

Free- free Scattering Theory of the Elastic Scattering of an Electron

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Abstract In the present work, the elastic scattering of an electron from the target by absorbing a photon from the laser field has been studied. Since the solution of the Schrödinger equation of whole three-body system has not been found, we consider such intensities of electromagnetic field (Laser field) that the electron field coupling is the dominant process and the target is transparent to the field such that photon- target coupling can be ignored. Internal structure of target can be ignored and represented just as a scattering potential. For number of photon, $l = -1$, i.e., absorption of a photon (inverse Bremsstrahlung), The differential scattering cross section of an electron depends upon the fourth power of the wavelength (λ^4) and the intensity of the Laser field. The certain values of laser parameters the differential scattering cross section of scattered electron decreases with increase in scattering angle and attains a minimum value of 0.1 barn and further increase in scattering angle also increases in differential scattering cross section and attains a maximum value of 0.3 barn.

Keywords: scattering, schrodinger, electron, plasma, laser

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1. Introduction

Electron atom interaction in the presence of a laser field attracted considerable theoretical attention in the recent year not only because of the importance in applied areas (such as plasma heating or laser driven fusion), but also in view of their interest in fundamental atomic theory. The problem of this process, is in general, very complex, since in addition to the difficulties associated with the treatment of electron atom collision, the presence of the laser introduces new parameters (for example, the laser photon energy $\hbar\omega$ and intensity I) which may influence the collision. Moreover, the laser photon can play the role of a "third body" during the collision, and "dressed" the atomic states. It is therefore of interest to begin the theoretical analysis by considering the simple problem of the scattering of an electron by a potential in the presence of a laser field. A fully realistic description of the target atom is quite difficult. We shall represent it here by a potential model.

Mason and Newell (1982) reported experimental evidence of simultaneous electron-photon excitation of atoms. However most experimental studies have been performed with noble gases [1] a recent one being with a Nd:YAG laser [2]. On the other hand, theoretical studies are not easy to perform with these atoms and the hydrogen atom has been studied extensively [3]. Hydrogen as a one electron atom is a simple to deal with and it is often interesting information concerning the main features of the problem. The free - free process can theoretically be

studied at various levels. As the target does not change states in this process, its own energy spectrum can be ignored and a simple potential can mimic the electron atom interaction. The collision process can then be treated either classically or quantum mechanically by means of the simple scattering theories. Furthermore, the collision can be treated as occurred at such intensities of electromagnetic field that the electron-field coupling is the dominant process and the target is transparent to the field such that photon-target coupling can be ignored. If, however, the frequency of the photon is such as to couple two stationary states of the target, then the target-field interaction becomes extremely important. Here we discussed such intensity of the electromagnetic field where the photon-field interaction can be neglected. The aim of the work is effect of various collision and laser parameters on the collision process. This is the motive of the paper. Furthermore, most theoretical studies for scattering of electron by atoms in an intense radiation field are based on perturbation [4] starting with the well known Kroll and Waston (1976) work on the soft photon approximation, there exist only a few non-perturbative approaches for this problem. Shakeshaft (1983) formulated a non- perturbative method of coupled integral equations for calculating the scattering cross section by assuming the potential to be separable. Rosenberg (1981) applied the variational method for coulomb scattering in a laser field using a low frequency approximation [6].

The reaction studied in the present work is,

$$e_{k_i}^- + H(i) + N(\omega, \hat{\epsilon}) \rightarrow e_{k_f}^- + H(j) + (N \pm 1)\gamma(\omega, \hat{\epsilon})$$

Representing the collision of an incoming electron with momentum k_i with hydrogen atom initially in the state i in the presence of a single mode laser beam moving to the excited state j with exchange of 1 photons between the electron and the laser field. We have used the Born approximation, to treat the electron- atom interaction as it is simple enough to allow calculation of a larger number of transitions, and it becomes exact at high energies.

