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Investigation of Characteristics of the Reflection of Stationary and Pulse Sound from Visco-Elastic Cylindrical Layers

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Abstract The scattering task of stationary and impulse sound signals by the viscous-elastic and elastic cylindrical layers is studied with the help of Fourier series and fundamental solutions of the Helmholtz equation in circular cylindrical coordinates system. The reflection's characteristics of sound (harmonic and impulse) are calculated in the large range of angles (including inverse direction).

Keywords: *viscous-elastic layer, impulse, Fourier series*

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1. Introduction

The problems of scattering of stationary and non-stationary sound signals discussed in various publications [1-13]. In these papers the ideal or elastic bodies of infinite circular cylinders form are substantially examined. In the present work the scattering of stationary and impulse sound signals by the viscous-elastic cylindrical layers are studied.

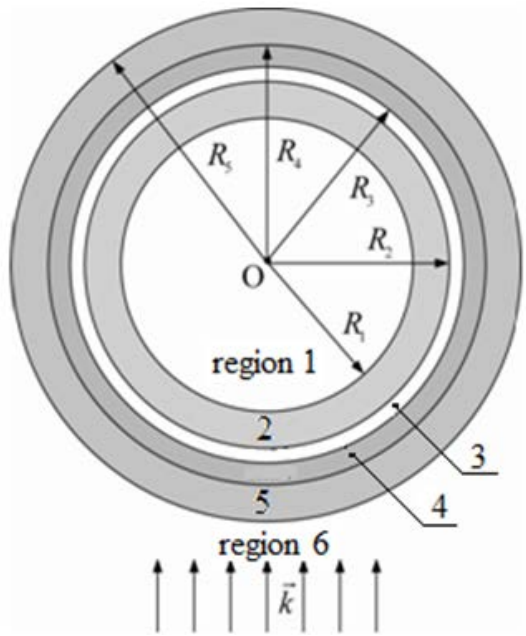

Figure 1. The system of the viscous-elastic and elastic cylindrical layers irradiated by plane sound wave in liquid

The first part of the article substantiates the scattering of the stationary and non-stationary (impulse) sound signals by the viscous-elastic cylindrical layers. Firstly we will consider a harmonic uninterrupted sound signal of frequency ω, which irradiates a system of the elastic and viscous-elastic cylindrical layers (Figure 1). The wave vector $\vec{k}$ of this wave is orthogonal as regards the axis z of system of the cylindrical layers (the plane task, Figure 1).

The system consists of four layers: external thin viscous-elastic layer (5), the viscous-elastic more thick layer (4), elastic layer (3), the inside viscous-elastic layer (2). There is vacuum on the inside region of the system (region 1), and water - on the outside region (region 6). All layers are of various characteristics of material: the density, the elastic modules and the loss coefficients. The wave numbers of flexural waves and the corresponding arguments of the Bessel's and the Naman's cylindrical functions by the loss in the viscous-elastic layers are complex.

The following designations of the sound pressures, scalar and vector displacement potentials one can find [2-7,11,12,13]:

1. on the inside region of the layers (region 1) - is the vacuum, the wave process is absent;
2. the viscous-elastic layer 2 is are described by scalar potential Φ_2 and vector potential $\vec{\Psi}_2$;
3. the elastic layer 3 - scalar potential Φ_3 and vector potential $\vec{\Psi}_3$;
4. the viscous-elastic layer 4 - Φ_4 and $\vec{\Psi}_4$;
5. the viscous-elastic layer 5 - Φ_5 and $\vec{\Psi}_5$;
6. outside region 6 is are described by irradiate sound pressures p_i and scattering sound pressures p_s.

For plane task all vector potentials have only one nonzero component ψ_z,named as ψ.

