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Evaluation of the Relative Efficiency of Gas Stations by Data Envelopment Analysis

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Abstract Performance measurement is an important part of management science and operation research. Data Envelopment Analysis is a powerful analytical tool that has been successfully applied for measuring and benchmarking the relative performance in a wide variety of activities. Data Envelopment Analysis assists decision makers to distinguish efficient and inefficient decision making units in a homogeneous group. Super-efficiency Data Envelopment Analysis models can be used in ranking the performance of efficient decision making units. In this paper, Data Envelopment Analysis is employed to present a mathematical model for evaluating the relative efficiency of gas stations of Iranian Oil products Company. Banker, Charnes and Cooper model is applied to determine the relative efficiency of the stations. Super efficiency model of Andersen and Petersen and Slack Based Measure of Super efficiency ranking method are used to determine the most efficient unit.

Keywords: *data envelopment analysis, decision making unit, ranking, super efficiency, input/output*

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1. Introduction

Today, energy and fuel are prominent elements in the progress of the industrialized countries. In Iran, because of having a lot of oil sources, oil and petroleum products play important and strategic role in the economy of the country. Gas stations are directly involved in distributing petroleum products. The main function of these stations is presenting desirable and high-grade products and services to consumers. So evaluating their performance of them is significant. In NIOPDC (National Iranian Oil Products Distribution Company), gas stations are ranked every six months (twice a year) based on some specific indexes. Most of these indexes are qualitative and they pay attention to the beauty and appearance of the stations. Measurable and quantitative indexes are used less. This method is required to spend a long time and eventually the result is not satisfactory. In this paper, Data Envelopment Analysis (DEA) is employed to present a mathematical model as a precise and assured method for measuring the performance of the stations and also ranking them. In This systematic and comprehensive approach, every gas station is considered as a system with specified and quantitative inputs and outputs and then their efficiencies will be evaluated.

DEA is a well established methodology used to evaluate the relative efficiency of a set of comparable entities called decision making units (DMUs) with multiple inputs and outputs by some specific mathematical programming models [1,2]. DEA was introduced in 1978 when Charnes et al. [3] (CCR approach) demonstrated how to change a fractional linear measurement of efficiency into a linear programming format. Since the first DEA model developed, many other DEA models and applications have been developed and extended (see [4,5,6]). In energy and environmental studies, DEA has been widely applied to estimate the technical efficiency of energy industries [7,8], assessing energy efficiencies of different organizations [9,10] and measuring ecological efficiency [11,12]. DEA can be used to optimize the performance measure of each DMU. It calculates a maximal performance measure for each DMU relative to all DMUs in the firms under observation [13]. Assessment of bank branch performance [14], examining bank efficiency [15], measuring the efficiency of higher education institutions [16], solving facility layout design (FLD) problem [17] and measuring the efficiency of organizational investments in information technology [18] are examples of using DEA in various areas.

Data Envelopment Analysis assists decision makers to distinguish efficient and inefficient decision making units in a homogeneous group. Standard DEA models cannot provide more information about efficient units. Super-efficiency DEA models can be used in ranking the performance of efficient DMUs and overcome this obstacle [19]. Super-efficiency DEA model is obtained when a DMU under evaluation is excluded from the reference set of the original DEA model. This model was developed by Banker et al. [20] and Andersen and Petersen [21].

The rest of the paper is organized as follows: Section 2 describes the DEA methodology. Section 3 points out the application of DEA in evaluating 26 gas stations of oil company in two northern cities of Iran. Section 4 contains the conclusion.

2. DEA Methodology

DEA is based on a linear programming. This method measures the relative efficiency of operational units with multiple inputs and outputs. The principal advantage of the DEA technique is that it does not require the specification of a particular functional form for the technology. This non-parametric approach solves a linear programming (LP) formulation per DMU and the weights assigned to each DMU are the results of the corresponding LP. The original model developed by Charnes, Cooper and Rhodes (CCR model) was applicable when characterized by constant returns to scale(CRS). Imperfect competition may cause a DMU not to operate at optimal scale. Banker, Charnes and Cooper (BCC model, 1984) extended the CCR model to account for technologies that show variable returns to scale(VRS). The technical efficiency score (in both CRS and VRS models) equal one implies full efficiency. On the other hand, if the score is less than one it indicates technical inefficiency.

Suppose that there are n DMUs, DMU$_j$: $j = 1,\dots,n$, and the performance of each DMU is characterized by a production process of m inputs ($x_{ij} : i = 1,\dots,m$) to produce s outputs ($y_{rj} : r = 1,\dots,s$). Relative efficiency is defined as the ratio of weighted sum of outputs to the weighted sum of inputs. The efficiency measure for DMU$_o$ is defined as

$$e_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}, \tag{1}$$

Where the weights u_r and v_i are non-negative.

