

Double Convection of a Binary Viscoelastic Fluid under Helical Force Effect: Linear and Weakly Nonlinear Analysis

Kpossa Gbedode Mathieu, Monwanou Vincent Adjimon *

Institut de Mathématiques et de Sciences Physiques (IMSP), Laboratoire de la Mécanique des Fluides, de la Dynamique Non linéaire et de la Modélisation des Systèmes Biologiques (LMFDNMSB), Porto-Novo, Bénin

*Corresponding author: vincent.monwanou@imsp-uac.org

Received August 11, 2022; Revised September 15, 2022; Accepted September 27, 2022

Abstract We used linear stability theory based on the normal mode decomposition technique to study the criterion of appearance of the stationary convection and the oscillatory convection in a binary viscoelastic fluid mixture in a porous medium under the effect of helical force. Nonlinear stability theory based on the minimum representation of double Fourier series is used to study the rate of heat and mass transfer. We have determined the analytical expression of the Rayleigh number of the system as a function of the dimensionless parameters. Expressions for heat and mass transfer rates are determined as a function of Nusselt and Sherwood number, respectively. The transient behaviors of the Nusselt number and the Sherwood number are studied by solving the finite amplitude equations using the Runge - Kutta method. Then, the effect of each dimensionless parameter on the system is studied pointed out interesting results.

Keywords: helical force, stationary convection, oscillatory convection, viscoelastic fluid, porous medium, stability theory

Cite This Article: KPOSSA Gbèdodé Mathieu, and MONWANOU Vincent Adjimon, “Double Convection of a Binary Viscoelastic Fluid under Helical Force Effect: Linear and Weakly Nonlinear Analysis.” *International Journal of Physics*, vol. 10, no. 4 (2022): 193-203. doi: 10.12691/ijp-10-4-2.

1. Introduction

The study of diffusive double convection in a porous medium is of great practical importance in many branches of science and engineering such as centrifugal filtration process, petroleum industry, food engineering, chemical engineering, geophysics and biomechanics, etc. Due to the increasing research in porous media, double-diffusive convection in porous media attracts many researchers in recent years due to its applications in atmospheric science, seawater flow, earth mantle convection, solidification of binary alloys. An excellent review of most of the related findings on diffusive double convection in a porous medium has been given by Nield and Bejan [1], Ingham [2] and Pop and Vafai [3]. The occurrence of thermal instability in a horizontal porous layer was first studied extensively by Horton and Rogers [4] and Lapwood [5]. However, Nield was the first to generalize the double convection problem studied by Horton, Rogers and Lapwood. Rees et al. in a series of studies investigated the thermal non-equilibrium effect (LTNE) on convective fluxes in a porous medium [6-9]. Several other authors have studied the problem of double diffusive convection in a porous medium [10-21]. Bahloul and his collaborators [22] carried out an analytical and numerical study of

double diffusive convection in a shallow horizontal porous layer under the influence of the Soret effect. Gaikwad et al. studied the problem of diffusive double convection in an anisotropic porous medium by taking into account cross-diffusion effects and using linear analyzes [23]. Malashety et al. studied how heat and mass transfer are affected in a fluid saturated porous layer using a local thermal non-equilibrium model [24]. In addition, Shivakumara et al. investigated the combined effects of boundary and thermal non-equilibrium on the onset of Darcy-Brinkman convection in a porous layer [25].

It has been shown that the instability can be stationary and oscillatory depending on the viscoelastic properties of the fluid [26]. Oscillatory convection in viscoelastic fluid study was also made by several researchers [27-38].

The classical problem of Rayleigh-Bénard convection of viscous fluids in a rotating layer is studied by Chandrasekhar and he showed that rotation stabilizes the system [39]. However, in the planetary atmosphere, the turbulent convection motion is destined to become helical. The properties of small-scale helical turbulence were first discovered in magneto-hydrodynamics by Steenbeck et al. [40].

This discovery fostered the development of the magneto-hydrodynamic (MHD) theory. The publications on the hydrodynamic alpha effect have been made by Levina et al. [41,42,43] following by many others works [44-48].

Although there are several works in the literature on diffusive double convection in a rotating porous medium saturated with ordinary fluid and viscoelastic fluid, very little attention has been devoted to the study of the effect of the helical force on the double convection of binary viscoelastic fluid. The aim of the present work is to study the effect of the helical force on the stationary and oscillatory convection of a binary viscoelastic fluid in a porous medium. To do this, we used the linear stability theory to determine the expression of the thermal Rayleigh-Darcy number as a function of the system parameters. Then we studied the effect of each parameter on the occurrence of stationary and oscillatory convection in a binary viscoelastic fluid under the effect of the helical force. We also used the nonlinear stability theory to study the heat and mass transfer criteria by calculating the Nusselt number and the Sherwood number. We provided main results and discussions of the work followed by the conclusions.

2. Mathematical Formulation of the Problem

Consider a porous horizontal layer saturated by a binary viscoelastic fluid, incompressible confined between two distant horizontal plates ($z = 0$ and $z = d$) of d and heated from below. The layer is subjected to a vertical temperature and concentration gradient. A temperature and concentration difference across the layer is imposed: $T(z = 0) = T_0 + \Delta T$, $C(z = 0) = C_0 + \Delta C$, $T(z = d) = T_0$ and $C(z = d) = C_0$. The configuration of the problem is shown in Figure 1.

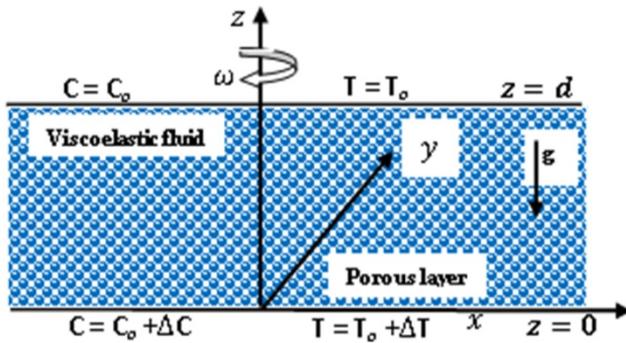


Figure 1. Vertical section through a porous layer saturated by a viscoelastic fluid

Under the Boussinesq approximation [38], the equations that govern the dynamics of a viscoelastic fluid in a porous medium in dimensionless form are:

$$\left(\lambda_1 \frac{\partial}{\partial t} + 1 \right) \left(\frac{1}{Va} \frac{\partial \mathbf{q}}{\partial t} + \nabla p - Ra_{TD} T \mathbf{k} \right) + \left(\lambda_2 \frac{\partial}{\partial t} + 1 \right) \mathbf{q} = 0 \quad (1)$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 \right) T + (\mathbf{q} \cdot \nabla) T - w = 0 \quad (2)$$

$$\left(\frac{\partial}{\partial t} - \tau \nabla^2 \right) C + (\mathbf{q} \cdot \nabla) C - w = 0. \quad (3)$$

In the above equations, $Ra_{TD} = Ra_T Da = \frac{\alpha_T \rho_0 g \Delta T K d}{\varepsilon \mu k_T}$ represents the thermal Rayleigh-Darcy number, $Ra_{CD} = Ra_C Da = \frac{\alpha_C \rho_0 g \Delta C K d}{\varepsilon \mu k_T}$ represents the mass Rayleigh-Darcy number, $S_D = S Da = \frac{\rho_0 a \omega K}{\varepsilon \mu}$ represents the product of the helical force intensity and the Darcy number, $S = \frac{\rho_0 a \omega d^2}{\varepsilon \mu}$ represents the intensity of the helical force, $Da = \frac{K}{d^2}$ represents the Darcy number, $Va = \frac{\varepsilon P_r}{Da}$ represents Vadasz number, $P_r = \frac{\mu}{\rho_0 k_T}$ represents the Prandtl number, $\tau = \frac{k_C}{k_T}$ the diffusivity ratio, $\mathbf{q} = (u, v, w)$ filtration rate field, $\mathbf{f} = \mathbf{e}(\text{curl} \mathbf{q})_z - \frac{\partial(\mathbf{e} \wedge \mathbf{q})}{\partial z}$ with $\mathbf{e} = (0, 0, 1)$ and ∇ the nabla operator.

