlapped-frames event.

3.1.1. Perception in K (“stationary frame”) of a proper time interval for events in K' (“traveling frame”)—receding frames

Suppose a signal of an event $E'(0,0,0)$ is emitted from K' origin at time t' with respect to K', which will be perceived at time t in K.

Let the speed of the light signal traveling from K' to K be $c'_{K'\to K}$ with respect to K', and $c_{K'\to K}$ with respect to K.

K' perspective

From the perspective of K', the origin of K' at the event occurring time is at a distance of vt' from that of K (Figure 1).

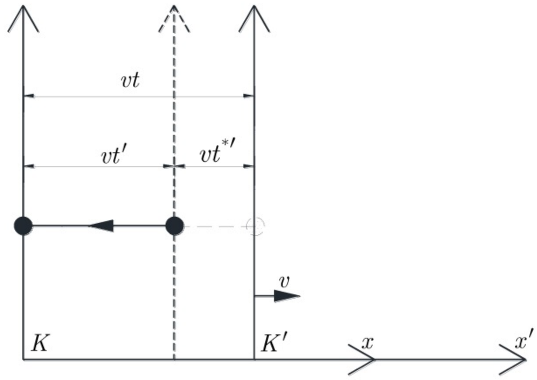

Figure 1. Signal propagation—receding frames

Let $t^{*\prime}$ be the time interval it takes the event signal to reach the origin of K, from K' perspective. By that time, K' will have moved a further distance of $vt^{*\prime}$ away from K, bringing the distance traveled by the signal to $vt' + vt^{*\prime}$ with respect to K'. The time interval $t^{*\prime}$ can then be expressed as

$$t^{*\prime} = \frac{vt' + vt^{*\prime}}{c'_{K'\to K}},$$

leading to

$$t^{*\prime} = \frac{vt'}{c'_{K'\to K} - v}.$$

It follows that the event perception time t in K with respect to K' will be given by

$$t = t' + t^{*\prime}; t = t'\left(\frac{c'_{K'\to K}}{c'_{K'\to K} - v}\right); \quad t = \frac{t'}{1 - \frac{v}{c'_{K'\to K}}}, \tag{8}$$

exhibiting time dilation.

K perspective

Now, from the perspective of K, the origin of K' at the event occurring time is at a distance of vt' from that of K. The signal will have traveled a distance of vt', at the speed of $c_{K'\to K}$ with respect to K, when it reaches K origin. Therefore, the event will be perceived at time t in K, given by

$$t = t' + \frac{vt'}{c_{K'\to K}};, t = t'\left(1 + \frac{v}{c_{K'\to K}}\right), \tag{9}$$

dilated with respect to t'.

3.2. General Time Alteration Factor Derivation for Approaching Frames

Suppose that the time is set to $t = t' = 0$ when K and K' are at a distance of d from each other.

3.2.1. Perception in K ("stationary frame") of a proper time interval for events in K' ("traveling frame")—approaching frames

Suppose a signal of an event $E'_o(0,0,0)$ is emitted from K' origin at time $t'_o = 0$ with respect to K', which will be perceived at time t_o in K, and another signal of an event $E'(0,0,0)$ is emitted at time t' from K' origin, which will be perceived at time t in K.

Let the speed of a light signal traveling from K' to K be $c'_{K'\to K}$ with respect to K', and $c_{K'\to K}$ with respect to K.

K' perspective

From the perspective of K', the origin of K' at the event E'_o occurring time is at a distance of d from that of K. Let $t_o^{*\prime}$ be the time interval it takes the event E'_o signal to reach the origin of K, from K' perspective. By that time, K' will have moved a distance of $vt_o^{*\prime}$ closer to K, bringing the distance traveled by the signal to $d - vt_o^{*\prime}$ with respect to K'. The time interval $t_o^{*\prime}$ can then be expressed as

$$t_o^{*\prime} = \frac{d - vt_o^{*\prime}}{c'_{K'\to K}},$$

leading to

$$t_o^{*\prime} = \frac{d}{c'_{K'\to K} + v},$$

and

$$t_o = t'_0 + t_o^{*\prime}; t_o = \frac{d}{c'_{K'\to K} + v}. \tag{10}$$

Similarly, with respect to K', the origin of K' at the event E' occurring time is at a distance of $d - vt'$ from that of K (Figure 2).