2. Theory

We consider a collision between an electron and hydrogen atom in the presence of a laser field. We begin this section by considering the simple case in which the target atom is modeled by a center of force and hence does not interact with the laser field. The field is assumed to be purely monochromatic with angular frequency ω , linearly polarized with linear polarization vector $\hat{\epsilon}$. We also assume that the dipole approximation is valid. The Hamiltonian of the electron-atom system in the presence of a laser beam can be written as,

$$H = H_f + H_t + V(r_o, r_a) \quad (1)$$

Where

$H_f \rightarrow$ Hamiltonian of the free electron

$H_t \rightarrow$ Hamiltonian of the target atom

And $V(r_o, r_a)$ is the interaction between the incident electron and atom, defined as,

$$V(r_o, r_a) = \frac{-Z}{|r_o|} + \sum_{j=1}^Z \frac{1}{|r_o - r_{aj}|} \quad (2)$$

Where,

$r_o \rightarrow$ position coordinate of the projectile electron

$Z \rightarrow$ atomic number of target or charge of the target atom

$r_a \rightarrow$ position of an atomic electron

For H-atom, $Z=1$ (i.e., one electron) equation (2) becomes

$$V(r_o, r_a) = \frac{-Z}{|r_o|} + \frac{1}{|r_o - r_1|}$$

In equation (2) Z is the charge of the target atom. Working in the coulomb gauge we have the electric field

$$\vec{E}(t) = \vec{E}_o \sin \omega t.$$

And the corresponding vector potential is

$$\vec{A}(t) = \vec{A}_o \cos \omega t \quad \text{with} \quad \vec{A}_o = \frac{c\vec{E}_o}{\omega}.$$

In the presence of a laser field the incidence electron of momentum k is represented as

$$\chi = \vec{r}_o, t = (2\pi)^{-3/2} \exp \left[\left(i\vec{k} \cdot \vec{r} - i\vec{k} \cdot \vec{\alpha}_o \sin \omega t \right) - \frac{iE_k t}{\hbar} \right] \quad (3)$$

Where, $E_k = \frac{\hbar^2 k^2}{2m}$

$$\vec{\alpha}_o = \frac{e\vec{A}_o}{m\omega}$$

Also, $\vec{\alpha}_o = \frac{e\vec{E}_o}{m\omega^2}$ is the measure of the coupling

between the field and the projectile.

Equation (3) is also known as Volkov wave function. This equation gives the states of a free electron in a laser field. Since there is no conservation of energy in free state of an electron. So the states represented by the Volkov wave function is also called as virtual state, to make the energy conservation there must be another parameter. Here in our case we take the static potential as the third parameter for energy conservation of electron [7].

The S-matrix (scattering matrix) element for the transition from initial state (i) to final state (f) ($i.e. i \rightarrow f$) is given by, [ref]

$$S = \frac{-i}{\hbar} \langle \chi_f | V \chi_i^+ \rangle$$

The angle bracket denotes both space and time integration. Where,

χ_f is the Volkov wave function for the final state of an electron

χ_i^+ is the exact formal solution of the following Schrödinger wave equation,

$$\frac{1}{2m} \left(\vec{p} + \frac{e\vec{A}(t)}{c} \right)^2 \psi(\vec{r}, t) + V(\vec{r})\psi(\vec{r}, t) = i\hbar \frac{\partial \psi(\vec{r}, t)}{\partial t}$$

where $\psi(\vec{r}, t)$ is the time dependent wave function in presence of the scattering potential $V(\vec{r})$ in the presence of laser field.

The solution to this equation (4.65) can be written as [ref],

$$\psi^+(\vec{r}, t) = \psi_i(\vec{r}, t) + \int_{-\infty}^{\infty} G_+(t, t') V \psi^+(\vec{r}, t') dt'$$

Now taking electron atom interaction upto the first Born approximation, we get scattering matrix (S-matrix) as,

$$S = \frac{-i}{\hbar} \langle \chi_f | V | \chi_i \rangle \quad (4)$$

χ_i is the Volkov wave function for the initial state of an electron

And V is the interaction potential defined by equation (2).

Initially the electron is free from potential and takes part in interaction for certain time and after that it scatters from the potential to the free state. So this interaction is also known as free-free scattering process [8].

