

Can Time in Special Relativity Appear Frozen despite the Clock Hypothesis Says it Cannot?

Arne Bergstrom*

B&E Scientific Ltd, United Kingdom

*Corresponding author: arne.bergstrom@physics.org

Received October 31, 2013; Revised November 20, 2013; Accepted November 22, 2013

Abstract According to general relativity, time in a gravitational field will appear slowed down, or close to a black hole even frozen to complete standstill. From an assumed equivalence between gravity and acceleration, one might thus expect that time in special relativity could similarly appear to be slowed down, or even frozen, when observing a system in strong acceleration even at moderate relativistic velocities. Specifically, this would seem to be the case for hyperbolic space time motion when accelerated motion takes place along a hyperbola corresponding to constant time in the Minkowski diagram. On the other hand, the original postulates in Einstein's theory of special relativity are today normally supplemented with a new postulate, the clock hypothesis, stating that time is unaffected by accelerations. The present study concludes that there is however no inconsistency here: Without being in conflict with the clock hypothesis, time can still appear to be slowed down or even frozen in the special case of hyperbolic motion. This is then due to the special scaling properties of this type of motion, which happen to imitate a constant acceleration. Slowing-down of time can thus occur not only at extreme velocities close to light speed, but also at moderate relativistic velocities for sufficiently powerful accelerations.

Keywords: *gravity-acceleration equivalence, hyperbolic motion, scaling*

Cite This Article: Arne Bergstrom, "Can Time in Special Relativity Appear Frozen despite the Clock Hypothesis Says it Cannot?" *International Journal of Physics* 1, no. 6 (2013): 146-150. doi: 10.12691/ijp-1-6-3.

1. Introduction

Presently there seems to be considerable consensus about the validity of the so-called clock hypothesis [1], which comparatively recently has been introduced into special relativity, and which states that accelerated motion of a system does not affect the passage of time in that system as observed from a reference frame at rest. Arguments for the clock hypothesis will be described below and shown to be persuasive. However, as with all statements based on generalisations of observations, it does not seem obvious that the clock hypothesis necessarily has the general validity normally attributed to it.

To early men on some tropical island, there would similarly have been convincing empirical evidence that water could never exist in any odd frozen, solid form – a conclusion based on their observations of water in the ocean, in lakes, small ponds, rivers, wells, rain, etc. But, although outside their experience and unknown to them, ice would still exist.

Could there analogously be special cases when time could be affected by accelerations and perhaps even be frozen to standstill when observed from a stationary system, and where thus the clock hypothesis might seem not be valid? We know that this can be the case in general relativity in the heavily curved spacetime at black holes, but could this perhaps also possibly happen in special relativity, *i.e.* in flat spacetime?

One such candidate for frozen time in special relativity is the peculiar properties of hyperbolic motion as studied below, where the spacetime trajectory of a relativistic motion might be made to coincide with the hyperbola representing constant time in the co-moving frame in a Minkowski diagram. This would then suggest that time in some particular accelerated systems could indeed appear to be arbitrarily slowed down [2] or maybe even frozen when observed from a stationary system. This is the type of situation considered in detail below as a candidate for a case in special relativity when time is indeed affected by accelerations.

The problem of accelerated systems in flat spacetime and hyperbolic motion has been extensively studied in the literature [3,4] (*cf* also Rindler coordinates [5,6]). In the following, special attention will be paid to the case of frozen time, since this is the case that most clearly differs from the normal relativistic time dilation occurring at velocities close to the speed of light. However, the reader is reminded that not just frozen time but even the slowing-down of time due to acceleration in hyperbolic motion as discussed below is in apparent conflict with the clock hypothesis.

2. The Case for Frozen Time

The Lorentz transformation relates a stationary frame $xyz\tau$ to a frame $x'y'z'\tau'$, moving with, say, its x' -axis with velocity in the positive direction along the x -axis of the

xyzτ frame. The corresponding Lorentz transformation is given as follows [7].