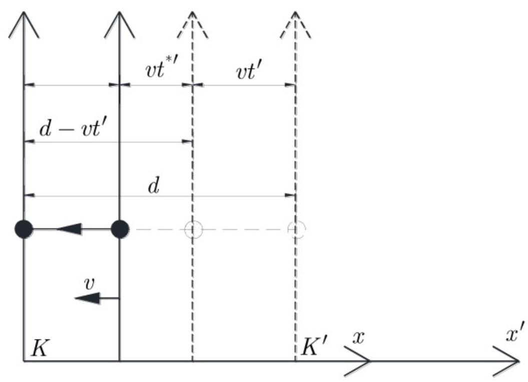

Figure 2. Signal propagation—approaching frames

Let $t^{*\prime}$ be the time interval it takes the event E' signal to reach the origin of K, from K' perspective. By that time, K' will have moved a distance of $vt^{*\prime}$ closer to K, bringing the distance traveled by the signal to $d - vt' - vt^{*\prime}$ with respect to K'. The time interval $t^{*\prime}$ can then be expressed as

$$t^{*\prime} = \frac{d - vt' - vt^{*\prime}}{c'_{K' \to K}},$$

leading to

$$t^{*\prime} = \frac{d - vt'}{c'_{K' \to K} + v} = \frac{d}{c'_{K' \to K} + v} - \frac{vt'}{c'_{K' \to K} + v}.$$

It follows that the event perception time t of E' in K with respect to K' will be given by

$$t = t' + t^{*\prime},$$
$$t = t' + \frac{d}{c'_{K' \to K} + v} - \frac{vt'}{c'_{K' \to K} + v}.$$

Therefore,

$$\Delta t = t - t_o ; \Delta t = \frac{t'}{1 + \frac{v}{c'_{K' \to K}}}.$$

Since $t'_o = 0,$ then

$$\Delta t = \frac{\Delta t'}{1 + \frac{v}{c'_{K' \to K}}}. \quad (11)$$

exhibiting time contraction.

K perspective

Now, from the perspective of K, the origin of K' at the event E'_o occurring time is at a distance of d from that of K. The signal will have traveled a distance d, at the speed $c_{K' \to K}$ with respect to K, when it reaches K origin. Therefore, the event will be perceived at time t_o in K, given by

$$t_o = \frac{d}{c_{K' \to K}}.$$

Similarly, with respect to K, the origin of K' at the event E' occurring time is at a distance of $d - vt'$ from that of K. The signal will have traveled this distance at the speed $c_{K' \to K}$ with respect to K when it reaches K origin. Therefore, the event will be perceived at time t in K, given by

$$t = t' + \frac{d}{c_{K' \to K}} - \frac{vt'}{c_{K' \to K}}.$$

Therefore,

$$\Delta t = t - t_o ; \Delta t = t' \left(1 - \frac{v}{c_{K' \to K}} \right).$$

Since $t'_o = 0,$ then

$$\Delta t = \Delta t' \left(1 - \frac{v}{c_{K' \to K}} \right), \quad (12)$$

exhibiting time contraction.

4. Classical Approach

Considering the classical assumption that the speed of light depends on the velocity of the emitter, the speed of light is constant, say c, with respect to the source rest frame. The speed of light relative to the other frame becomes $c \pm v$, according to the classical addition of velocities.

4.1. Case of Receding Reference Frames—Classical Approach

4.1.1. Change of duration for events occurring at K' ("traveling frame") origin—Classical Approach—receding frames

The speed $c'_{K' \to K}$ and $c_{K' \to K}$ of a light signal traveling from K' to K with respect to K' and K, would be c and $c - v$, respectively.

Applying Eq. (8) for the perceived time interval in K from the K' perspective, we get

$$t = \frac{t'}{1 - \frac{v}{c'_{K' \to K}}} = \frac{t'}{1 - \frac{v}{c}}.$$

Whereas, the same perceived time interval in K from the perspective of K, is given by Eq. (9) as

$$t = t' \left(1 + \frac{v}{c_{K' \to K}} \right) = t' \left(1 + \frac{v}{c - v} \right);$$
$$t = \frac{t'}{1 - \frac{v}{c}}. \quad (13)$$

Therefore, the perceived time interval in K is the same from the perspective of both frames.