Now we can write equation (4) as,

$$S_{k_f k_i} = \frac{-i}{\hbar} \int \int_{-\infty}^{\infty} \left\{ \begin{array}{l} (2\pi)^{-3/2} \exp \left[\left(i\vec{k}_f \cdot \vec{r} - i\vec{k}_f \cdot \vec{\alpha}_o \sin \omega t \right) - \frac{iE_{k_f} t}{\hbar} \right] V(\vec{r}) \\ (2\pi)^{-3/2} \exp \left[\left(i\vec{k}_i \cdot \vec{r} - i\vec{k}_i \cdot \vec{\alpha}_o \sin \omega t \right) - \frac{iE_{k_i} t}{\hbar} \right] d^3 r \end{array} \right\} dt$$

Where,

$k_i \rightarrow$ initial wave vector of projectile particle

$k_f \rightarrow$ final wave vector of the scattered particle

$$S_{k_f k_i} = \frac{-i}{\hbar} (2\pi)^{-3} \int_{-\infty}^t e^{-i\Delta \cdot \vec{r}} e^{i\Delta \cdot \vec{\alpha}_o \sin \omega t} e^{i(E_{k_f} - E_{k_i}) \frac{t}{\hbar}} V(\vec{r}) d^3 r dt$$

Here, $\Delta \rightarrow (k_f - k_i)$

(momentum transfer during scattering process)

$$S_{k_f k_i} = \frac{-i}{\hbar} \hat{V}(\Delta) \int_{-\infty}^t e^{i\Delta \cdot \vec{\alpha}_o \sin \omega t} e^{i(E_{k_f} - E_{k_i}) \frac{t}{\hbar}} dt$$

where,

$$\hat{V}(\Delta) = (2\pi)^{-3} \int e^{-i\Delta \cdot \vec{r}} V(\vec{r}) d^3 r$$

This is the Fourier transformation of the potential in the momentum space.

Since $V(\vec{r})$ is independent of time (t), we can separate time and space integration so that $\hat{V}(\Delta)$ can be taken outside of time integration [9].

Using the generating function of the Bessel Polynomial We get,

$$e^{i\Delta \cdot \vec{\alpha}_o \sin \omega t} = \sum_{l=-\infty}^{\infty} J_l(\Delta \cdot \alpha_o) e^{il\omega t}$$

So the S-matrix element becomes

$$S_{k_f k_i} = \frac{-i}{\hbar} \hat{V}(\Delta) \int_{-\infty}^t \sum_{l=-\infty}^{\infty} J_l(\Delta \cdot \alpha_o) e^{il\omega t} e^{i(E_{k_f} - E_{k_i}) \frac{t}{\hbar}} dt$$

where l is the no of photons exchange during the scattering process. It may takes the values,

$$l = 0, \pm 1, \pm 2, \pm 3, \dots$$

Where, positive l describes the photon emission stimulated Bremsstrahlung and negative l describes the photon absorption or inverse Bremsstrahlung and $l=0$ corresponds to the pure elastic scattering in the presence of the laser field.

$$S_{k_f k_i} = \frac{-i}{\hbar} T_{k_f k_i}^l \int_{-\infty}^t e^{i(E_{k_f} - E_{k_i} + l\hbar\omega) \frac{t}{\hbar}} dt \quad (5)$$

where,

$$T_{k_f k_i}^l = \hat{V}(\Delta) \sum_l J_l(\Delta \cdot \alpha_o)$$

Using the idea of integral form of Dirac delta function

$$\delta(x - x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik(x-x')} dk$$

We reduce equation (5) in the following form,

$$S_{k_f k_i} = -2\pi i T_{k_f k_i}^l \delta(E_{k_f} - E_{k_i} + l\hbar\omega) \quad (6)$$

The delta function in above equation ensures energy conservation,

$$i.e., E_{k_f} = E_{k_i} - l\hbar\omega$$

$$\frac{k_f}{k_i} = \left(1 - \frac{l\hbar\omega}{E_{k_i}} \right)^{\frac{1}{2}}$$

Here $\hbar\omega$ is the photon energy and E_{k_i} is the kinetic energy of a projectile electron.