All potentials and sound pressures are also expanded by independent solutions of the Helmholtz equation in circular cylindrical coordinates system [1-7, 10,11,12,13]:

$$\Phi_2(r,\varphi)=\sum_{m=0}^{\infty}[I_m J_m(k_2 r)+K_m N_m(k_2 r)]\cos m\varphi \quad (1)$$

$$\Psi_2(r,\varphi)=\sum_{m=1}^{\infty}[L_m J_m(k_2' r)+Q_m N_m(k_2' r)]\sin m\varphi \quad (2)$$

$$\Phi_3(r,\varphi)=\sum_{m=0}^{\infty}[P_m J_m(k_3 r)+R_m N_m(k_3 r)]\cos m\varphi \quad (3)$$

$$\Psi_3(r,\varphi)=\sum_{m=1}^{\infty}[S_m J_m(k_3' r)+T_m N_m(k_3' r)]\sin m\varphi \quad (4)$$

$$\Phi_4(r,\varphi)=\sum_{m=0}^{\infty}[V_m J_m(k_4 r)+W_m N_m(k_4 r)]\cos m\varphi \quad (5)$$

$$\Psi_4(r,\varphi)=\sum_{m=1}^{\infty}[X_m J_m(k_4' r)+Y_m N_m(k_4' r)]\sin m\varphi \quad (6)$$

$$\Phi_5(r,\varphi)=\sum_{m=0}^{\infty}[V_{1m} J_m(k_5 r)+W_{1m} N_m(k_5 r)]\cos m\varphi \quad (7)$$

$$\Psi_5(r,\varphi)=\sum_{m=1}^{\infty}[X_{1m} J_m(k_5' r)+Y_{1m} N_m(k_5' r)]\sin m\varphi \quad (8)$$

$$p_i(r,\varphi)=\sum_{m=0}^{\infty}\varepsilon_m(-i)^m J_m(k_6 r)\cos m\varphi \quad (9)$$

$$p_s(r,\varphi)=\sum_{m=0}^{\infty}A_m H_m^{(1)}(k_6 r)\cos m\varphi \quad (10)$$

where k_2-k_6 and $k_2'-k_5'$ – is a longitudinal and flexural wave numbers in elastic, viscous-elastic layers and outside liquid region correspondingly; $\varepsilon_m=1$ for $m=0$, $\varepsilon_m=2$ for $m\neq 0$; $I_m, K_m, L_m, Q_m, P_m, R_m, S_m, T_m, V_m, W_m, X_m, Y_m, V_{1m}, W_{1m}, X_{1m}, Y_{1m}$ – are the unknown coefficients of expanding, which are calculated basing at the boundary conditions [3-13]:

1. a strains on the boundary (vacuum - viscous-elastic layer 2 , $r=R_1$): is equal to zero:

$$\lambda_2 k_2^2\Phi_2+2\mu_2\left(-\frac{\partial^2\Phi_2}{\partial r^2}-r^{-2}\frac{\partial\Psi_2}{\partial\varphi}+r^{-1}\frac{\partial^2\Psi_2}{\partial r\partial\varphi}\right)=0\Big|_{r=R_1} \quad (11)$$

$$-2r^{-1}\frac{\partial^2\Phi_2}{\partial r\partial\varphi}+2r^{-2}\frac{\partial\Phi_2}{\partial\varphi}-k_2'^2\Psi_2-2\frac{\partial^2\Psi_2}{\partial r^2}=0\Big|_{r=R_1} \quad (12)$$

2. continuity of displacements and strains at external boundary of the viscous-elastic layer 2 and at boundary of the elastic layer 3 ($r=R_2$):

$$-\frac{\partial\Phi_2}{\partial r}+r^{-1}\frac{\partial\Psi_2}{\partial\varphi}=-\frac{\partial\Phi_3}{\partial r}+r^{-1}\frac{\partial\Psi_3}{\partial\varphi}\Big|_{r=R_2}; \quad (13)$$

$$-r^{-1}\frac{\partial\Phi_2}{\partial\varphi}-\frac{\partial\Psi_2}{\partial r}=-r^{-1}\frac{\partial\Phi_3}{\partial\varphi}-\frac{\partial\Psi_3}{\partial r}\Big|_{r=R_2} \quad (14)$$