3. Linear Stability Theory

In this section, we predict the thresholds of steady and oscillatory convection using linear stability theory. To study the linear stability of the system, we need the linear parts of the equations (1)-(3). The pressure and the two components of the velocity field can be eliminated easily by applying $\nabla \wedge$ and $\nabla \wedge \nabla \wedge$ to Navier-Stokes equations and by considering the vertical components of the resulting equations. After some calculations, we obtain the following linear equations:

$$\left(\lambda_1 \frac{\partial}{\partial t} + 1 \right) \left(\frac{1}{Va} \frac{\partial}{\partial t} \nabla^2 w - Ra_{TD} \nabla_{\perp}^2 T + Ra_{CD} \nabla_{\perp}^2 C - S_D \left(\frac{\partial^2}{\partial z^2} - \nabla_{\perp}^2 \right) \xi \right) \quad (4)$$

$$+ \left(\lambda_2 \frac{\partial}{\partial t} + 1 \right) \nabla^2 w = 0$$

$$\left(\lambda_1 \frac{\partial}{\partial t} + 1 \right) \left(\frac{1}{Va} \frac{\partial \xi}{\partial t} - S_D \frac{\partial^2 w}{\partial z^2} \right) + \left(\lambda_2 \frac{\partial}{\partial t} + 1 \right) \xi = 0 \quad (5)$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 \right) T - w = 0 \quad (6)$$

$$\left(\frac{\partial}{\partial t} - \tau \nabla^2 \right) C - w = 0. \quad (7)$$

The eigenvalue problem defined by the equations (4) - (7) is solved using periodic time-dependent perturbations in the horizontal plane, assuming that the amplitude is small enough and can be expressed by normal mode decomposition [29].

$$\begin{pmatrix} w \\ T \\ C \\ \xi \end{pmatrix} = \begin{pmatrix} W(z) \\ T(z) \\ C(z) \\ Z(z) \end{pmatrix} e^{i(k_x x + k_y y) + st} \tag{8}$$

where k_x and k_y represent respectively the wave numbers in the directions x and y . $s = \sigma + i\omega$ is a complex number which represents the growth rate of disturbances; with σ the disturbance growth factor and ω the disturbance frequency.

By replacing the equations (8) in the equations (4) - (7) then setting $D = \frac{\partial}{\partial z}$, we obtain the following ordinary differential equations:

$$\begin{aligned} &(\lambda_1 s + 1) \left(\frac{1}{Va} (D^2 - k^2) Ws + k^2 Ra_{TD} T - k^2 Ra_{CD} C \right) \\ &+ S_D (\lambda_1 s + 1) (D^2 + k^2) Z \end{aligned} \tag{9}$$

$$\begin{aligned} &+ (\lambda_2 s + 1) (D^2 - k^2) W = 0 \\ &- S_D (\lambda_1 s + 1) D^2 W \\ &+ \left(\frac{1}{Va} (\lambda_1 s + 1) s + (\lambda_2 s + 1) \right) Z = 0 \end{aligned} \tag{10}$$

$$(s - (D^2 - k^2)) T - w = 0 \tag{11}$$

$$(s - \tau (D^2 - k^2)) C - w = 0. \tag{12}$$

By eliminating Z from the equations (9) and (11), we obtain the equation below:

$$\begin{aligned} &\left(\frac{s}{Va} + \frac{\lambda_2 s + 1}{\lambda_1 s + 1} \right) \left(\frac{1}{Va} (D^2 - k^2) Ws \right. \\ &\left. + k^2 Ra_{TD} T - k^2 Ra_{CD} C \right) \\ &+ S_D^2 D^2 (D^2 + k^2) W \\ &+ \left(\frac{\lambda_2 s + 1}{\lambda_1 s + 1} \right) \left(\frac{s}{Va} + \frac{\lambda_2 s + 1}{\lambda_1 s + 1} \right) (D^2 - k^2) W = 0. \end{aligned} \tag{13}$$

In order to analytically solve the equations (11)-(13), the boundary conditions below are used at $z = 0$ and $z = 1$.

$$T = D^2 W = W = C = 0. \tag{14}$$

The exact solutions that satisfy the equations (11)-(13) using the boundary conditions Eq. (14) are

$$W = A \sin(\pi z), T = B \sin(\pi z), \text{ and } C = E \sin(\pi z) \tag{15}$$

where A , B and E are constants. By replacing these solutions in the equations (11)-(13), we obtain the following equations:

$$\begin{aligned} &- \left((\pi^2 + k^2) A_{11}^2 + \pi^2 S_D^2 (-\pi^2 + k^2) \right) A \\ &+ k^2 A_{11} Ra_{TD} B - k^2 A_{11} Ra_{CD} E = 0. \end{aligned} \tag{16}$$

$$(s + (\pi^2 + k^2)) B - A = 0 \tag{17}$$

$$(s + \tau (\pi^2 + k^2)) E - A = 0 \tag{18}$$

with $A_{11} = \left(\frac{s}{Va} + \frac{\lambda_2 s + 1}{\lambda_1 s + 1} \right)$.

The system formed by equations (16)-(18) only admits a non-trivial solution if the determinant of the matrix whose elements are the coefficients of the constants A , B and E is zero. Thus, setting $\delta^2 = \pi^2 + k^2$, the eigenvalue problem leads to the following characteristic polynomial :

$$\begin{aligned} p(s) = &a_0 + a_1 s + a_2 s^2 + a_3 s^3 \\ &+ a_4 s^4 + a_5 s^5 + a_6 s^6 = 0 \end{aligned} \tag{19}$$

where the coefficients a_0 , a_1 , a_2 , a_3 , a_4 , a_5 and a_6 are known constants.

We thus obtain the expression of the thermal Rayleigh- Darcy number of the system which is presented as follows:

$$\begin{aligned} Ra_{TD} = &\frac{(s + \delta^2) (\delta^2 A_{11}^2 + \pi^2 S_D^2 (-\pi^2 + k^2))}{k^2 A_{11}} \\ &+ \frac{(s + \delta^2)}{(s + \tau \delta^2)} Ra_{CD}. \end{aligned} \tag{20}$$

3.1. Stationary Stability Analysis

For the existence of neutral stability, the real part of the growth rate of disturbances s is zero. Moreover, for the appearance of stationary convection, the growth rate of disturbances is zero. That is, $s = \sigma = i\omega = 0$. By replacing $s = \sigma = i\omega = 0$ in equation (20), we obtain the expression of the thermal Rayleigh-Darcy number in the case of stationary convection which is written:

$$Ra_{TD}^{st} = \frac{\delta^2 (\delta^2 + \pi^2 S_D^2 (-\pi^2 + k^2))}{k^2} + \frac{1}{\tau} Ra_{CD}. \tag{21}$$

It should therefore be noted that the expression of the stationary thermal Rayleigh-Darcy number Ra_{TD}^{st} is independent of the parameters of the viscoelastic fluid. This is explained by the fact that the fluid in its basic state is not flowing and viscoelasticity cannot therefore affect the onset of stationary convection when the fluid is at rest or when its flow is weak [20].

