The Critical Error in the Formulation of the Special Relativity

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Abstract The perception of events in two inertial reference frames in relative motion was analyzed from the perspective of the Special Relativity postulates, leading to the Lorentz transformation equations for the time and space coordinate in the relative motion direction. Yet, straightforward inconsistencies were identified upon examining the conversion of the time interval between two co-local events in the traveling reference frame. The approach used in the Special Relativity formulation to get around the identified inconsistencies was revealed. Subsequent mathematical contradictions in the Lorentz transformation equations, disproving the Special Relativity predictions, were shown.

Keywords: *special relativity, time dilation, length contraction, Lorentz transformation, mathematical contradictions, co-local and simultaneous events*

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1. Introduction

Einstein [1] formulated his theory of Special Relativity (SR) on the basis of two postulates; the principle of relativity (i.e., the equations describing the laws of physics have the same form in all proper frames of reference), and the principle of the constancy of the speed of light in all reference frames. The Lorentz transformation (LT), a set of space-time equations to convert coordinates between two inertial frames of reference in relative motion, predicting time dilation and length contraction under particular interpretations, was the principal result of the SR. There are, however, many theoretical difficulties with the SR, not always recognized in mainstream physics. Several researches have identified inconsistencies between experimental findings and the theory's predictions. For instance, faster than light transfer of information by microwaves has been realized by Nimz. [2,3] Moreover, the famous Michelson-Morley experiment, often claimed to support the SR validation, has been reinterpreted to show it hadn't been properly evaluated [4,5] The SR predictions have led to numerous paradoxes, consistently generating critical publications on the SR validity, [6,7,8,9] particularly the clock paradox expressed in what's become known as the twin paradox, discussed in details in a critical study [10] challenging the viability of the SR. Some researches [11,12,13] have shown that alternative versions of SR can be realized without the speed of light postulate and give rise to perfectly coherent theories, provided that space-time transformation different than the Lorentz's one is adopted. Most critics argue that the SR is mathematically sound, yet its mathematical formulation is based on faulty assumptions, particularly the speed of light postulate. In response, according to some authors, [14,15,16,17] the SR can be derived from the principle of relativity postulate only, and the light speed constancy would then rather be deduced from the theory, as being a consequence of the first postulate. In this paper, mathematical anomalies in the SR formulation have been revealed, and contradictions in the LT have been identified, regardless of whether the speed of light postulate is used in its derivation or not, adding a further reason why reconsidering the concepts of SR is needed, and an alternative version of SR should be adopted.

The SR time dilation prediction is based on the transformation, in the "stationary" reference frame, of a proper time interval between two events occurring at the origin (or co-local events) of the "traveling" reference frame-in relative translational motion with respect to the "stationary" one. It has been shown in earlier works that such transformation is invalid, as it involves co-local events-occurring successively, at a reference frame origin (zero spatial coordinates), or with zero spatial interval. [18,19] In this paper, further analysis of event perceptions relative to both frames reconfirm the invalidity of such transformation, hence the invalidity of the SR prediction of the time dilation. Similar analysis for simultaneous events proves the invalidity of the SR length contraction prediction.

2. Temporal Events Analysis

Consider two inertial frames of reference, $K(x, y, z, t)$ and $K'(x', y', z', t')$, in translational relative motion with

parallel corresponding axes, and let their origins be aligned along the overlapped x- and x'- axes. Let v be the relative motion velocity in the x-x' direction. K and K' coordinate systems are assumed to be overlapping at the time $t = t' = 0$; so as event coordinates in K and K' can be considered as space and time intervals measured from the initial zero coordinates of the overlapped-frames event.

2.1. Arbitrary Non-origin Events

Let's suppose that at the frames overlapping instant, an event $E_1(x', 0, 0, 0)$ $[E_1(x, 0, 0, 0)]$ takes place at a distance x' with respect to K' origin (x with respect to K origin) on the x-x' axis. According to the SR light speed postulate (i.e., the constancy of the speed of light in all inertial reference frames), this event is perceived by an observer at K' origin at the time:

$$t' = \frac{x'}{c}, \tag{1}$$

and by an observer at K origin at the time:

$$t = \frac{x}{c}. \tag{2}$$

With respect to the K' observer, the origin of K' at the event perception time is at a distance of vt' from that of K. Therefore, using the SR speed of light postulate, the event perception times in K and K' shall be related by the following equation, with respect to the K' observer:

$$t = t' + \frac{vt'}{c}. \tag{3}$$

To account for any time scaling distortion due to the relative motion between the inertial frames K and K', let's write equation (3) in the form:

$$t = \gamma\left(t' + \frac{vt'}{c}\right), \tag{4}$$

where γ is a real positive factor depending on v, and which will be determined.