Now the differential scattering cross section in terms of transition matrix is given by,

$$\frac{d\sigma^{B1}}{d\Omega} = \frac{m^2}{(2\pi)^2 \hbar^4} \frac{k_f}{k_i} \left| T_{k_f k_i}^l \right|^2$$

This is the required relation of differential cross section of electron with transfer of l photon.

where,

$$\left| T_{k_f k_i}^l \right|^2 = \sum_l J_l^2(\Delta \cdot \alpha_o) |\hat{V}(\Delta)|^2$$

$$\hat{V}(\Delta) = (2\pi)^{-3} \int e^{-i\Delta \cdot \vec{r}} V(\vec{r}) d^3 r$$

For spherically symmetric potential,

$$V(\vec{r}) = V(r)$$

So that,

$$\hat{V}(\Delta) = (2\pi)^{-3} \int_0^{\infty} V(r) r^2 dr \int_0^{\pi} e^{-i\Delta r \cos \theta} \sin \theta d\theta \int_0^{2\pi} d\phi$$

This gives,

$$|\hat{V}(\Delta)|^2 = \frac{e^4}{4\pi^4 \Delta^4}$$

To calculate $\sum_l J_l(\Delta \cdot \alpha_o)$ we have the expression for the Bessel function which is, [ref]

$$J_n(x) = \frac{x^n}{2^n \Gamma(n+1)} \left[1 - \frac{x^2}{2 \cdot 2(n+1)} + \frac{x^4}{2 \cdot 4 \cdot 2(n+1)(n+2)} - \dots \right]$$

Here we take

$$n = l \text{ and } x = \Delta \cdot \alpha_o \text{ then}$$

$$J_l(x) = \frac{(\Delta \cdot \alpha_o)^l}{2^l \Gamma(l+1)} \left[1 - \frac{(\Delta \cdot \alpha_o)^2}{2 \cdot 2(l+1)} + \frac{(\Delta \cdot \alpha_o)^4}{2 \cdot 4 \cdot 2(l+1)(l+2)} - \dots \right]$$

For low intensity and low frequency we get,

$$\left[J_l(\Delta \cdot \alpha_o) \right]^2 = \frac{\Delta^2 \alpha_o^2}{4}$$

Thus we get square modulus of the transition matrix element as,

$$\left| T_{k_f k_i}^l \right|^2 = \frac{\Delta^2 \alpha_o^2}{4} \frac{e^4}{4\pi^4 \Delta^4}$$

Substituting this values we get the value of the differential cross section as,

$$\frac{d\sigma^{B1}}{d\Omega} = \frac{e^6}{(2\pi)^6 \hbar^4} \frac{k_f}{k_i} \frac{E_o^2}{\omega^4 (k_f - k_i)^2}$$

$$\frac{d\sigma^{B1}}{d\Omega} = \frac{e^6}{(2\pi)^6 \hbar^4} \frac{k_f}{k_i} \frac{E_o^2}{\omega^4 (\Delta)^2} \quad (7)$$

We have, $\Delta = \overline{k_f} - \overline{k_i}$

$$\Delta^2 = k_f^2 + k_i^2 - 2\overline{k_f k_i} \cos \theta$$

Where, θ is the angle between k_f and k_i , that means θ is the scattering angle between initial and final momentum of the projectile particle.

Thus we have,

$$\Delta^2 = k_i^2 \left[\left(1 - \frac{l\hbar\omega}{E_{k_i}}\right) - 2 \left(1 - \frac{l\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \cos \theta + 1 \right]$$

Substituting this value in equation (7) we get,

$$\frac{d\sigma^{B1}}{d\Omega} = \frac{e^6}{(2\pi)^6 \hbar^4} \frac{k_f}{k_i} \frac{E_o^2}{\omega^4 k_i^2 \left[\left(1 - \frac{\hbar\omega}{E_{k_i}}\right) - 2 \left(1 - \frac{\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \cos \theta + 1 \right]}$$

$$\frac{d\sigma^{B1}}{d\Omega} = \frac{e^6}{\hbar^2 (2\pi)^6} \left(1 - \frac{\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \frac{E_o^2}{\omega^4 2mE_{k_i} \left[\left(1 - \frac{\hbar\omega}{E_{k_i}}\right) - 2 \left(1 - \frac{\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \cos \theta + 1 \right]}$$

$$\frac{d\sigma^{B1}}{d\Omega} = \frac{e^6}{(2)^8 m\hbar^2 (\pi)^{10} c^5} \left(1 - \frac{l\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \frac{\lambda^4 I_o}{E_{k_i} \left[\left(1 - \frac{l\hbar\omega}{E_{k_i}}\right) - 2 \left(1 - \frac{l\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \cos \theta + 1 \right]}$$

We have the relation

$$\omega = \frac{2\pi c}{\lambda} \quad \text{and} \quad E_o^2 = \frac{8\pi I_o}{c}$$

where, I_o is the intensity of the laser field.