$$\lambda_2 k_2^2\Phi_2+2\mu_2\left(-\frac{\partial^2\Phi_2}{\partial r^2}-r^{-2}\frac{\partial\Psi_2}{\partial\varphi}+r^{-1}\frac{\partial^2\Psi_2}{\partial r\partial\varphi}\right)$$
$$=\lambda_3 k_3^2\Phi_3+2\mu_3\left(-\frac{\partial^2\Phi_2}{\partial r^2}-r^{-2}\frac{\partial\Psi_3}{\partial\varphi}+r^{-1}\frac{\partial^2\Psi_3}{\partial r\partial\varphi}\right)\Big|_{r=R_2} \quad (15)$$

$$\mu_2\left(-2r^{-1}\frac{\partial^2\Phi_2}{\partial r\partial\varphi}+2r^{-2}\frac{\partial\Phi_2}{\partial\varphi}-k_2'^2\Psi_2-2\frac{\partial^2\Psi_2}{\partial r^2}\right)$$
$$=\mu_3\left(-2r^{-1}\frac{\partial^2\Phi_3}{\partial r\partial\varphi}+2r^{-2}\frac{\partial\Phi_3}{\partial\varphi}-k_3'^2\Psi_3-2\frac{\partial^2\Psi_3}{\partial r^2}\right)\Big|_{r=R_2} \quad (16)$$

3. continuity of displacements and strains at external boundary of the elastic layer 3 and at boundary of the viscous-elastic layer 4 ($r=R_3$):

$$-\frac{\partial\Phi_3}{\partial r}+r^{-1}\frac{\partial\Psi_3}{\partial\varphi}=-\frac{\partial\Phi_4}{\partial r}+r^{-1}\frac{\partial\Psi_4}{\partial\varphi}\Big|_{r=R_3}; \quad (17)$$

$$-r^{-1}\frac{\partial\Phi_3}{\partial\varphi}-\frac{\partial\Psi_3}{\partial r}=-r^{-1}\frac{\partial\Phi_4}{\partial\varphi}-\frac{\partial\Psi_4}{\partial r}\Big|_{r=R_3}; \quad (18)$$

$$\lambda_3 k_3^2\Phi_3+2\mu_3\left(-\frac{\partial^2\Phi_3}{\partial r^2}-r^{-2}\frac{\partial\Psi_3}{\partial\varphi}+r^{-1}\frac{\partial^2\Psi_3}{\partial r\partial\varphi}\right)$$
$$=\lambda_4 k_4^2\Phi_4+2\mu_4\left(-\frac{\partial^2\Phi_4}{\partial r^2}-r^{-2}\frac{\partial\Psi_4}{\partial\varphi}+r^{-1}-\frac{\partial^2\Psi_4}{\partial r\partial\varphi}\right)\Big|_{r=R_3} \quad (19)$$

$$\mu_3\left(-2r^{-1}\frac{\partial^2\Phi_3}{\partial r\partial\varphi}+2r^{-2}\frac{\partial\Phi_3}{\partial\varphi}-k_3'^2\Psi_3-2\frac{\partial^2\Psi_3}{\partial r^2}\right)$$
$$=\mu_4\left(-2r^{-1}\frac{\partial^2\Phi_4}{\partial r\partial\varphi}+2r^{-2}\frac{\partial\Phi_4}{\partial\varphi}-k_4'^2\Psi_4-2\frac{\partial^2\Psi_4}{\partial r^2}\right)\Big|_{r=R_3} \quad (20)$$

4. continuity of displacements and strains at external boundary of the viscous-elastic layer 4 and at boundary of the viscous-elastic layer 5 ($r=R_4$):

$$-\frac{\partial\Phi_4}{\partial r}+r^{-1}\frac{\partial\Psi_4}{\partial\varphi}=-\frac{\partial\Phi_5}{\partial r}+r^{-1}\frac{\partial\Psi_5}{\partial\varphi}\Big|_{r=R_4}; \quad (21)$$

$$-r^{-1}\frac{\partial\Phi_4}{\partial\varphi}-\frac{\partial\Psi_4}{\partial r}=-r^{-1}\frac{\partial\Phi_5}{\partial\varphi}-\frac{\partial\Psi_5}{\partial r}\Big|_{r=R_4}; \quad (22)$$