The efficiency of a specific DMU$_0$ Can be evaluated by the BCC model of DEA which is presented in multiplier form as follows:

$$\begin{aligned} Max \quad & \theta_0 = \sum_{r=1}^{s} u_r y_{r0} - u_0 \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_0 \le 0, \quad j = 1,\dots,n, \\ & \sum_{i=1}^{m} v_i x_{i0} = 1, \\ & v_i \ge \varepsilon,\ u_r \ge \varepsilon,\ for\ all\ i,r\ and\ u_0\ free\ in\ sign. \end{aligned} \tag{2}$$

The above formulations assume that $x_{ij}, y_{rj} \ge 0 \quad \forall i, j.$ All variables in (2) are also constrained to be non-negative except for u_0 which may be positive, negative or zero with consequences that make it possible to use optimal values of this variable to identify RTS. The term $\varepsilon > 0$ in the constraints of (2) is not a real number. It is, instead, a non-Archimedean infinitesimal which is smaller than any positive real number. The entire frontier DMUs (efficient DMUs) has $\theta_0 = 1$. In order to discriminate the performance of efficient DMUs, Andersen and Petersen [21] developed a procedure for ranking efficient units. Their methodology enables an extreme efficient unit o to achieve an efficiency score greater than one by removing the constraint corresponding to DMU$_o$ in (2) as shown in model (3):

$$\begin{aligned} Max \quad & \varphi_0 = \sum_{r=1}^{s} u_r y_{r0} - u_0 \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_0 \le 0, \quad j = 1,\dots,n, \quad j \ne 0. \\ & \sum_{i=1}^{m} v_i x_{i0} = 1, \\ & v_i \ge \varepsilon,\ u_r \ge \varepsilon,\ for\ all\ i,r\ and\ u_0\ free\ in\ sign. \end{aligned} \tag{3}$$

Let the optimal objective value to (3) be φ_0. For an efficient *DMU$_o$*, φ_0 is not less than unity and this value indicates *super-efficiency* of *DMU$_o$*.

Tone [24] has defined the slack based measure of super efficiency of *DMU$_o$* as the optimal objective function value δo of the following program:

$$\begin{aligned} Min \quad & \delta_0 = \frac{\frac{1}{m}\sum_{i=1}^{m} \frac{\bar{x}_i}{x_o}}{\frac{1}{s}\sum_{r=1}^{s} \frac{\bar{y}_r}{y_o}} \\ s.t. \quad & \sum_{\substack{j=1 \\ j \ne 0}}^{n} \lambda_j x_{ij} \le \bar{x}_i, \quad i = 1,\dots m, \\ & \sum_{\substack{j=1 \\ j \ne 0}}^{n} \lambda_j y_{rj} \ge \bar{y}_r, \quad r = 1,\dots s, \\ & \bar{x}_i \ge x_{io}, \quad i = 1,\dots m, \\ & 0 \le \bar{y}_r \le y_{ro}, \quad r = 1,\dots s, \\ & \lambda_j \ge 0, \quad j = 1,\dots n. \end{aligned} \tag{4}$$

δ_o is a weighted L$_1$ distance from (x_0, y_0) to the production possibility set spanned by (x_0, y_0), $j = 1,\dots,n,\ j \ne 0.$

3. Application of DEA in Gas Stations

In this section**,** DEA method is applied to evaluate the efficiency of 26 gas stations of two cities in the north of Iran. Data of the model have been derived from available documents in NIOPDC (National Iranian Oil Products Distribution Company). Seven variables from the data set as inputs and outputs have been used. Inputs include capacity of the tanks (x_1)(liter), number of nozzles (x_2), number of staff (x_3) and area (x_4)(m^2). The output variables are sold-out products (y_1)(this money variable is stated as current Iranian million Rials), Automatic power generator (y_2) and Automated teller machine (ATM)(y_3). The chosen input-output data used in the application are available over first and second periods of solar year, 1388 and they are displayed in Table 1. In this table, y_1^1 is the variable sold-out products for the first period of the year, y_1^2 is the variable sold-out products for the second period of the year, number 1 for two variables y_2 and y_3 shows the existence of the technology and number 0 shows non-existence. The problem is solved by using a BCC model and the super efficiency models of Andersen and Petersen [22], and SBM. The results are reported in Table 2 and Table 3.

Table 1. Input/output data of NIOPDC

Station	x_1	x_2	x_3	x_4	y_1^1	y_1^2	y_2	y_3
1	157000	6	8	2350	699913333	523717500	1	0
2	140000	7	9	1700	1006051250	817430833	1	1
3	61920	12	10	2400	1443842500	1161450000	0	1
4	106000	8	5	1000	833737500	583571250	1	1
5	225000	15	12	2600	1919315416	1521854166	1	0
6	185000	8	3	2000	11157430833	922810833	0	0
7	135000	12	11	1540	677804166	634358333	1	0
8	180000	10	9	2000	796336666	713984166	1	0
9	90000	6	5	1200	421283333	351310000	1	0
10	225000	21	12	1400	867883333	838262500	1	0
11	100000	6	5	500	520480833	442768333	0	0
12	187000	13	11	1348	1093674166	950575000	0	1
13	225000	10	8	1374	892810000	750239166	1	1
14	240000	12	6	1500	908190833	844565833	0	0
15	165000	14	7	2150	2023383333	1913783333	1	1
16	84000	3	4	1400	381770833	289864166	1	0
17	90000	32	12	1270	1014266666	490700000	1	1
18	225000	12	11	2300	1149968333	960133333	0	1
19	48500	7	6	2050	1331900000	1098833333	1	1
20	84000	5	4	300	299524166	256952500	1	0
21	180000	7	6	2150	510232500	377775000	1	0
22	135000	8	3	3800	335839166	305242500	0	0
23	98000	8	5	1300	518133333	435833333	0	0
24	90000	8	8	1300	905000000	777733333	1	1
25	135000	7	6	1200	441935000	433509166	1	0
26	135000	13	11	1978	1071883333	1004216666	0	1