From this expression of the stationary Rayleigh- Darcy number found, we find some results known in the literature. Thus, if $S_D = 0$ and $Ra_{CD} = 0$, we find the expression of the stationary thermal Rayleigh-Darcy number in the case [1]. If $S_D = 0$, we find the expression of the stationary thermal Rayleigh-Darcy number in the case [37].

We then determine the stationary critical thermal Rayleigh-Darcy number Ra_{TDc}^{st} and the critical wave number k_c . The equation verifying the critical wave

number k_c is obtained by minimizing the expression of the stationary thermal Rayleigh-Darcy number Ra_{TDc}^{st} with respect to the wave number; i.e. $\left(\frac{\partial Ra_{TDc}^{st}}{\partial k} = 0\right)$. By doing so, we find the critical thermal Rayleigh-Darcy number Ra_{TDc}^{st} given by the expression:

$$Ra_{TDc}^{st} = \pi^2 \left[\left(1 - \pi^4 S_D^4\right)^{\frac{1}{2}} + \left(1 - \pi^2 S_D^2\right) \right] \times \left[1 + \left(\frac{1 - \pi^2 S_D^2}{1 + \pi^2 S_D^2}\right)^{-\frac{1}{2}} \right] + \frac{1}{\tau} Ra_{CD}. \tag{22}$$

3.2. Oscillatory Stability Analysis

In the case of the oscillatory neutral stability study, the disturbance growth rate is non-zero. That is $s = i\omega$. By replacing $s = i\omega$ in the equation (20), we obtain the expression of the thermal Rayleigh-Darcy number on the form:

$$Ra_{TD} = \Delta_1 + i\omega\Delta_2. \tag{23}$$

The Rayleigh number being a real physical quantity, so the imaginary part of the expression of the thermal Rayleigh-Darcy number given by $\Delta_2 = 0$ which leads to the equation below:

$$b_1(\omega^2)^5 + b_2(\omega^2)^4 + b_3(\omega^2)^3 + b_4(\omega^2)^2 + b_5(\omega^2) + b_6 = 0. \tag{24}$$

The thermal oscillatory Rayleigh-Darcy number of the system is then given by the expression:

$$Ra_{TD}^{osc} = \frac{\pi^2 S_D^2 \left(-\pi^2 + k^2\right) \left[\omega^2 \left(\frac{1}{Va} + \frac{\lambda_2 - \lambda_1}{1 + \lambda_1^2 \omega^2}\right) + \delta^2 \left(\frac{1 + \lambda_1 \lambda_2 \omega^2}{1 + \lambda_1^2 \omega^2}\right) \right]}{k^2 \left[\omega^2 \left(\frac{1}{Va} + \frac{\lambda_2 - \lambda_1}{1 + \lambda_1^2 \omega^2}\right)^2 + \left(\frac{1 + \lambda_1 \lambda_2 \omega^2}{1 + \lambda_1^2 \omega^2}\right)^2 \right]} + \frac{\delta^2 \left[\delta^2 \left(\frac{1 + \lambda_1 \lambda_2 \omega^2}{1 + \lambda_1^2 \omega^2}\right) - \omega^2 \left(\frac{1}{Va} + \frac{\lambda_2 - \lambda_1}{1 + \lambda_1^2 \omega^2}\right) \right]}{k^2} + \left(\frac{\omega^2 + \tau\delta^4}{\omega^2 + \tau^2\delta^4}\right) Ra_{CD}. \tag{25}$$

If $S_D = 0$ and $Ra_{CD} = 0$, we find the expression of the oscillatory thermal Rayleigh-Darcy number in the case [1]. If $S_D = 0$, we find the expression of the oscillatory thermal Rayleigh-Darcy number in the case [37].

4. Weakly Nonlinear Analysis

The linear stability theory of the system made above aims to determine the critical values of the Rayleigh-Darcy number above which the system is unstable and below which the system is stable. Although this theory of linear stability is important, it does not allow the calculation of certain important physical quantities such as the amplitude of convective motions, the rate of heat transfer and the rate of mass transfer. So we need to do a non-linear stability analysis of the system [12]. In most applications of traditional porous media, the Vadasz number is large enough [15]. For this reason, we will subsequently neglect it in the calculations that follow. To further simplify the calculations, we have considered the case of two-dimensional rolls. Thus, all physical quantities are considered to be independent of y and the current function ψ such as $u = \frac{\partial\psi}{\partial z}$ and $w = -\frac{\partial\psi}{\partial z}$ will therefore be used. A local nonlinear stability analysis will be performed using a strongly truncated representation of the Fourier series for the current function, temperature and concentration. The minimal system that describes this convection of finite amplitude is written as follows [13]:

$$\psi = A_1(t) \sin(kx) \sin(\pi z) \tag{26}$$

$$T = B_1(t) \cos(kx) \sin(\pi z) + S_1(t) \sin(2\pi z) \tag{27}$$

$$C = D_1(t) \cos(kx) \sin(\pi z) + E_1(t) \sin(2\pi z) \tag{28}$$

where the amplitudes $A_1(t)$, $B_1(t)$, $S_1(t)$, $D_1(t)$ and $E_1(t)$ are time dependent functions and are to be determined from the dynamics of the system.

By applying $\nabla\Lambda$ to the equation (1) to eliminate the pressure and by introducing the current function ψ in the resulting equation and in the equations (1)-(3), we obtain at least:

$$\frac{dA_1}{dt} = H_1 \tag{29}$$

$$\frac{dB_1}{dt} = -k\pi A_1 S_1 - \delta^2 B_1 - kA_1 \tag{30}$$

$$\frac{dS_1}{dt} = -4\pi^2 S_1 + \frac{k\pi}{2} A_1 B_1 \tag{31}$$

$$\frac{dD_1}{dt} = -k\pi A_1 E_1 - \tau\delta^2 D_1 - kA_1 \tag{32}$$

$$\frac{dE_1}{dt} = -4\tau\pi^2 E_1 + \frac{k\pi}{2} A_1 D_1 \tag{33}$$

$$\frac{dH_1}{dt} = \frac{1}{L} \left[\begin{array}{l} -X_1 H_1 - k\lambda_1 \lambda_2 Ra_{TD} \frac{d^2 B_1}{dt^2} \\ + k\lambda_1 \lambda_2 Ra_{CD} \frac{d^2 D_1}{dt^2} - kRa_{TD} (\lambda_1 + \lambda_2) \frac{dB_1}{dt} \\ + kRa_{CD} (\lambda_1 + \lambda_2) \frac{dD_1}{dt} - X_2 A_1 \\ - kRa_{TD} B_1 + kRa_{CD} D_1 \end{array} \right] \tag{34}$$

With $L = \lambda_2^2 \delta^2 + \lambda_1^2 \pi^2 S_D^2 (-\pi^2 + k^2)$, $X_1 = 2(\lambda_2 \delta^2 + \lambda_1 \pi^2 S_D^2 (-\pi^2 + k^2))$ and $X_2 = \delta^2 + \pi^2 S_D^2 (-\pi^2 + k^2)$.