$$\lambda_4 k_4^2\Phi_4+2\mu_4\left(-\frac{\partial^2\Phi_4}{\partial r^2}+r^{-2}\frac{\partial\Psi_4}{\partial\varphi}+r^{-1}\frac{\partial^2\Psi_4}{\partial r\partial\varphi}\right)$$
$$=\lambda_5 k_5^2\Phi_5+2\mu_5\left(-\frac{\partial^2\Phi_5}{\partial r^2}-r^{-2}\frac{\partial\Psi_5}{\partial\varphi}+r^{-1}\frac{\partial^2\Psi_5}{\partial r\partial\varphi}\right)\Big|_{r=R_4} \quad (23)$$

$$\mu_4\left(-2r^{-1}\frac{\partial^2\Phi_4}{\partial r\partial\varphi}+2r^{-2}\frac{\partial\Phi_4}{\partial\varphi}-k_4'^2\Psi_4-2\frac{\partial^2\Psi_4}{\partial r^2}\right)$$
$$=\mu_5\left(-2r^{-1}\frac{\partial^2\Phi_5}{\partial r\partial\varphi}+2r^{-2}\frac{\partial\Phi_5}{\partial\varphi}-k_5'^2\Psi_5-2\frac{\partial^2\Psi_5}{\partial r^2}\right)\Big|_{r=R_4} \quad (24)$$

5. tangent strains are equal to zero, a normal component of a displacement vector is continuous, a normal strains is equal to total scattering pressure in liquid at an external boundary of the viscous-elastic layer 5 and liquid ($r=R_5$):

$$-\frac{\partial\Phi_5}{\partial r}+r^{-1}\frac{\partial\Psi_5}{\partial\varphi}=\omega^{-2}\rho_0^{-1}\left(\frac{\partial p_i}{\partial r}+\frac{\partial p_s}{\partial r}\right)\Big|_{r=R_5}; \quad (25)$$

$$\lambda_5 k_5^2 \Phi_5 + 2\mu_3 \left(\frac{\partial^2 \Phi_5}{\partial r^2} - r^{-2}\frac{\partial \Psi_5}{\partial \varphi} + r^{-1}\frac{\partial^2 \Psi_5}{\partial r \partial \varphi}\right) = p_i + p_s \Big|_{r=R_5}; \quad (26)$$

$$-2r^{-1}\frac{\partial^2 \Phi_5}{\partial r \partial \varphi} + 2r^{-2}\frac{\partial \Phi_5}{\partial \varphi} - k_5'^2 \Psi_5 - 2\frac{\partial^2 \Psi_5}{\partial r^2} = 0\Big|_{r=R_5}, \quad (27)$$

where $\lambda_2, \mu_2, \lambda_4, \mu_4, \lambda_5, \mu_5$ – Lame constants of the viscous-elastic layers; λ_3, μ_3 – Lame constants of the elastic layer; ρ_0 – the solidity of the liquid environment; ω – the circular frequency.

If we substitute $(1) \div (10)$ in the boundary conditions $(11) \div (27)$, and the trigonometrical functions $\cos m\varphi$ and $\sin m\varphi$ are opthogonal, we'll get the algebraical system of 17 order to define the unknown coefficients of expanding of the fixed index. We are interested in the coefficients A_m (10). Based in (10) and by using the asymptotic of Hankel function of the first kind $H_m^{(1)}(kr)$, we get the far sound field characteristic of the Fraunhofer region (the angular scattering characteristic) $D(\varphi)$ [9,10,11]:

$$D(\varphi) = -\exp(-i\pi/4)(\pi k_6)^{-1} \sum_{m=0}^{\infty} (-i)^m A_m \cos m\varphi. \quad (28)$$

At Figure 2 are shown the modulus of the angular scattering characteristics $D(\varphi)$ for the different wave sizes $k_6 R_5$. Figure 2 (a): $k_6 R_5 \approx 3{,}78$; Figure 2 (b): $k_6 R_5 \approx 7{,}56$; Figure 2 (c): $k_6 R_5 \approx 11{,}35$.