In fact, this scaling factor is essential for the speed of light postulate to be retained, since using the light speed postulate, the inverse of Eq. (3) (i.e., from the perspective of K) can be written as:

$$t' = t - \frac{vt}{c},$$

which, when substituted in Eq. (3) will lead to $t = t'$, and consequently to $v = 0$.

Now, replacing Eq. (1) into Eq. (4) yields:

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right). \tag{5}$$

Multiplying both sides of Eq. (4) by c, and using Eqs. (1) and (2) leads to:

$$x = \gamma(x' + vt'). \tag{6}$$

Using the SR first postulate and the isotropic property of space, the inverse of the transformation Eqs. (5) and (6) can be obtained by swapping the primed and unprimed coordinates, and replacing v with $-v$:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right). \tag{7}$$

$$x' = \gamma(x - vt). \tag{8}$$

Solving Eqs. (5), (6), (7), and (8) for γ results in ([18], [19]):

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{9}$$

Eqs. (5) and (6) are therefore the inverse-(7) and (8) the direct Lorentz transformation equations for the time and space coordinate in the relative motion direction (i.e., in the x-x' direction).

2.2. Co-local Events at K' Origin

Now, suppose an event $E_2(0, 0, 0, t')$ $[E_2(vt, 0, 0, t)]$ occurs at K' origin,

$$x' = 0, \tag{10}$$

at the time t' with respect to K' (t with respect to K).

Again, with respect to the K' observer, the origin of K' at the event perception time is at a distance of vt' from that of K. Therefore, using the SR speed of light postulate, the event perception times in K and K' shall be related by the following equation, with respect to the K' observer:

$$t = t' + \frac{vt'}{c}. \tag{11}$$

Taking account of the time scaling factor (γ) determined earlier, Eq. (11) should be written in the following corrected form:

$$t = \gamma\left(t' + \frac{vt'}{c}\right). \tag{12}$$

However, in this case Eq. (1) doesn't hold, and therefore Eq. (5) doesn't follow. Yet, in SR it is customary for such events (occurring at K' origin) to replace Eq. (10) ($x' = 0$) in LT Eq. (5), inapplicable in this case, since it is deduced from Eq. (4) (corresponding to Eq. (12)) for events having $x' = ct'$ invalid for co-local events having $x' = 0$ and $t' > 0$. Replacing t' with x'/c in Eq. (12) while $x' = 0$ will result in a truncated equation with a missing vital term, vt'/c.

Therefore, for an event occurring at K' origin ($x' = 0$) at time t', the SR-predicted time t with respect to K is concluded from the invalid (for this case) Eq. (5) as:

$$t = \gamma t', \tag{13}$$

interpreted in SR as a time dilation with respect to K since $\gamma > 1$, of course.

Whereas, Eq. (12) predicts this time dilation to be:

$$t = \gamma t'\left(1 + \frac{v}{c}\right). \qquad (14)$$

Comparing Eqs. (13) and (14) results in the contradiction:

$$v = 0. \qquad (15)$$

It follows that the SR conversion $x' = 0$; $t = \gamma t'$, physically predicting time dilation in K for co-local events time interval in K', is invalid.

The same analysis of the above two events can be performed from the perspective of an observer at K origin, with a similar contradiction being obtained.

2.3. Simultaneous Events

Similarly, LT Eq. (6) is not applicable for events having $t' = 0$ and $x' \neq 0$ as it is derived under Eqs. (1) and (2), requiring $x' = 0$ for $t' = 0$. However, in SR interpretation of LT Eq. (6), length contraction from the perspective of K' is predicted by setting $t' = 0$ (for simultaneous events duration) to get the relation $x = \gamma x'$, ignoring the restriction imposed by the basic speed of light constancy Eqs. (1) and (2). Hence follows the invalidity of the SR length contraction prediction. The same reasoning is applicable to Eq. (8) to show the invalidity of the SR time contraction prediction from the perspective of K.