The above system of nonlinear autonomous differential equations whose time variables cannot be solved analytically. It will be solved numerically. After determining the values of the amplitude functions $A_1(t)$, $B_1(t)$, $S_1(t)$, $D_1(t)$, $E_1(t)$ and $H_1(t)$ we get the expressions of Nusselt number and Sherwood number as a function of time. From the qualitative point of view, we perform a stationary analysis by fixing the left side of the equations (29)-(34) equal to zero by considering that all the amplitudes are constant. By eliminating all the amplitudes except A_1 , we obtain an equation as a function of A_1 and which takes the form:

$$A_1 \begin{bmatrix} \left(\delta^2 + \pi^2 S_D^2 (-\pi^2 + k^2) \right) \\ \times \left(\delta^2 + \frac{A_1^2}{8} k^2 \right) \left(\tau \delta^2 + \frac{A_1^2}{8} \tau^{-1} k^2 \right) \\ -k^2 \left(\tau \delta^2 + \frac{A_1^2}{8} \tau^{-1} k^2 \right) Ra_{TD} \\ +k^2 \left(\delta^2 + \frac{A_1^2}{8} k^2 \right) Ra_{CD} \end{bmatrix} = 0. \quad (35)$$

The solution $A_1 = 0$ only gives the pure conduction solution when Ra_{TD} is large enough. Therefore, the remaining solutions with $x = \frac{A_1^2}{8}$ are given by

$$F_1 x^2 + F_2 x + F_3 = 0. \quad (36)$$

With

$$F_1 = \tau^{-1} k^4 \left(\delta^2 + \pi^2 S_D^2 (-\pi^2 + k^2) \right)$$

$$F_2 = \tau^{-1} k^2 \left[\delta^2 \left(\delta^2 + \pi^2 S_D^2 (-\pi^2 + k^2) \right) (1 + \tau^2) \right. \\ \left. + k^2 (\tau Ra_{CD} - Ra_{TD}) \right]$$

$$F_3 = \delta^2 \left[\tau \delta^2 \left(\delta^2 + \pi^2 S_D^2 (-\pi^2 + k^2) \right) \right. \\ \left. + k^2 (Ra_{CD} - \tau Ra_{TD}) \right].$$

The equation (36) gives a positive root for some fixed values of system parameters:

$$\frac{A_1^2}{8} = \frac{1}{2F_1} \left[-F_2 + \sqrt{F_2^2 - 4F_1 F_3} \right]. \quad (37)$$

Considering now that the expression which is under the radical of the equation (37) is zero, we find the expression of the thermal Rayleigh-Darcy number Ra_{TD}^F of finite amplitude which describes the beginning of stationary motions of finite amplitude. The thermal Rayleigh-Darcy number Ra_{TD}^F of finite amplitude is given by

$$Ra_{TD}^F = \frac{1}{2G_1} \left[-G_2 + \sqrt{G_2^2 - 4G_1 G_3} \right] \quad (38)$$

$$G_1 = k^4$$

$$T_{Total} = T_l - \frac{\Delta T}{d} z + T(x, z, t)$$

$$\bar{H} = \frac{k_T \Delta T}{d} (1 - 2\pi C_1)$$

$$\bar{J} = \frac{k_C \Delta C}{d} (1 - 2\pi E_1)$$

$$Nu = \frac{d\bar{H}}{k_T \Delta T} = (1 - 2\pi C_1)$$

$$Sh = 1 + \frac{2k^2 A_1^2}{k^2 A_1^2 + 8\tau^2 (\pi^2 + k^2)}$$

$$G_3 = \left[\delta^2 \left(\delta^2 + \pi^2 S_D^2 (-\pi^2 + k^2) \right) (\tau^2 - 1) + \tau k^2 Ra_{CD} \right]^2.$$

4.1. Heat and Mass Transport

After determining the amplitudes of the equations (29)-(34), we can determine the heat and mass transport of a viscoelastic fluid under the effect of the helical force. This is very important for the study of convection in fluids. In the ground state, heat and mass transport are solely determined by conduction. If \bar{H} and \bar{J} are the heat and mass transfer rates per unit area, respectively, then

$$\bar{H} = -k_T \frac{\partial T_{Total}}{\partial z} \Big|_{z=0} \quad (39)$$

$$\bar{J} = -k_c \frac{\partial C_{Total}}{\partial z} \Big|_{z=0} \quad (40)$$

where the brackets represent the horizontal mean and T_{Total} and C_{Total} are given by

$$T_{Total} = T_l - \frac{\Delta T}{d} z + T(x, z, t) \quad (41)$$

$$C_{Total} = C_l - \frac{\Delta C}{d} z + C(x, z, t). \quad (42)$$

By replacing the equations (26)-(28) in the equations (41)-(42) then the resulting equations in the (39)-(40), we get the expressions for heat and mass transfer rate:

$$\bar{H} = \frac{k_T \Delta T}{d} (1 - 2\pi C_1) \quad (43)$$

$$\bar{J} = \frac{k_C \Delta C}{d} (1 - 2\pi E_1). \quad (44)$$

The Nusselt and Sherwood numbers are defined by

$$Nu = \frac{d\bar{H}}{k_T \Delta T} = (1 - 2\pi C_1) \quad (45)$$

$$Sh = \frac{d\bar{J}}{k_C \Delta C} = (1 - 2\pi E_1). \quad (46)$$

By using the equations (29)-(34) to express C_1 and E_1 as a function of A_1 , the equations (45)-(46) become

$$Nu = 1 + \frac{2k^2 A_1^2}{k^2 A_1^2 + 8(\pi^2 + k^2)} \quad (47)$$

$$Sh = 1 + \frac{2k^2 A_1^2}{k^2 A_1^2 + 8\tau^2(\pi^2 + k^2)} \quad (48)$$

5. Results and Discussions

Our analysis consists in studying theoretically the criterion of appearance of convection in a porous layer saturated by a binary mixture of viscoelastic fluid heated from below in the presence of the helical force.

Figure 2 show the marginal stability curves in the plane (k, Ra_{TD}) in the case of stationary convection.

The thermal Rayleigh-Darcy number of the system in the stationary case is independent of the parameters of the viscoelastic fluid. These results therefore boil down to the effect of the helical force on the stationary double convection of a binary Newtonian fluid in a porous

medium. Figure 2-a shows that increasing the intensity of the helical force causes the critical thermal Rayleigh- Darcy number to decrease. This shows that the helical force destabilizes the system by accelerating the onset of stationary convection. According to Figure 2-b, we noticed that the critical thermal Rayleigh-Darcy number increases with the growth of the mass Rayleigh-Darcy number. So the mass Rayleigh-Darcy number stabilizes the system by delaying the onset of stationary convection. Figure 2-c shows that the increase in the ratio of diffusivities causes the decrease in the critical thermal Rayleigh-Darcy number. Which shows that the ratio of diffusivities destabilizes the system by accelerating the onset of stationary convection.

Figure 3 presents the marginal stability curves in the plane (k, Ra_{TD}) in the case of oscillatory convection for distinct values of the intensity of the helical force, the relaxation time parameter, the retardation time parameter, the mass Rayleigh-Darcy number, the Vadasz number and the ratio of diffusivities for some fixed values of the system. parameters.