λ is the wave length of the radiation.

c is the velocity of light.

$$\frac{d\sigma^{B1}}{d\Omega} = C \left(1 - \frac{l\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \frac{\lambda^4 I_o}{E_{k_i} \left[\left(1 - \frac{l\hbar\omega}{E_{k_i}}\right) - 2 \left(1 - \frac{l\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \cos \theta + 1 \right]}$$

Where, $C = \frac{e^6}{(2)^8 m\hbar^2 (\pi)^{10} c^5}$ is the constant quantity.

Now, for $l=1$ i.e, one photon emission (stimulated emission)

$$\frac{d\sigma^{B1}}{d\Omega} = C \left(1 - \frac{\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \frac{\lambda^4 I_o}{E_{k_i} \left[\left(1 - \frac{\hbar\omega}{E_{k_i}}\right) - 2 \left(1 - \frac{\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \cos \theta + 1 \right]} \quad (8)$$

This is the differential cross section for the scattering of an electron with one photon emission (stimulated Bremsstrahlung) in the presence of potential.

also for $l=-1$ i.e, one photon absorption (inverse Bremsstrahlung),

$$\frac{d\sigma^{B1}}{d\Omega} = C \left(1 + \frac{\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \frac{\lambda^4 I_o}{E_{k_i} \left[\left(1 + \frac{\hbar\omega}{E_{k_i}}\right) - 2 \left(1 + \frac{\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}} \cos \theta + 1 \right]} \quad (9)$$

This is the differential cross section for the scattering of an electron with one photon absorption (inverse Bremsstrahlung) in the presence of potential.

3. Result and Discussion

In the present work, we have studied the elastic scattering of an electron-atom interaction by absorbing a photon from the laser field. For simplicity, it has been neglected the dressing effect of an atom and the field-atom interaction so that we can neglect the internal structure of an atom because the intensity of the field in such a way that the target atom is transparent for the field. [10] Here, the differential cross-section equations (8) and (9) is found to be proportional to λ^4 (where λ is the wavelength of the radiation) which explains why it has been easier to observe laser-assisted cross section using infrared laser (high wave length and low frequency) than those operating in the visible or ultraviolet spectral regions [11]. Equation (9) is plotted as the functions of scattering angle, kinetic energy of an electron, wavelength and intensity of the laser field as shown below:

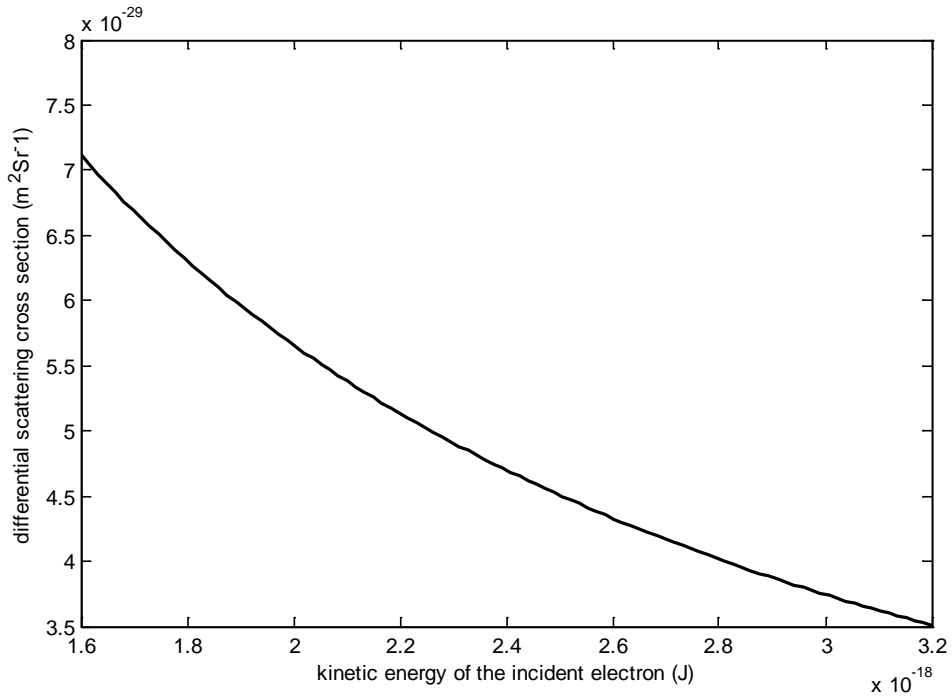


Figure 1. Variation of differential scattering cross section with kinetic energy of the incident electron

From the plot, we see that the differential scattering cross section decrease as the kinetic energy of the incident electron increases. Going on increasing the kinetic energy

of an incident electron we will get zero value of scattering cross section.

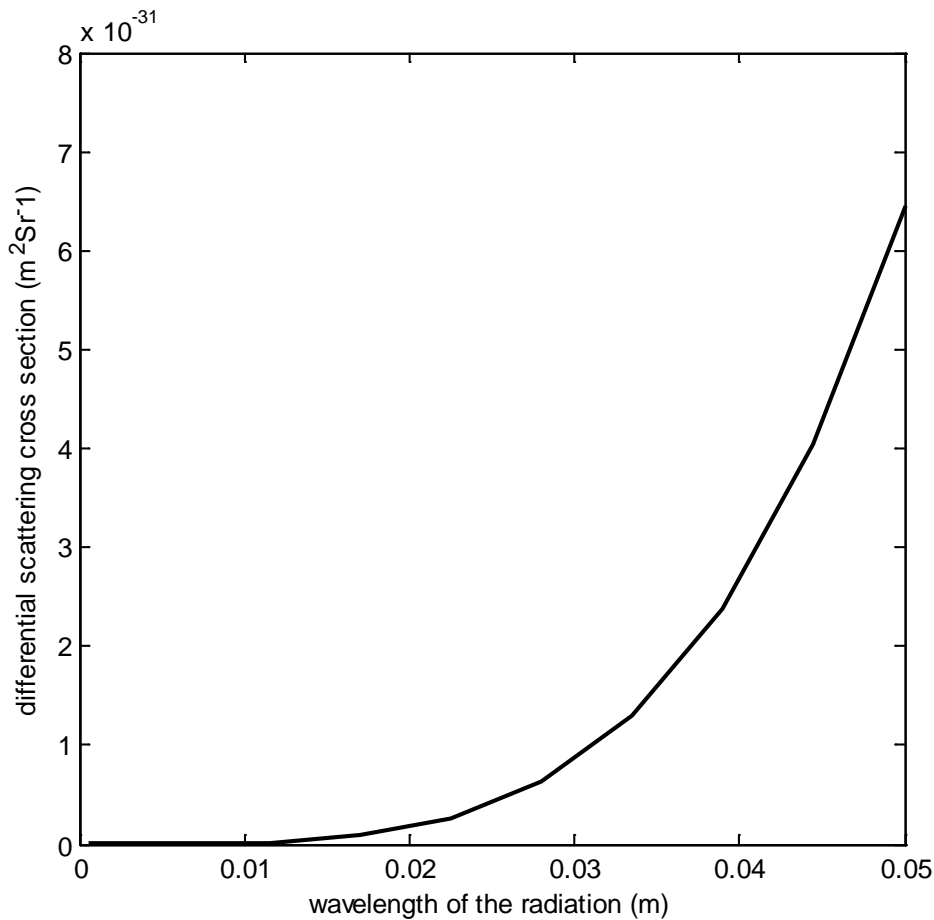


Figure 2. Variation of differential cross section with wavelength of the field

From the plot, it is found that the differential scattering cross section increases with increase in wavelength of the

laser field [12]. It shows that the differential scattering cross section is zero below the wavelength 0.01m.