3. The Special Relativity Approach

It is ascertained in the previous sections that the LT time equations:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right), t = \gamma\left(t' + \frac{vx'}{c^2}\right),$$

are principally derived on the basis of events having $x = ct$; $x' = ct'$, implicitly incorporated in the equations. In fact, regardless of the derivation method, a simple physical analysis of the above LT time equations reveals that with respect to K, the time t' it takes a light signal, emitted from the point of the coinciding origins at $t = t' = 0$, to travel a distance x' in K' is equal to the time t for the signal to travel the corresponding distance x in K less the signal travel time of the distance vt travelled by the origin of K' at the time t, corrected by the relativistic factor γ. In other words, an event occurring in K' [origin] at the time t with respect to K has already occurred at the time t' equal to t less the signal time of travel from the position of K' [origin] at the time t to K origin, corrected by the relativistic factor γ. Therefore, the term vx/c^2 in the Lorentz time transformation must be the [uncorrected] time it takes the light signal to travel the distance between the origins at the time t with respect to K, or

$$\frac{vx}{c^2} = \frac{vt}{c},$$

leading to $x = ct$. Similarly, the inverse LT leads to $x' = ct'$.

Hence, the expressions $x = ct$ and $x' = ct'$, invalid for co-local events having $x = 0$ and $t > 0$ ($x' = 0$ and $t' > 0$) are an intrinsic part of the LT equations. These restrictions are obviously fatal for the SR formulation requiring such co-local events-separated by a time interval—for the interpretation of the LT. In order to overcome this obstacle in the SR formulation, the equations:

$$x = ct, \qquad (16)$$

and

$$x' = ct', \qquad (17)$$

expressing the basic speed of light constancy principle, were manipulated and combined into the equation:

$$x^2 - c^2t^2 = x'^2 - c^2t'^2, \qquad (18)$$

set as the principle equation representing the SR speed of light postulate [20]. Setting $x = 0$ with $t > 0$ (or $x' = 0$ with $t' > 0$), or $t = 0$ with $x \neq 0$ (or $t' = 0$ with $x' \neq 0$), is made now possible with the constructed Eq. (18), while the conditions $x = 0$; $t = 0$ ($x' = 0$; $t' = 0$) imposed by the original light speed constancy Eqs. (16) and (17), are ignored!

It should be noted that Eq. (18) can also be obtained from the light sphere equations, namely:

$$x^2 + y^2 + z^2 = c^2t^2, \qquad (19)$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2, \qquad (20)$$

representing the light speed constancy principle in the three-dimensional space, by subtracting the two equations from each other, and using the invariance of the y and z coordinates (i.e., $y = y'$, $z = z'$). However, Eqs. (19) and (20) also require that at the instant of time $t = t' = 0$ the moment when the spherical light wave front is emitted from the coinciding frame origins the spatial coordinates must be zero as well, i.e. $x = x' = 0$, $y = y' = 0$ and $z = z' = 0$; these initial conditions are not attributed to the resulting Eq. (18) in the SR formulation.

Eq. (18) forms the basis of the LT derivation in the SR formulation [20]. The LT equations are indeed derivable, yet more tediously, from Eq.(18) being mathematically equivalent to the deriving Eqs. (16) and (17) except with no consideration given to the coordinate values obtained from these equations at the space and time origins (i.e., ignoring the initial conditions required by equations (16) and (17)). Such a critical violation undermines the validity of the SR predictions, in agreement with the findings of earlier studies. [18,19] In fact, these studies demonstrate that the LT equations result in mathematical contradictions when applied for co-local or simultaneous events.