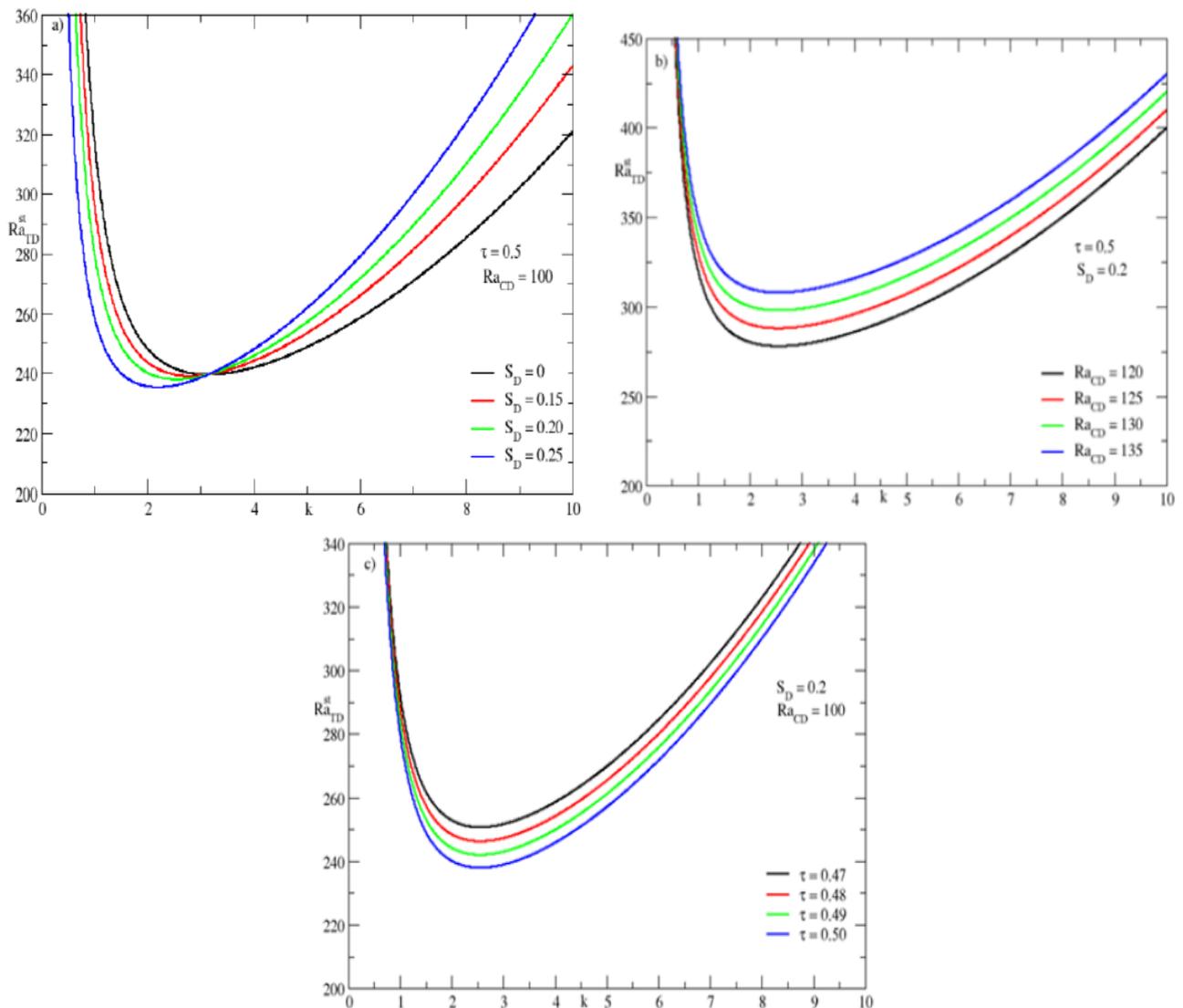


Figure 2. Marginal stability curves

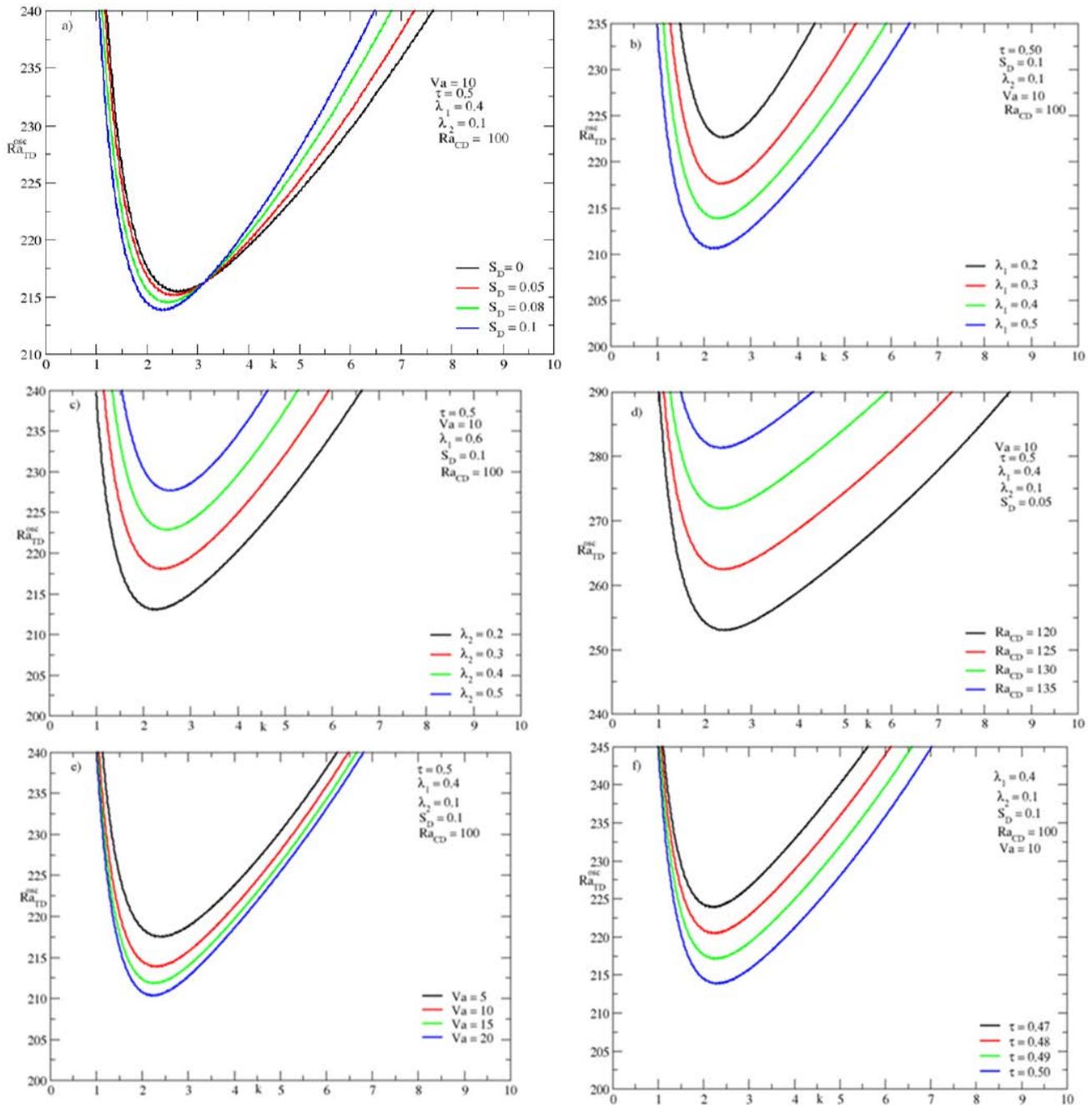


Figure 3. Marginal stability curves

Figure 3-a shows that when the intensity of the helical force increases, the critical thermal Rayleigh-Darcy number decreases. This means that the intensity of the helical force destabilizes the system by accelerating the onset of oscillatory convection. Figure 3-b shows that when the relaxation time parameter increases, the critical thermal Rayleigh-Darcy number decreases. So the relaxation time parameter destabilizes the system by accelerating the onset of oscillatory convection. According to the Figure 3-c the critical thermal Rayleigh-Darcy number increases with the growth of the delay time parameter. This means that the delay time parameter stabilizes the system by delaying the onset of oscillatory convection. Figure 3-d shows that the critical thermal Rayleigh-Darcy number increases with the growth of the mass Rayleigh-Darcy number. So the