It follows that the time interval measured at the origin of the traveling frame K' between two events will be perceived as a dilated time interval in the stationary frame K, by a factor of $(1 - v/c)^{-1}$.

4.1.2. Doppler Effect

If the proper time interval in K' represents the period of a periodic event (e.g., wave, vibration or rotation period), then the relation between the actual and perceived frequency of the event can be determined from Eq. (13) as

$$f = f' \left(1 - \frac{v}{c} \right), \quad (14)$$

where, f and f' are the perceived and actual frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is lower than the proper frequency in the receding source frame.

Equation (14) expresses the classical Doppler effect for the case of a receding source from the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to the receding source, $c-v$ with respect to the observer, when light travels towards the observer.

4.2. Case of Approaching Reference Frames—Classical Approach

4.2.1. Change of duration for events occurring at K' (traveling frame) origin—Classical Approach—approaching frames

In this case, $c'_{K'\to K}=c$, and $c_{K'\to K}=c+v$. Applying Eq. (11) for the perceived time interval in K from the K' perspective, we get

$$\Delta t=\frac{\Delta t'}{1+\dfrac{v}{c'_{K'\to K}}}=\frac{\Delta t'}{1+\dfrac{v}{c}}; \tag{15}$$

whereas, the same perceived time interval in K from the perspective of K, is given by Eq. (12) as

$$\Delta t=\Delta t'\left(1-\frac{v}{c_{K'\to K}}\right)=\Delta t'\left(1-\frac{v}{c+v}\right). \quad \Delta t=\frac{\Delta t'}{1+\dfrac{v}{c}}. \tag{16}$$

Therefore, the perceived time interval in K is the same from the perspective of both frames.

4.2.2. Doppler Effect

If the proper time interval in K' represents the period of a periodic event (e.g., wave, vibration or rotation period), then the relation between the actual and perceived frequency of the event can be determined from Eq. (16) as

$$f=f'\left(1+\frac{v}{c}\right), \tag{17}$$

where, f and f' are the perceived and actual frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is higher than the proper frequency in the approaching frame.

Equation (17) expresses the classical Doppler effect for the case of a source approaching the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to the source, $c+v$ with respect to the observer, when light travels towards the observer.

5. Special Relativity Case

According to the Special Relativity second postulate, light travels in the absence of a propagating medium at a constant speed with respect to all inertial reference frames. Let c be the absolute speed of light with respect to both frames, K and K'.

5.1. Case of Receding Reference Frames—Special Relativity

5.1.1. Change of duration for events occurring at K' ("traveling frame") origin—Special Relativity—receding frames

The speed of light postulate requires that the speed $c'_{K'\to K}$ or $c_{K'\to K}$ of a light signal traveling from K' to K with respect to K' or K would always be c.

Applying Eq. (8) for the perceived time interval in K from the K' perspective, we get

$$t=\frac{t'}{1-\dfrac{v}{c'_{K'\to K}}}=\frac{t'}{1-\dfrac{v}{c}}. \tag{18}$$

Whereas, the same perceived time interval in K from the perspective of K, is given by Eq. (9) as

$$t=t'\left(1+\frac{v}{c_{K'\to K}}\right)=t'\left(1+\frac{v}{c}\right). \tag{19}$$

Each of the parameters t' and represents the same entity in Eqs. (18) and (19) (i.e., t' represents the time interval between two successive events, measured in the traveling frame K', and the corresponding time interval as perceived in the stationary frame K. It follows that

$$t=\frac{t'}{1-\dfrac{v}{c}}=t'\left(1+\frac{v}{c}\right),$$

leading to $v=0$, unless an ad hoc assumption is made such that t is transformed by a certain factor (say γ) with respect to K, and by another factor (say β) with respect to K', leading to the equation

$$t=\frac{\beta t'}{\left(1-\dfrac{v}{c}\right)}=\gamma t'\left(1+\frac{v}{c}\right);$$

which can be satisfied only if $\beta=1/\gamma$, resulting in

$$\gamma=\frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}.$$

Therefore,

$$t=\frac{t'}{\gamma\left(1-\dfrac{v}{c}\right)}=\gamma t'\left(1+\frac{v}{c}\right); \tag{20}$$

It follows that the proper time interval measured at the origin of the traveling frame K' between two events will be perceived as a time interval dilated in K by a factor of $\gamma(1+v/c)$, or $[\gamma(1-v/c)]^{-1}$.