4. Mathematical Contradictions

Indeed, substituting LT Eq. (7) into LT Eq. (5), returns:

$$t = \gamma\left(\gamma\left(t - \frac{vx}{c^2}\right) + \frac{vx'}{c^2}\right),$$

which can be simplified to:

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma x'}{x}\right). \qquad (21)$$

Since, as shown earlier, the conditions of $x = ct$ and $x' = ct'$ are embedded in Eqs. (7) and (5), then Eq. (21) can be written as:

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma t'}{t}\right). \qquad (22)$$

If Eqs. (7), (5) and (22) were generalized (i.e., applied to conversions other than $x = ct$; $x' = ct'$) and particularly applied to an event with the time $t' = 0$, then according to Eq. (7), the transformed t- coordinate with respect to K would be $t = vx / c^2$. Consequently, for $t' = 0$ (and $t \neq 0$), Eq. (22) would reduce to:

$$t\left(\gamma^2 - 1\right) = t\gamma^2, \qquad (23)$$

yielding the contradiction:

$$\gamma^2 - 1 = \gamma^2, \text{ or } 0 = 1.$$

It follows that the physical conversion of the time coordinate $t' = 0$ to $t = vx / c^2$, for $x \neq 0$, by LT Eq. (7), is proved to be invalid, since it leads to a contradiction when used in Eq. (22), resulting from the LT equations for $t \neq 0$ (i.e., beyond the initial overlaid-frames instant satisfying $t = 0$ for $t' = 0$) -Letting $t = 0$ would satisfy Eq. (23), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $t' = 0$ would be $t = vx / c^2 = 0$, yielding $v = 0$, as we're addressing the conversion of $t' = 0$ to $t = vx / c^2$ for $x \neq 0$.

A similar contradiction is obtained by substituting Eq. (5) into Eq. (7), and applying Eq. (5) for the conversion $t = 0$; $t' = -vx' / c^2$ of the time coordinate $t = 0$.

Furthermore, substituting Eq. (8) into Eq. (6), yields:

$$x = \gamma\left(\gamma\left(x - vt\right) + vt'\right);$$
$$x\left(\gamma^2 - 1\right) = \gamma v\left(\gamma t - t'\right); \qquad (24)$$
$$x\left(\gamma^2 - 1\right) = \gamma vt\left(\gamma - \frac{t'}{t}\right).$$

Since Eqs. (8) and (6), along with Eqs. (7) and (5), are embedding the conditions of $x = ct$ and $x' = ct'$, Eq. (24) can be written as:

$$x\left(\gamma^2 - 1\right) = \gamma vt\left(\gamma - \frac{x'}{x}\right). \qquad (25)$$

If Eqs. (8), (6) and (25) were generalized (i.e., applied to conversions other than $x = ct$; $x' = ct'$), and particularly applied to an event with the coordinate $x' = 0$ then according to Eq. (8), the transformed x- coordinate with respect to K would be $x = vt$. Consequently, for $x' = 0$ (and $x \neq 0$), Eq. (25) would reduce to:

$$x\left(\gamma^2 - 1\right) = x\gamma^2, \qquad (26)$$
$$\gamma^2 - 1 = \gamma^2, \text{ or } 0 = 1.$$

Consequently, the physical conversion of the space coordinate $x' = 0$ of K' origin to $x = vt$, at time $t > 0$, with respect to K by LT Eq. (8), is invalid under the SR formulation, since it leads to a contradiction when used in Eq. (25), resulting from LT equations, for $x \neq 0$ (i.e., beyond the initial overlaid-frames position satisfying $x = 0$ for $x' = 0$) Letting $x = 0$ would satisfy Eq. (26), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $x' = 0$ would be $x = vt = 0$, yielding $v = 0$, as we're addressing the conversion of $x' = 0$ to $x = vt$ for $t > 0$.

A similar contradiction would follow upon substituting Eq. (6) into Eq. (8), and applying Eq. (6) for the conversion $x = 0$; $x' = -vt'$ of the space coordinate $x = 0$.

5. Conclusion

The LT equations are shown to be merely applicable for events satisfying the basic light speed constancy equations $x = ct$ and $x' = ct'$. The erroneous application of the LT on co-local events ($x' = 0$; $t' > 0$, in K', or $x = 0$; $t > 0$, in K), or simultaneous events ($t' = 0$; $x' \neq 0$, in K', or $t = 0$; $x \neq 0$, in K), is shown to result in mathematical contradictions and invalid predictions of time dilation, or length contraction, respectively.

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