mass Rayleigh-Darcy number stabilizes the system by delaying the onset of oscillatory convection. According to the Figure 3-e the critical thermal Rayleigh-Darcy number decreases with increasing Vadasz number, which shows that the Vadasz number destabilizes the system by accelerating the onset of oscillatory convection. According to the Figure 3-f, the critical thermal Rayleigh-Darcy number decreases when the ratio of diffusivities increases. This means that the diffusivity ratio destabilizes the system by accelerating the onset of oscillatory convection.

Figure 4 shows the variations of the Nusselt number and the Sherwood number as a function of the thermal Rayleigh-Darcy number for several values of the system parameters in the case of the nonlinear stability study for amplitude.

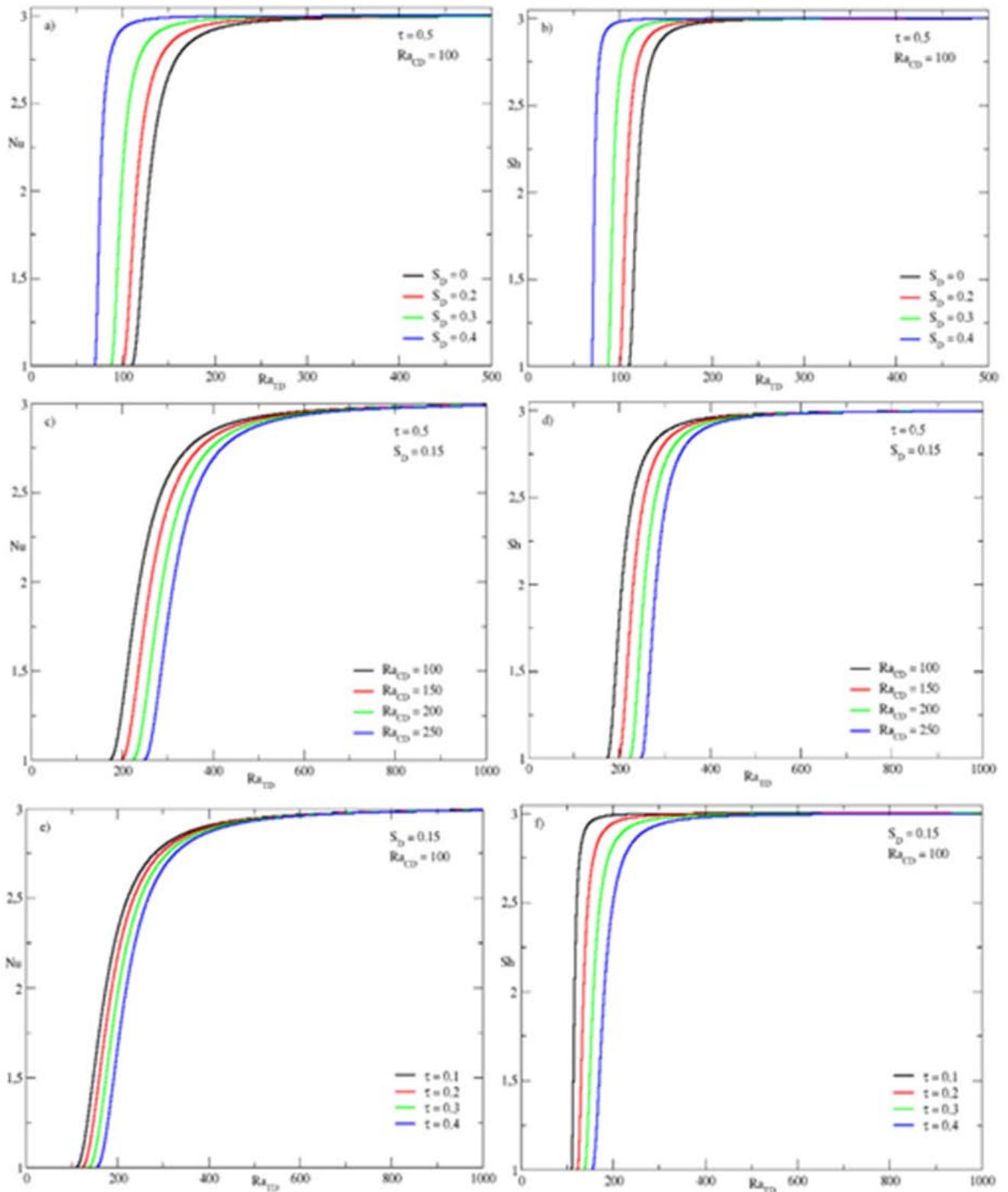


Figure 4. Curves of variation of the Nusselt number and the Sherwood number in the case of the stationary nonlinear stability study

From Figure 4-a and Figure 4-b, we noticed that the Nusselt and Sherwood numbers increase with increasing helical force intensity. The effect of this force is therefore to accelerate the rates of mass and heat transfer. Throughout Figure 4-c, 4-d, 4-e and 4-g, as the Rayleigh-Darcy number and the ratio of diffusivities increase, the Nusselt and Sherwood numbers decrease.

The effect of the Rayleigh-Darcy number and the ratio of diffusivities is therefore to reduce the rates of heat and mass transfer.

Figure 5 shows the evolution of the Nusselt number and the Sherwood number as a function of time when one of the parameters of the system is varied by fixing the other parameters.