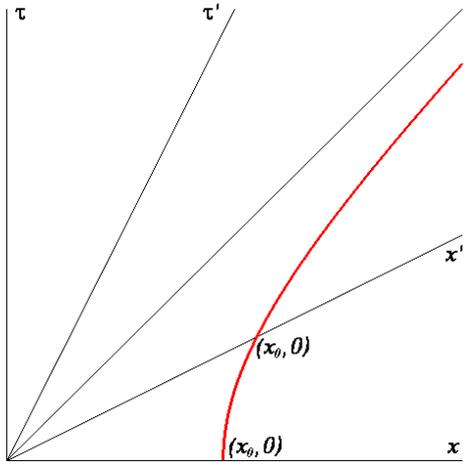


Figure 1. Minkowski diagram showing how the x' and τ' axes in a moving frame relate to the x and τ axes in a stationary frame. For increasing relative velocities, the x' and τ' axes tilt more and more towards the light ray $\tau = x$, with a spacetime point $(x_0, 0)$ scaling as a hyperbola for increasing velocities. Note that this hyperbola may be identical to the hyperbolic trajectory (red) observed in the stationary frame due to a particle in constant acceleration in its co-moving frame. Hence the time shown by a clock moving along such a hyperbolic spacetime trajectory may appear frozen to a stationary observer as discussed in the text.

$$x = \frac{x' + \beta \tau'}{\sqrt{1 - \beta^2}}, y = y', z = z', \tau = \frac{\tau' + \beta x'}{\sqrt{1 - \beta^2}}, \quad (1)$$

where $\tau = t/c$ and $\beta = v/c$, with t and v being the time and velocity in ordinary units, and c being the speed of light.

The Lorentz transformation has the counterintuitive property that events that are simultaneous in one system are not simultaneous in a system moving with some velocity relative to this system. This is clearly illustrated in a Minkowski diagram [8] as in Figure 1.

Note that from (1) follows that [9]

$$x^2 - \tau^2 = x'^2 - \tau'^2. \quad (2)$$

As an example, we study the case when we assume the time τ' to be frozen in the $x'y'z'\tau'$ frame so that, e.g., $\tau' = 0$ and we have a constant x' , say $x' = x_0$. Then (1) gives

$$x = \frac{x_0}{\sqrt{1 - \beta^2}}, y = y', z = z', \tau = \frac{\beta x_0}{\sqrt{1 - \beta^2}}, \quad (3)$$

from which we thus have

$$\tau / x = \beta. \quad (4)$$

From (3) we get a hyperbolic trajectory in the stationary $x \tau$ frame (x_0 constant),

$$x^2 - \tau^2 = x_0^2, \quad (5)$$

which combined with (4) gives

$$\tau^2 / \beta^2 - \tau^2 = x_0^2, \quad (6)$$

and we thus get the following expression for the velocity,

$$\beta = \frac{\tau}{\sqrt{\tau^2 + x_0^2}}. \quad (7)$$

and thus

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{\sqrt{\tau^2 + x_0^2}}{x_0} \quad (8)$$

Differentiating (7) with respect to τ , we then get for the acceleration α in the stationary reference frame,

$$\alpha = \frac{x_0^2}{(\tau^2 + x_0^2)^{3/2}} = \frac{1}{x_0 \gamma^3} \quad (9)$$