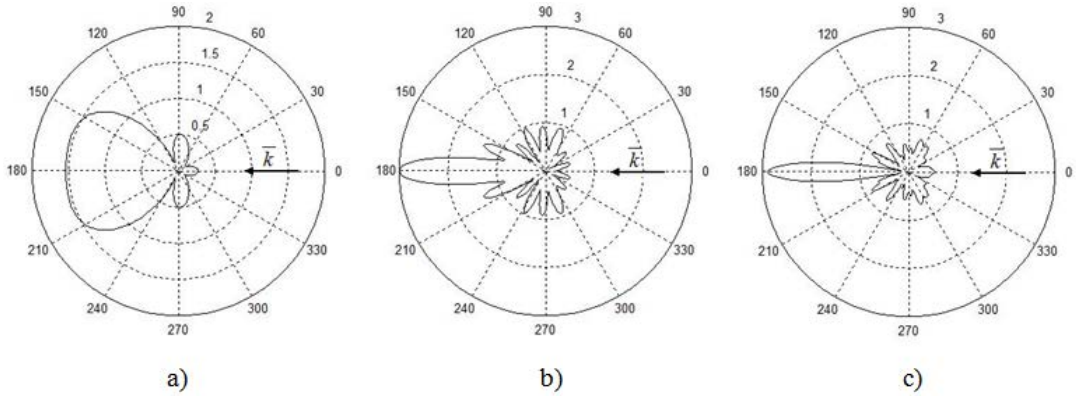

Figure 2. The modulus of the angular scattering characteristics of cylindrical layers $\|D(\varphi)\|$ for the different wave sizes

The angular scattering characteristics in frequency range $\Delta\nu = 200 \div 2000$ Hz (by step $h_{\Delta\nu} = 20$ Hz) and in angle range $\Delta\varphi = 0^0 \div 150^0$ (by step $h_{\Delta\varphi} = 30^0$). are represented for the calculation of impulse signals scattering charact $\nu_0 = (\nu_1 \nu_2)^{1.2} \approx \nu_1 = 200$ Hz, $\nu_2 = 2000$ Hz). Period number of the filling frequency is equal to 30.

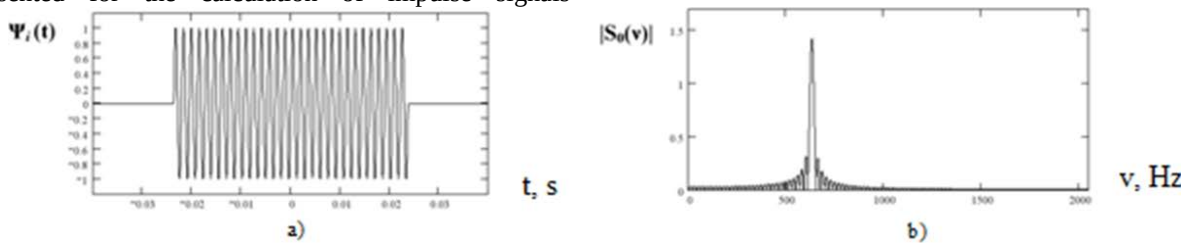

Figure 3. The irradiate impulse $\Psi_i(t)$ (a); the modulus of his spectrum $|S_0(\nu)|$ (b)

The irradiate impulse $\Psi_i(t)$ and the modulus of his spectrum $|S_0(\nu)|$ it connect [14]:

$$\Psi_i(t) = \pi^{-1} \operatorname{Re} \int_{2\pi\nu_1}^{2\pi\nu_2} S_0(\nu) \exp(i2\pi\nu t) d(2\pi\nu). \quad (29)$$

The scattering impulse $\Psi_s(t)$ are represented [14]:

$$\Psi_s(t,\varphi) = \pi^{-1} \int_{2\pi\nu_1}^{2\pi\nu_2} S(\nu,\varphi) \exp(i2\pi\nu t) d(2\pi\nu), \quad (30)$$

where $S(\nu,\varphi) = S_0(\nu) D(\nu,\varphi)$ - the spectral characteristic of the scattering impulse.