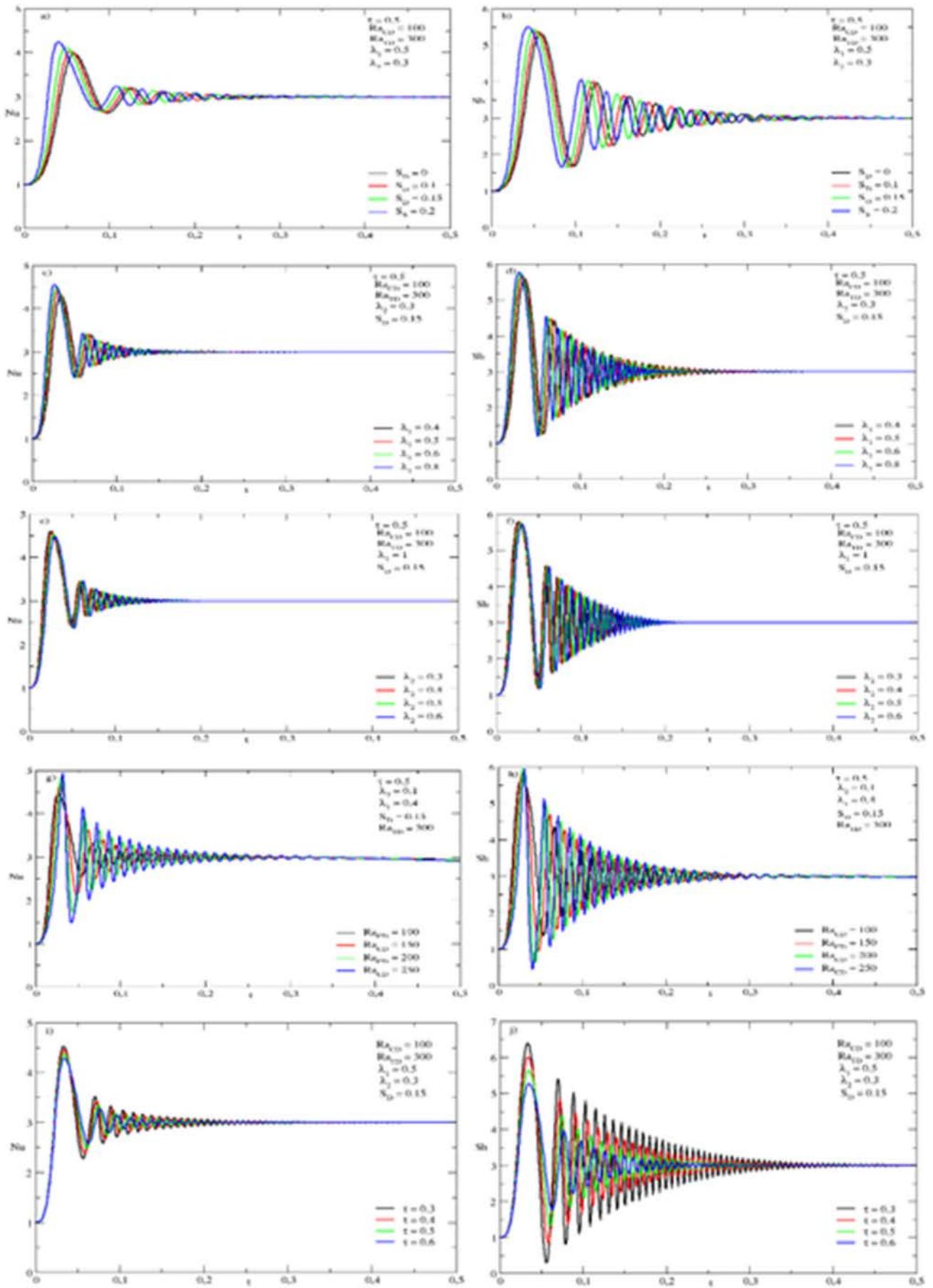


Figure 5. Curves of variation of the Nusselt number and the Sherwood number as a function of time in the case of the non-linear unsteady stability study

According to these different figures, we note that the heat and mass transfer rates increase for certain parameters and decrease for others when the value of the chosen parameter increases. From these different figures, we find that when the time is small, the values of Nusselt and Sherwood numbers oscillate and approach the value of the steady state when the time is large, thus showing the state stationary. Figure 5-a, 5-b, 5-c, 5-d, 5-g and 5-h show that transient Nusselt and Sherwood numbers increase with increasing helical force intensity, relaxation time parameter, and mass Rayleigh-Darcy number. The effect of these different parameters is to accelerate the rates of heat and mass transfer. From Figure 5-e, 5-f, 5-i and 5-j, increasing the delay time parameter and the diffusivity ratio causes the transient Nusselt and Sherwood numbers to decrease. The effect of the delay time parameter and the ratio of diffusivities is to reduce the rates of heat and mass transfer.

6. Conclusion

We used the linear stability theory and the nonlinear stability theory to study the criterion of appearance of the double diffusive convection and the heat and mass transfer rates in a horizontal porous layer saturated by a binary mixture of fluid viscoelastic in the presence of the helical force. According to the various results obtained, it should be noted that in a porous medium saturated by a binary mixture of viscoelastic fluid, in the stationary case, the viscoelastic fluid behaves like a Newtonian fluid. The mass Rayleigh-Darcy number delays the onset of steady convection by stabilizing the system. On the other hand, the intensity of the helical force and the ratio of diffusivities accelerate the onset of stationary convection by destabilizing the system. In the oscillatory case, the mass Rayleigh-Darcy number and the delay time parameter delay the onset of oscillatory convection by stabilizing the system when the helical force intensity, Vadasz number, diffusivities ratio and the relaxation time parameter accelerate the onset of oscillatory convection by destabilizing the system. In the case of motions of finite amplitude, the stationary Nusselt and Sherwood numbers increase when the intensity of the helical force increases. The rates of heat and mass transfer are therefore accelerated by increasing the intensity of the helical force. On the other hand, increasing the ratio of diffusivities and the mass Rayleigh-Darcy number reduces the heat and mass transfer rates. Transient Nusselt and Sherwood numbers increase with increasing helical force intensity, relaxation time parameter, and mass Rayleigh-Darcy number. At the onset of convection, the transient Nusselt and Sherwood numbers oscillate when the weather is weak. But as time increases, they gradually decrease and approach the steady state value of the system.

Conflict of Interest

The authors declare that they have no conflict of interest.