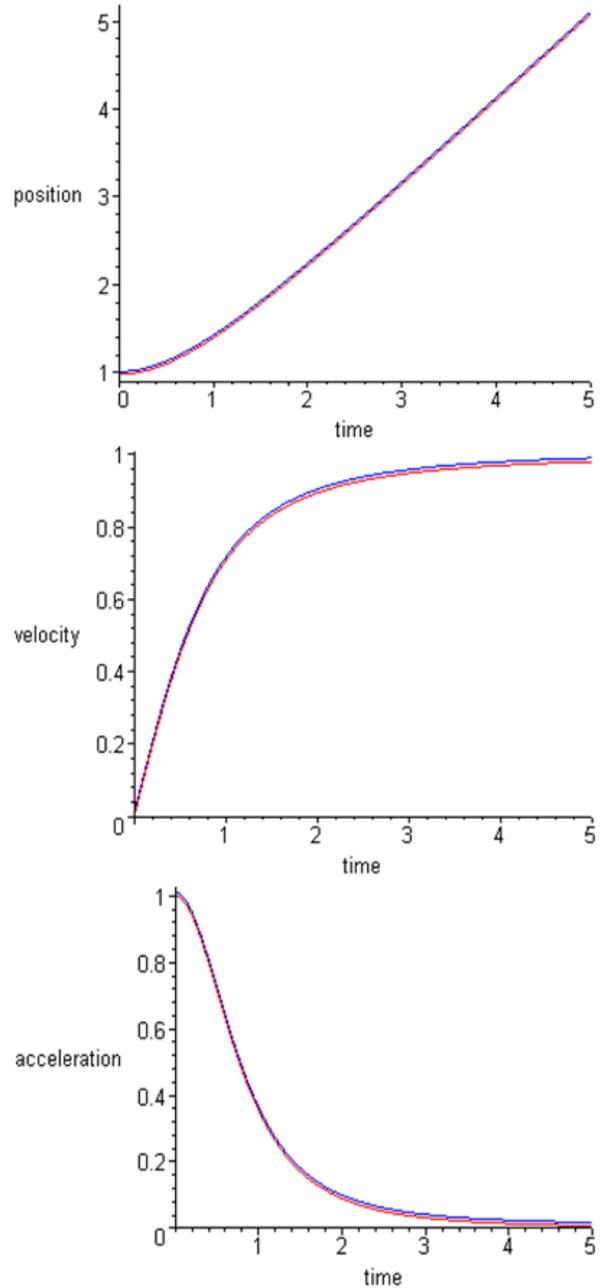


Figure 2. The position $x(\tau)$ as calculated from (5), the velocity $\beta(\tau)$ as given in (7), and the acceleration $\alpha(\tau)$ as given in (9), all calculated as functions of time τ (blue) in the stationary frame (with $x_0 = 1$) in the frozen-time case (3). Note that these curves are mathematically identical to the corresponding curves (red, coinciding) calculated from a different starting point using (10) through (15) with $\alpha' = 1/x_0$

Figure 2 shows the position $x(\tau)$ calculated from (5), the velocity $\beta(\tau)$ according to (7), and the acceleration $\alpha(\tau)$ according to (9), all as functions of time τ in the stationary frame for the case of frozen time in (3).

3. Main Argument against Frozen Time

However, the case of frozen time derived above would seem to be contradicted by the following argument. The relationship between rest-frame time τ and time τ' for a particle subjected to a constant acceleration α' in the co-moving frame $x' \tau'$ can be derived [4,10] to be as follows,

$$\tau = \sinh(\alpha' \tau') / \alpha' \quad (10)$$

The relationship between the space coordinate x in the stationary frame and the time τ' in the co-moving frame is correspondingly [10],

$$x = \cosh(\alpha' \tau') / \alpha'. \quad (11)$$

We note that (10) and (11) give a hyperbolic motion [10] in the $x\tau$ frame (α' is a constant, cf(5) above),

$$x^2 - \tau^2 = 1/\alpha'^2. \quad (12)$$

The velocity β and acceleration α in the rest frame can be calculated from (10) and (11) to be as follows [4] (cf the identical expressions in the more general case in (19)-(21) below, and in particular also (4) and (9) above),

$$\beta = \tanh(\alpha' \tau') = \tau/x, \quad (13)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh(\alpha' \tau'), \quad (14)$$

$$\alpha = \frac{\partial}{\partial \tau} \beta = \frac{\alpha'}{\gamma^3}. \quad (15)$$

Note that the time-dependent acceleration α in (15) as measured in the rest frame $x\tau$ transforms [4] to a constant acceleration α' as measured in the co-moving frame $x' \tau'$ (as it should).