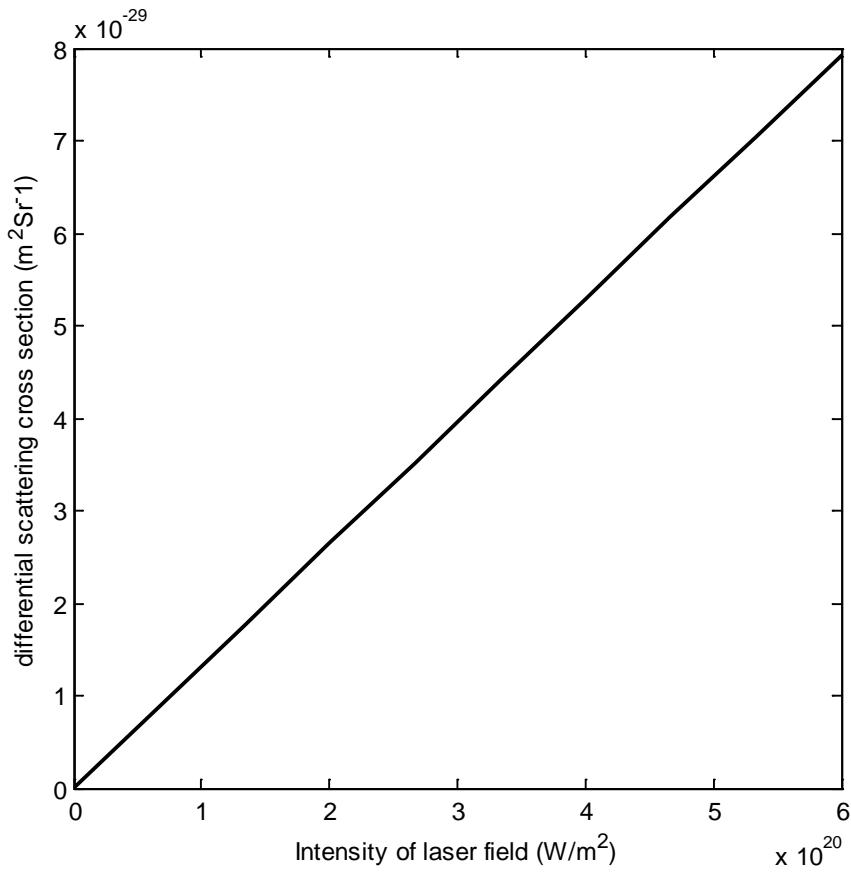


Figure 3. variation of differential cross section with intensity of the laser field

This figure is indicated that the differential scattering cross section increase with increase in the intensity of the laser field. It shows that the differential scattering cross section varies linearly with intensity.