5.1.2. Lorentz Transformation

Now, back to Eq. (20), if the time t' measured at K' origin was for an event that has initially taken place at a point of coordinate $x'>0$ $(x>0)$ on the x-x' axis, then t' could be replaced by x'/c, in the last term of Eq. (20), to yield

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right). \quad (21)$$

Equations (21) is limited to K' proper events with positive x' coordinates, and not applicable for events having $x' = 0$ when $t' > 0$ —in which case Eq. (20) should be used—since they were obtained under $x' = ct'$ and $x = ct$, which returns $t' = 0$ and $t = 0$ for $x' = 0$ and $x = 0$, respectively. Letting $x' = 0$ in Eq. (21)returns the wrong result $t = \gamma t'$.

Now, replacing t and t' with x/c and x'/c in Eqs. (20), yields

$$x = \gamma(x' + vt'), \quad (22)$$

exhibiting perceived length expansion in K.

Equations (21) and (22) are the Lorentz transformation equations, the basis of the special relativity theory.[2,5] However, these equations are derived under the assumption $x' = ct'$ and $x = ct$. i.e. they are limited to K' proper events with positive x and x' coordinates. Indeed, it has been shown in related studies that the Lorentz transformation equations result in mathematical contradictions when applied for co-local or simultaneous events [6,7,8].

It follows that Eq. (20) is a time dilation equation verified within the Special Relativity frame.

5.1.3. Doppler Effect

If the time interval represents the period of a periodic event (e.g., wave, vibration or rotation period) in K' (source), then the relation between the actual and perceived frequency of the event can be determined from Eq. (20) as

$$f = \gamma f'\left(1 - \frac{v}{c}\right). \quad (23)$$

where, f and f' are the perceived and actual (proper) frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is lower than the proper frequency in the receding source frame.

Equation (23) expresses the relativistic Doppler Effect [2] for the case of a receding source from the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to both the source and the observer when light travels towards the observer.

It follows that, the relativistic Doppler Effect Eq. (23) is the consequence of the time dilation given by Eq. (20), verified within the Special Relativity frame. Hence, the experimental validation of Eq. (23) would be a confirmation of the time dilation Eq. (20).

5.2. Case of Approaching Reference Frames—Special Relativity

5.2.1. Change of duration for events occurring at K' (traveling frame) origin—Special Relativity—approaching frames

In this case, $c'_{K'\to K} = c_{K'\to K} = c$. Applying Eq. (11) for the perceived time interval in K from the K' perspective, we get

$$\Delta t = \frac{\Delta t'}{1 + \frac{v}{c'_{K'\to K}}} = \frac{\Delta t'}{1 + \frac{v}{c}}; \quad (24)$$

Whereas, the same perceived time interval in K from the perspective of K, is given by Eq. (12) as

$$\Delta t = \Delta t'\left(1 - \frac{v}{c_{K'\to K}}\right) = \Delta t'\left(1 - \frac{v}{c}\right). \quad (25)$$

Each of the parameters $\Delta t'$ and Δt represent the same entities in Eqs. (24) and (25) (i.e., $\Delta t'$ represents the proper time interval between two successive events, measured in the traveling frame K', and Δt the corresponding time interval as perceived in K. It follows that

$$\Delta t = \Delta t'\left(1 - \frac{v}{c}\right) = \frac{\Delta t'}{\left(1 + \frac{v}{c}\right)},$$

leading to $v = 0$, unless an ad hoc assumption is made such that Δt is transformed by a certain factor (say γ) with respect to K, and by another factor (say β) with respect to K', which leads to the equation

$$\Delta t = \gamma\Delta t'\left(1 - \frac{v}{c}\right) = \frac{\beta\Delta t'}{\left(1 + \frac{v}{c}\right)},$$

which can be satisfied only if $\beta = 1/\gamma$, resulting in

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Therefore,

$$\Delta t = \gamma\Delta t'\left(1 - \frac{v}{c}\right) = \frac{\Delta t'}{\gamma\left(1 + \frac{v}{c}\right)}; \quad (26)$$

It follows that the time interval measured at the origin of the traveling frame K' between two events will be perceived as a time interval contracted in K by a factor of $\gamma(1 - v/c)$, or $[\gamma(1 + v/c)]^{-1}$, in the case of approaching frames.