We now want to consider the possibility of frozen time in the moving $x' \tau'$ frame as observed from the stationary $x \tau$ frame. Solving for τ' in (10) then gives [4]

$$\tau' = ar \sinh(\alpha' \tau) / \alpha'. \quad (16)$$

From (16) follows that there seems to be no way (except asymptotically for $\alpha' \rightarrow \infty$ corresponding to the trivial case $\beta = 1$) for the time τ' in the co-moving frame to appear frozen (*i.e.* have constant τ') when observed during some interval in times τ in the stationary frame, as is suggested in the previous section. This can thus be regarded as the main argument against frozen time in hyperbolic motion.

4. Counterargument in Favour of Frozen Time

However, the above expression (16) for τ' is derived assuming hyperbolic motion with constant acceleration α' in the co-moving frame $x' \tau'$. *But that is not the only way to get hyperbolic motion.* It is important to note that spacetime motion according to (10) and (11) above is not equivalent to hyperbolic motion: The motion defined by (10) and (11) above implies the hyperbolic motion in (12),

but a hyperbolic motion as in (12) does not necessarily imply a motion according to (10) and (11) above.

Thus (10) and (11) above represent only one example of possible relationships $x(\tau')$, $\tau(\tau')$ between the coordinates $x \tau$ in the stationary frame and the coordinates $x' \tau'$ in the co-moving frame that result in hyperbolic motion as in (12). Consider for instance the following modification of (10) and (11),

$$x = \cosh(f(\tau')) / \alpha', \tau = \sinh(f(\tau')) / \alpha', \quad (17)$$

which, with a suitably chosen $f(\tau') \neq \alpha' \tau'$ in the relativistic regime, would obviously not correspond to the motion discussed in (10) and (11) above, but which nevertheless gives the same hyperbola as in (12),

$$\begin{aligned} x^2 - \tau^2 &= \\ &= \cosh^2(f(\tau')) / \alpha'^2 - \sinh^2(f(\tau')) / \alpha'^2 = \\ &= 1/\alpha'^2. \end{aligned} \quad (18)$$

Differentiating (17) we get for the velocity as measured in the stationary frame,

$$\beta = dx/d\tau = (dx/d\tau') / (d\tau'/d\tau) = \tanh(f(\tau')), \quad (19)$$

and thus also

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh(f(\tau')). \quad (20)$$

Differentiating (19) and (17) with respect to time τ' in order to calculate

$$\begin{aligned} d\beta/d\tau &= \left(d\beta/d\tau' \right) / \left(d\tau'/d\tau \right) \\ &= \left(1/\cosh^2(f(\tau')) \right) / \left(\cosh(f(\tau')) / \alpha' \right), \end{aligned}$$

and using (20), we then get for the acceleration α as measured in the stationary frame,

$$\alpha = \frac{\partial}{\partial \tau} \beta = \frac{\alpha'}{\gamma^3}, \quad (21)$$

which, like in (15), thus again transforms to a constant acceleration α' as measured in the co-moving frame (see the explanation below).

Specifically, the relationship with frozen time in (3) can be regarded as a limiting case of (17) with $f(\tau') \rightarrow 0$ (and $\alpha' = 1/x_0$). Hence, even though (3) does not describe a legitimate trajectory resulting from a constant acceleration in the co-moving frame as derived in (10) and (11), it nevertheless gives the same hyperbola as shown by the equivalence between (5), (12), and (18) above.

Hence constant accelerations in the co-moving frame lead to hyperbolic motion as in (12), but also other types of motion like (17), or the frozen-time case (3), lead to the same hyperbolic motion in the stationary frame, and also (surprisingly, cf explanation below) with seemingly the same acceleration (if in (5) we set the constant $x_0 = 1/\alpha'$).