In Table 2 and Table 3, the 2nd and 3rd columns report the optimal value to models (2) and (3). The BCC model indicates that 7 stations #4, #5, #6, #15, #17, #19, and #26 are full efficient in the first period and 11 stations #2, #4, #5, #6, #9, #12, #15, #17, #19, #24, and #26 are full efficient in the second period (see column 2 in Table 2 and Table 3). The forth column of each Table 2 and Table 3 reports the super-SBM measure of efficiency defined in (4). By the super efficiencies of the stations, in the 1st period, station #19 is the top-ranked station and the other 6 stations are ranked as 6> 26> 4> 5> 17> 15 and in the 2nd period, station #19 is the top-ranked station and the other 10 stations are ranked as 6> 15> 9> 26> 4> 5> 12> 2> 24>17. It is to be noted that based on the results reported in the third column in model (3) station #6 is the top-ranked followed by 4> 19> 15> 26> 5> 17 in Table 2 and station #9 is the top-ranked station in Table 3 followed by 19> 6> 15> 26> 5> 24> 4> 12> 2> 17. Consider a specific station, Say station #6. The super efficiency measures AP and SBM to this station are respectively 2.1733 and 1.9614. This station is the top-ranked station using the super efficiency model (3) proposed by Andersen and Petersen [22], whereas the top-ranked station in SBM methodology is station #19.

Table 2. Results for the first period

Station	E	SE-AP	SE-SBM
1	0.6195	-	-
2	0.6725	-	-
3	0.8613	-	-
4	1	2.1614(2)	1.4957(4)
5	1	1.4158(6)	1.0631(5)
6	1	2.1733 (1)	1.9614(2)
7	0.6817	-	-
8	0.7114	-	-
9	0.4	-	-
10	0.6668	-	-
11	0.4	-	-
12	0.9282	-	-
13	0.802	-	-
14	0.3444	-	-
15	1	1.7143(4)	1.0091(7)
16	0.5	-	-
17	1	1.0754(7)	1.0407(6)
18	0.6738	-	-
19	1	2.0669(3)	2.4038(1)
20	0.7872	-	-
21	0.4838	-	-
22	0.6715	-	-
23	0.4062	-	-
24	0.9973	-	-
25	0.453	-	-
26	1	1.6209(5)	1.7013(3)

Table 3. Results for the second period

Station	E	SE-AP	SE-SBM
1	0.6311	-	-
2	1	1.0444(10)	1.0387(9)
3	0.9278	-	-
4	1	1.4975(8)	1.4957(6)
5	1	1.7660(10)	1.2695(7)
6	1	2.2156(3)	1.901(2)
7	0.6117	-	-
8	0.9817	-	-
9	1	2.6495(1)	1.1769(4)
10	0.2359	-	-
11	0.4555	-	-
12	1	1.0772(9)	1.1163(8)
13	0.8549	-	-
14	0. 4389	-	-
15	1	1.8124(4)	1.82(3)
16	0.5354	-	-
17	1	1.0076(11)	1.0076(11)
18	0.6853	-	-
19	1	2.4042(2)	2.2928 (1)
20	0.5281	-	-
21	0.37	-	-
22	0.7185	-	-
23	0.4542	-	-
24	1	1.5090(7)	1.0268(10)
25	0.3782	-	-
26	1	1.7830(5)	1.7552(5)

4. Conclusion

There is a method in NIOPDC (National Oil Products Distribution Company) to evaluate the performance of gas stations and determine the efficient stations. This evaluation is performed every six month. In this paper, data envelopment analysis method has been applied to evaluate the relative efficiency of 26 gas stations of two Northern cities of Iran. Data of the model have been derived from available documents in NIOPDC. BCC model was used for evaluating the relative efficiency. In This approach, each gas station was considered as a system with specified and quantitative inputs and outputs. By using this method the efficient station and also inefficient stations have been identified. Super efficiency (AP-model) and slack based measure of super efficiency were used for ranking gas stations. In both first and second period, station #19 is the top-ranked station by SBM methodology whereas the top-ranked station in AP methodology is station #6 in the first period and station #9 in the second period. Both SBM and AP methodologies are applicable but as matter of fact, comparing with data in NIOPDC, SBM methodology is more accurate and reliable.

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