5.2.2. Lorentz Transformation

Now, back to Eq. (26), If the time t' measured at K' origin was for an event that has initially taken place at a point of coordinate $x' > 0$ $(x > 0)$ on the x-x' axis, then $\Delta t'$ could be replaced by $\Delta x'/c$, in the last term of Eq. (26), to yield

$$\Delta t = \gamma\left(\Delta t' - \frac{v\Delta x'}{c^2}\right). \quad (27)$$

Equations (27), Lorentz transformation time equation for approaching frames, is limited to K' proper events with positive x and x' coordinates, and are not applicable for $\Delta x' = 0$ with $\Delta t' > 0$ $(\Delta x = 0$ with $\Delta t > 0)$ — in which case Eq. (26) should be used—since they were obtained under $\Delta x' = c\Delta t'$ and $\Delta x = c\Delta t$, which returns $\Delta t' = 0$ and $\Delta t = 0$ for $\Delta x' = 0$ and $\Delta x = 0$, respectively. Letting $\Delta x' = 0$ in Eq. (27) returns the wrong result $\Delta t = \gamma \Delta t'$.

Now, replacing Δt and $\Delta t'$ with $\Delta x / c$ and $\Delta x' / c$ in Eq. (26), yields

$$\Delta x = \gamma(\Delta x' - v\Delta t'). \tag{28}$$

exhibiting perceived length contracted in K.

It follows that Eq. (26) is a time contraction equation verified within the Special Relativity frame.

5.2.3. Doppler Effect

If the time interval represents the period of a periodic event (e.g., wave, vibration or rotation period), then the relation between the actual and perceived frequency of the event can be determined from Eq. (26) as

$$f = \gamma f' \left(1 + \frac{v}{c}\right). \tag{29}$$

where, f and f' are the perceived and actual (proper) frequency with respect to an observer in K and K', respectively. Hence, the perceived frequency is higher than the proper frequency in the approaching source frame.

Equation (29) expresses the relativistic Doppler effect for the case of an approaching source to the observer at a uniform velocity v, under the assumption that the speed of light is c with respect to both the source and the observer when light travels towards the observer.

It follows that the relativistic Doppler Effect in the case of approaching reference frames exhibits blue shift for light, in line with the determined time (period) contraction [Eq. (26)]. Thus, the Special Relativity prediction of time dilation for both receding and approaching frames is contradicted.

It follows that, the relativistic Doppler Effect Eq. (29) is the consequence of the time contraction given by Eq. (26) for events occurring at K' origin, verified within the Special Relativity frame. Hence, the experimental validation in the “referenced study” of Eq. (29) would be a confirmation of the time contraction, contradicting the Special Relativity prediction of time dilation.

6. Conclusion

The Ives-Stilwell type experiments adopted criterion to validate the time dilation was shown to be invalid. An infinite number of frequency shift equations would satisfy the used validation criterion. Even if the time dilation test conducted through the experimental verification of the relativistic Doppler frequency shift equations was assumed to be valid, it would be, within the frame of the Special Relativity, asserting a time contraction for approaching inertial frames (i.e., the contraction in the stationary frame of a proper time interval measured at the origin of the traveling frame), which is in contradiction with the Special Relativity. This time contraction was asserted via its thorough derivation according to the Special Relativity assumptions (based on the speed of light postulate), and validation within the Special Relativity frame.

On the other hand, the classical relativity resulted in perceived time contraction for approaching frames, in line with the classical Doppler Effect. If the “referenced study” test result was presumed to be valid, it could be attributed to the classical Doppler Effect, within 10 % difference.

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