A somewhat bizarre fact is thus the following: Differentiating in order to get the accelerations in the three cases (3), (10) & (11), and (17), is thus shown above to

give the same constant acceleration α' when measured in the co-moving frame in all three cases. This thus despite the fact that only the case with constant proper acceleration in (10) & (11) is actually derived from such a constant acceleration in the co-moving frame.

That also the other cases (3) and (17) seemingly give the same result for the acceleration is due to the fact that the constant $1/\alpha'$ on the right-hand side in (5), (12), and (18), has the dual property of also being a scale factor, as clearly seen in (17). This is why it appears in (5) and (18) as if it would be an acceleration (since acceleration scales as length/time² and we thus specifically in this case get a net scale factor $1/\alpha'$ from the square of time).

In summary, hyperbolic motion is thus more general than just the result of the specific constant proper acceleration given in (10) and (11) above. Time may thus indeed seem to be frozen when observing a system in hyperbolic motion from a stationary reference system. This thus regardless of restrictions seemingly imposed by reference to results as in (10) and (16), which are derived for the particular constant proper acceleration discussed above. There are also other cases when we can have hyperbolic motion with what looks like a mathematically equivalent, constant proper acceleration α' . But this is then due to the appearance of α' as a scale factor in these equations, cf (17), which in the case of hyperbolic motion as in (18) then happens to imitate a constant proper acceleration (as is also illustrated in Figure 2).

5. Frozen Time is a Limiting Case

It should to be noted that frozen time τ' in the accelerating frame corresponds to the special limiting case $f(\tau') \rightarrow 0$ of the function $f(\tau')$ as discussed earlier. In accordance with (17) above, other functions $f(\tau')$ would correspond to hyperbolic motions with times τ' just slowed down [2], but not frozen, when observed from the stationary frame.

The clock hypothesis introduced [11] into special relativity in the 1990s would, if generally valid, exclude that accelerated motion of a system could at all affect the apparent passage of time in that system as observed from a system at rest. On the other hand, some variant of a local acceleration/gravity equivalence [12] as originally introduced by Einstein [13] would still be expected to exist between a system in a uniform gravitational field and a system in constant acceleration, *i e* in hyperbolic motion.

If so, then a system in hyperbolic motion would be expected to show an apparent slowing-down of time as discussed above, just as a system in a gravitational field does. In the limiting case, this slowing-down could then in both cases even result in an apparent freezing of time in systems subjected, respectively, to strong accelerations or extreme gravitational fields when observed from a reference system.

The argument involving scaling as discussed above thus explains how time, despite the clock hypothesis, can still appear to be slowed down – or in the special case $f(\tau') \rightarrow 0$ even be frozen – when observing accelerated systems in hyperbolic motion from a reference frame at rest. The scaling effect discussed above is then the mechanism by which this slowing-down or freezing avoids being in

conflict with the clock hypothesis, or in conflict with the traditional formula (16) for time in an accelerated system.

6. Experimental Results Vs Frozen Time

The clock hypothesis [1,11], *i e* the assumption that the passage of time is unaffected by accelerations, is supported by some experiments. In these experiments, particles experiencing even as extreme transverse accelerations as of the order of 10^{19} m/s² [14], or longitudinal accelerations of the order of 10^{17} m/s² [15], respectively, have shown no effects of the acceleration in addition to the normal relativistic time dilation.

These experimental results thus indicate that perhaps most types (cf the tropical island in the Introduction) of accelerations do not influence the passage of (dilated) time – not even larger ones than considered necessary in this paper.