References

- [1] D. A. Nield and A. Bejan, *Convection in Porous Media*, 3rd ed. (Springer-Verlag, New York, 2006).
- [2] D. B. Ingham and I. Pop, *Transport Phenomena in Porous Media* (Elsevier, Oxford, 2005), Vol. III.
- [3] K. Vafai, *Handbook of Porous Media* (Taylor and Francis, London, 2006).
- [4] C. W. Horton and F. T. Rogers, *Convection currents in a porous medium*, J. Appl. Phys. 16, 367 (1945).
- [5] E. R. Lapwood, *Convection of a fluid in a porous medium*, Proc. Cambridge Philos. Soc. 44, 508 (1948).
- [6] D. A. S. Rees and I. Pop, *Local thermal non-equilibrium in porous medium convection*. In Transport Phenomena in Porous Media, edited by D.B. Ingham and I. Pop (Elsevier, Oxford, 2005), Vol. III, p. 147.
- [7] D. A. S. Rees and I. Pop, *Vertical free convective boundary layer flow in a porous medium using a thermal non-equilibrium model*, J. Porous Media 3, 31 (2000).
- [8] D. A. S. Rees, *Vertical free convective boundary layer flow in a porous medium using a thermal non-equilibrium model: Elliptic effects*, J. Appl. Math. Phys. 54, 437 (2003).
- [9] N. Banu and D. A. S. Rees, *Onset of Darcy Benard convection using a thermal non-equilibrium model*, Int. J. Heat Mass Transfer 45, 2221 (2002).
- [10] O. V. Trevisan and A. Bejan, *Combined heat and mass transfer by natural convection in a porous medium*, Adv. Heat Transf. 20, 315 (1999).
- [11] A. Mojtabi and M. C. Charrier-Mojtabi, *Double diffusive convection in porous media*, in *Hand Book of Porous Media*, edited by K. Vafai (Marcel Dekker, New York, 2000), p. 559.
- [12] M. Mamou, *Stability analysis of double diffusive convection in porous enclosures*, in *Transport Phenomena in Porous Media II*, edited by D. B. Ingham and I. Pop (Elsevier, Oxford, 2002), p. 113.
- [13] J. W. Taunton, E. N. Lightfoot, and T. Green, *Thermohaline instability and salt fingers in a porous medium*, Phys. Fluids 15, 748 (1972).
- [14] P. R. Patil and N. Rudraiah, *Linear convective stability and thermal diffusion of a horizontal quiescent layer of a two component fluid in a porous medium*, Int. J. Eng. Sci. 18, 1055 (1980).
- [15] P. Vadasz, *Coriolis effect on gravity-driven convection in a rotating porous layer heated from below*, J. Fluid Mech. 376 (1998) 351375.
- [16] M. Mamou, P. Vasseur, and M. Hasnaoui, *On numerical stability analysis of double diffusive convection in confined enclosures*, J. Fluid Mech. 433, 209 (2001).
- [17] M. S. Malashetty, *Anisotropic thermoconvective effects on the onset of double diffusive convection in a porous medium*, Int. J. Heat Mass Transfer 36(9), 2397 (1993).
- [18] N. Rudraiah, P. K. Shrimani, and R. Friedrich, *Finite amplitude convection in a two component fluid saturated porous layer*, Int. J. Heat Mass Transfer 25, 715 (1982).
- [19] N. Rudraiah and P. G. Siddheshwar, *A weak nonlinear stability analysis of double diffusive convection with cross-diffusion in a fluid saturated porous medium*, Heat Mass Transfer 33(4), 287 (1998).
- [20] A. N. Borujerdi, A. R. Noghrehabadi, and D. A. S. Rees, *Onset of convection in a horizontal porous channel with uniform heat generation using a thermal non-equilibrium model*, Transp. Porous Media 69, 343 (2007).
- [21] B. T. Murray and C. F. Chen, *Double diffusive convection in a porous medium*, J. Fluid Mech. 201, 147 (1989).
- [22] A. Bahloul, N. Boutana, and P. Vasseur, *Double diffusive and Soret induced convection in a shallow horizontal porous layer*, J. Fluid Mech. 491, 325 (2003).
- [23] S. N. Gaikwad, M. S. Malashetty, and K. Rama Prasad, *Linear and non-linear double diffusive convection in a fluid saturated anisotropic porous layer with cross-diffusion effects*, Transp. Porous Media 80(3), 537 (2009).
- [24] M. S. Malashetty, M. Swamy, and R. Heera, *Double diffusive convection in a porous layer using a thermal non-equilibrium model*, Int. J. Therm.Sci. 47, 1131 (2008).

- [25] I. S. Shivakumara, A. L. Mamatha, and M. Ravisha, *Boundary and thermal non-equilibrium effects on the onset of Darcy Brinkman convection in a porous layer*, J. Eng. Math. 67, 317 (2010).
- [26] Anoj Kumar and B. S. Bhadauria, *Double diffusive convection in a porous layer saturated with viscoelastic fluid using a thermal non-equilibrium model*, J. Phys. Fluids 23, 054101 (2011).
- [27] D. M. Herbert, *On the stability of viscoelastic liquids in heated plane Couette flow*, J. Fluid Mech. 17, 353 (1963).
- [28] T. Green, III, *Oscillating convection in an elasticoviscous liquid*, Phys. Fluids 11, 1410 (1968).
- [29] N. Rudraiah, P. N. Kaloni, and P. V. Radhadevi, *Oscillatory convection in a viscoelastic fluid through a porous layer heated from below*, Rheol. Acta 28, 48 (1989).
- [30] N. Rudraiah, P. V. Radhadevi, and P. N. Kaloni, *Convection in a viscoelastic fluid saturated sparsely packed porous layer*, Can. J. Phys. 68, 1446 (1990).
- [31] Laroze, D., Maritnez-Mardones, J., Bragard, J., *Thermal convection in a rotating binary viscoelastic liquid mixture*, Eur. Phys. J. Special Top. 146, 291300 (2007).
- [32] Kim, M.C., Lee, S.B., Kim, S., Chung, B.J., *Thermal instability of viscoelastic fluids in porous media*, Int. J. Heat Mass Transf. 46, 50655072 (2003).
- [33] Yoon, D.Y., Kim, M.C., Choi, C.K. *The onset of oscillatory convection in a horizontal porous layer saturated with viscoelastic liquid*, Transp. Porous Media 55, 275284 (2004).
- [34] Sheu, L.J., Tam, L.M., Chen, J.H., Chen, H.K., Lin, K.T., Kang, Y. *Chaotic convection of viscoelastic fluids in porous media*, Chaos Solitons Fractals 37, 113124 (2008).
- [35] Wang, S., Tan, W. *Stability analysis of double-diffusive convection of Maxwell fluid in a porous medium heated from below*, Phys. Lett. A 372, 30463050 (2008).
- [36] Gaikwad, S.N., Malashetty, M.S., Prasad, K.R, *Linear and non-linear double diffusive convection in a fluid saturated anisotropic porous layer with cross-diffusion effects*, Transp. Porous Media 80, 537-560 (2009).
- [37] Malashetty, M.S.; Tan, W.C.; Swamy M., *The onset of double diffusive convection in a binary viscoelastic fluid saturated anisotropic porous layer*, Phys. Fluids 2009, 21, 084101.
- [38] Kumar, A., Bhadauria, B.S., *Thermal instability in a rotating anisotropic porous layer saturated by a viscoelastic fluid*, Int. J. Non-linear Mech. 453454 (2010).
- [39] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Dover, New-York, 1981.
- [40] M. Steenbeck, F. Krause, K.H. Rdlr, *Zur Berechnung der mittleren Lorentz-Feldstrke $u \times B$ furein elektrisch leitendes Medium in turbulenter, durch Coriolis-Krfte beeinflusster Bewegung*, Z. Naturforsch 21a (1966) 369376.
- [41] G.V. Levina, S.S. Moiseev, P.B. Rutkevich, *Hydrodynamic alpha-effect in a convective system*, in: L. Debnath, D.N. Riahi (Eds.), *Advances in Fluid Mechanics, Nonlinear Instability, Chaos and Turbulence*, WIT. Press, Southampton, 2000.
- [42] G.V. Levina, I.A. Burylov, A. V. Firylyov, L.V. Shestakova, *Helical-vortex instability in a convectively unstable fluid: origin and numerical simulation*, ICM- MAUBRAS, Perm (2004).
- [43] G.V. Levina, *Parameterization of helical turbulence in numerical models of intense atmospheric vortices*, Dokl. Earth Sci. 411A (2006) 14171420.
- [44] G. V. Levina, I. A. Burylov, *Numerical simulation of helical-vortex effects in Rayleigh-Benard convection*, Non-linear Process Geophys. 13 (2006) 205-222.
- [45] M. Essoun, J.B. Chabi Orou, *Effects of rotation and helical force on the onset of Rayleigh-Benard convection with free-free boundaries*, Afr. Phys. Rev.4 (2010) 65-71.
- [46] P. Hounsou, A.V. Monwanou, C.H. Miwadinou, J.B. Chabi Orou, *Effect of helical force on the stationary convection in a ferrofluid*, Indian J. Phys (2020) 94:1581-1590.
- [47] P. Hounsou, A.V. Monwanou, C.H. Miwadinou, J.B. Chabi Orou, *Effect of helical force on the stationary convection in a rotating ferrofluid*, Chinese Journal of Physics 65 (2020) 526-537.
- [48] P. Hounsou, A. V. Monwanou, C. H. Miwadinou and J. B. Chabi Orou, *Stationary convection in a binary mixture of ferrofluids in a porous medium*, Fluid Dyn. Res. 52 (2020) 035501 (21pp).