The time characteristic of the scattering impulse it normalize: the dependence from distance to observation point is absent (for plane task the dependence are represented: $1/r^{1/2}$.

At Figures 3 – 9 the time and spectral characteristics of irradiate and scattering impulses for the cylindrical viscous-elastic and elastic layers are shown. The scattering impulses are calculated by step $h_{\Delta\varphi} = 30^0$ in angle range φ : 0^0 (inverse direction) - 150^0. The scattering maximum is for $\varphi = 90^0$.

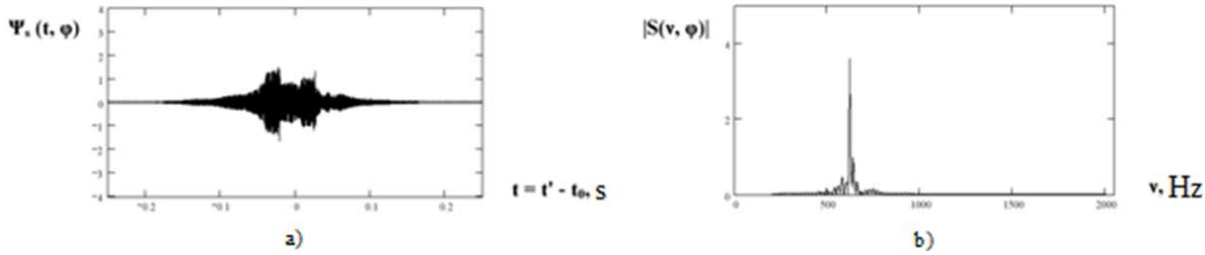

Figure 4. The normalized scattering impulse (inverse direction) $\Psi_s(t,\varphi)$, $\varphi = 0^0$ (a), the normalized modulus spectrum $|S(\nu,\varphi)|$ (b)

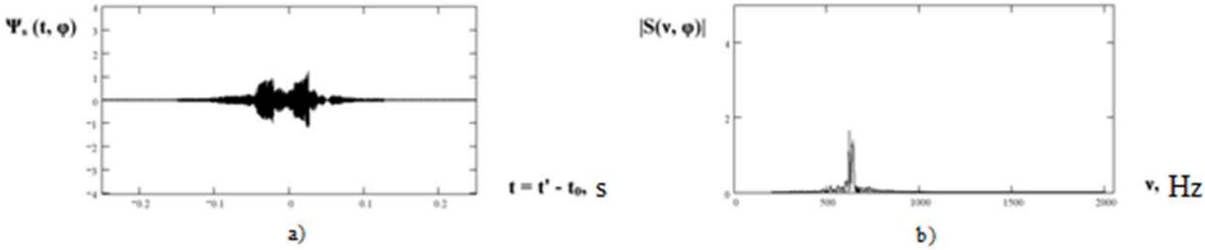

Figure 5. The normalized scattering impulse $\Psi_s(t,\varphi)$, $\varphi = 30^0$ (a); the normalized modulus spectrum $|S(\nu,\varphi)|$ (b)

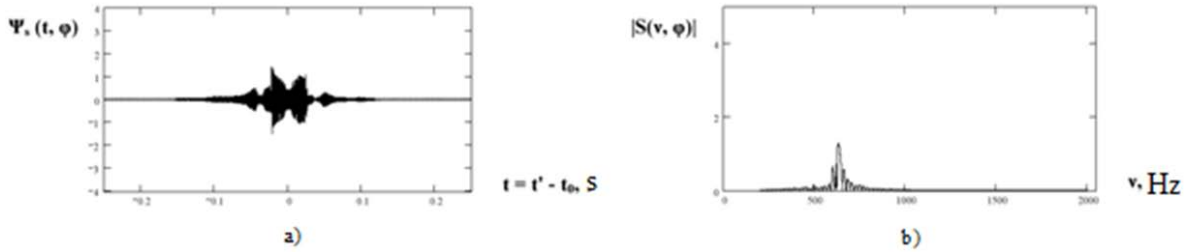

Figure 6. The normalized scattering impulse $\Psi_s(t,\varphi)$, $\varphi = 60^0$ (a); the normalized modulus spectrum $|S(\nu,\varphi)|$ (b)