However, experimental results like those just mentioned do not contradict that time could be frozen or dilated in the special case of hyperbolic spacetime motion as discussed in this paper. In hyperbolic motion one starts, *e g*, with velocity zero and then increases this velocity with a constant proper acceleration. This is thus a very special type of acceleration, compared to which results from the types of accelerations used in the experiments mentioned above do not seem to be relevant. In these experiments there seems to be no correspondence to hyperbolic spacetime motion with its especially designed, constant proper longitudinal acceleration, which is the essence of the mechanism for frozen or dilated time discussed in this paper.

7. Conclusion

So can time in a system in hyperbolic motion appear frozen when observed from a stationary system as the discussion above seems to suggest? Or is this not the case as the clock hypothesis states and as is also discussed above?

Actually, the two alternatives could indeed both be right and not be in conflict, as will now be described in the following summary of the above arguments.

Assume the clock hypothesis to be true. Then the successive inertial frames used in the derivation above would strictly behave as an accelerating frame as assumed in the derivation. According to the clock hypothesis, time would then not be affected by this acceleration.

Completely unrelated to the clock hypothesis, the special case of hyperbolic motion would by definition give a spacetime trajectory in the form of a hyperbola as in (5) or (18),

$$x^2 - \tau^2 = 1/\alpha^2. \quad (22)$$

The hyperbola in (22) is thus described by the parameter α on the right-hand side, which parameter thus defines a scale factor determining the size of the hyperbola.

Although this scale factor then only determines the size of the hyperbola and thus in principle has nothing to do with any acceleration, it nevertheless appears in the hyperbolic expression in exactly the same way as an

acceleration, as seen by the equivalence between (12) and (22). And since it does exactly that, it also indeed happens to define a constant acceleration in the co-moving frame – even though it is actually only a scale factor.

In the case this hyperbolic spacetime trajectory of the motion is arranged to coincide with a hyperbola in the Minkowski diagram describing constant time in the co-moving frame, then time in this co-moving frame would appear to a stationary observer to be frozen to standstill as discussed in this paper, and equivalently to what happens at a black hole in general relativity. But, as discussed above, this is then an effect only of the particular scaling properties of the hyperbolic motion and is thus not in conflict with the clock hypothesis as such.

The above analysis thus shows that dilated or frozen time may occur even at moderate relativistic velocities for a special type of strong accelerations. This could in some future have important technological implications, and it seems essential that this fact should not be forgotten and buried in some general view that the clock hypothesis excludes also dilated or frozen time at moderate relativistic velocities in the special case of hyperbolic spacetime motion.

References

- [1] Barton G 1999, *Introduction to the Relativity Principle* (Wiley), Ch. 6.
- [2] Misner C, Thorne K, and Wheeler J 1973, *Gravitation* (Freeman), Exercise 6.3 (b), p. 167.
- [3] Misner C, Thorne K, and Wheeler J 1973, *Gravitation* (Freeman), Ch. 6.
- [4] Barton G 1999, *Introduction to the Relativity Principle* (Wiley), Ch. 8.
- [5] Rindler W 2001, *Relativity: Special, General and Cosmological* (Oxford University Press).
- [6] http://en.wikipedia.org/wiki/Rindler_coordinates.
- [7] Pauli W 1981, *Theory of Relativity* (Dover), p. 10.
- [8] Aharoni J 1985, *The Special Theory of Relativity* (Dover), Ch. 1.9.
- [9] Møller C, *The Theory of Relativity* 2nd Ed (Oxford University Press), p. 37.
- [10] Misner C, Thorne K, and Wheeler J 1972, *Gravitation* (Freeman 1973), p. 166.
- [11] Mainwaring S and Stedman G 1993, *Phys. Rev. A*, 47 3611.
- [12] http://en.wikipedia.org/wiki/Equivalence_principle.
- [13] Einstein A 1911, *Ann. Phys.*, 35 898, Eng. transl. in *The Principle of Relativity* (Dover 1952).
- [14] Bailey J et al. 1977, *Nature*, 268 301.
- [15] Roos C et al. 1980, *Nature*, 286 24.