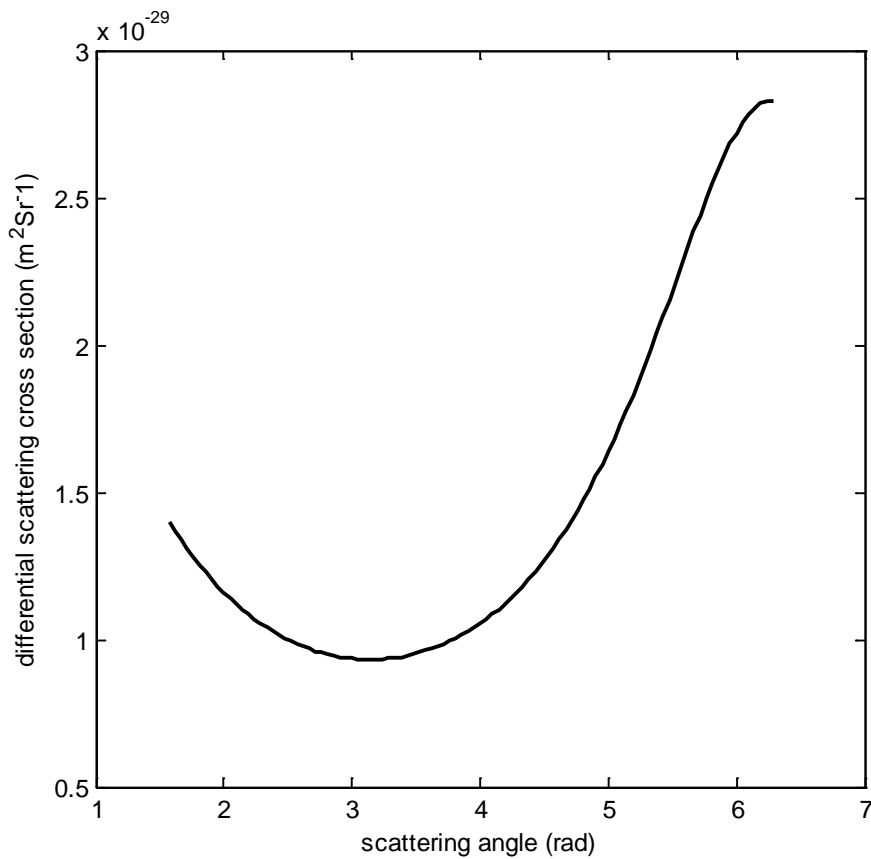


Figure 4. Variation of differential cross section with scattering angle

From the plot, it is clear that differential cross section decreases as the scattering angle increase and attains the minimum value $1 \times 10^{-29} m^2$ and further increase in

scattering angle also increase in differential scattering cross section which is clearly shown in polar plot.

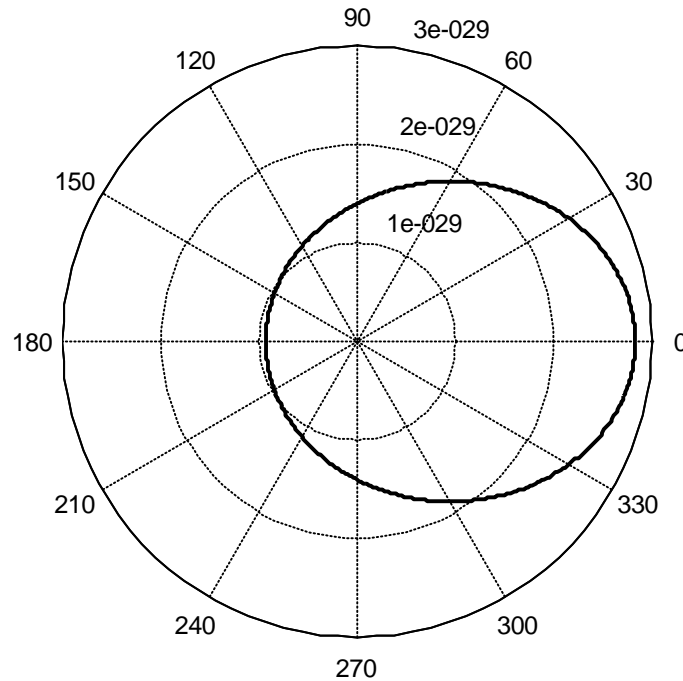


Figure 5. Polar plot of differential cross section with scattering angle

From the polar plot, we know that the differential scattering cross section of an electron decreases with increase in scattering angle and attains minimum value and further increase in scattering angle also increase in cross section and attains maximum value $3 \times 10^{-29} m^2$.

4. Conclusion

It is generally observed that when electrons are scattered from the atom in the presence of a laser field, a new effect is observed which are not accessible in ordinary electron –atom scattering. This collision have the basic peculiarity of being processes in which three subsystem are present (i) the electron (ii) the target atom (iii) the radiation field. The last one provided energy and momentum and is characterized by the polarization of its electric field, which introduces in this collision process a new physical axis an electron from the target atom by absorption of a photon from the laser field to the differential scattering cross section of an electron depends upon the intensity and wavelength of the laser field where as in ordinary electron -atom scattering, cross section only depends upon the scattering angle. The differential scattering cross section increases with increase in wavelength and intensity of the laser field. For a fixed value of a laser parameters and kinetic energy of an incident electron, the differential scattering cross section of an scattered electron decreases with increase in scattering angle and attains a minimum value of $1 \times 10^{-29} m^2 = 0.1 \text{ barn}$ and further increase in scattering angle also increase in differential scattering cross section and attains a maximum value of $3 \times 10^{-29} m^2 = 0.3 \text{ barn}$. Also from this study we see that, the differential scattering

cross section for the electric field perpendicular to the direction of momentum transfer is zero. So we concluded that the differential scattering cross section is also greatly depends upon the polarization of the laser field.

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