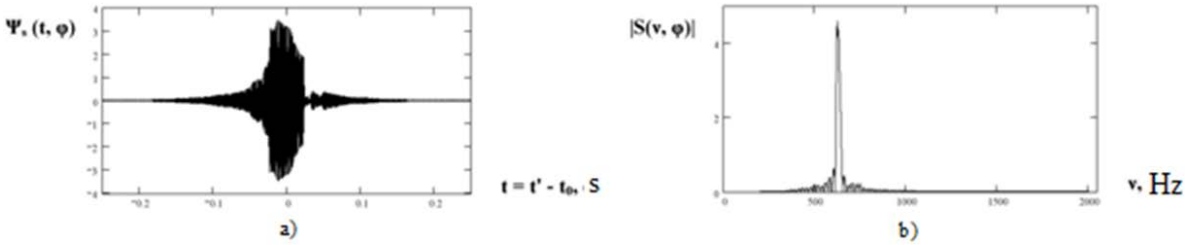

Figure 7. The normalized scattering impulse $\Psi_s(t,\varphi)$, $\varphi = 90^0$ (a); the normalized modulus spectrum $|S(\nu,\varphi)|$ (b)

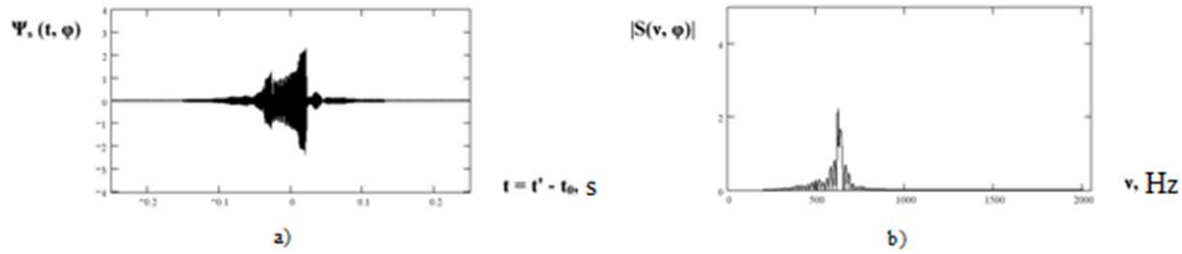

Figure 8. The normalized scattering impulse $\Psi_s(t,\varphi)$, $\varphi = 120^0$ (a); the normalized modulus spectrum $|S(\nu,\varphi)|$ (b)

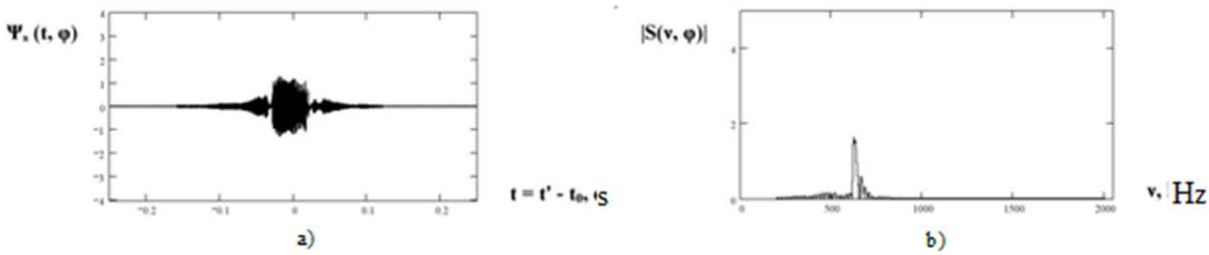

Figure 9. The normalized scattering impulse $\Psi_s(t,\varphi)$, $\varphi = 150^0$ (a); the normalized modulus spectrum $|S(\nu,\varphi)|$ (b)

The interaction of two systems of cylindrical layers (second parallel by first) is defined by the method for elliptic cylinders [15].

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