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Measuring Efficiency and Effectiveness for Non-Storable Commodities: A Mixed Separate Data Envelopment Analysis Spproaches with Real and Fuzzy Data

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Abstract Data Envelopment Analysis (DEA) is a technique for measuring the relative efficiency of Decision Making Units (DMUs) which produce similar products. Measures of both technical efficiency and service effectiveness for storable commodities are essentially the same. However, these measures for non-storable commodities, such as transport services, represent two distinct dimensions and a joint measurement of both or measurement with their impression mutual is necessary to fully capture the overall performance. In this paper, a Mixed Separate Data Envelopment Analysis (MSDEA) approach is introduced to analyze the overall performance of non-storable commodities. Then, the case of ten intercity car companies is described as the application of this novel approach. Moreover, when some observations are fuzzy, the efficiencies and effectiveness become fuzzy as well. For more extension, MSDEA approach with fuzzy observations called Fuzzy Mixed Separate Data Envelopment Analysis (FMSDEA) approach will be presented and illustrated with a numerical example.

Keywords: *Data Envelopment Analysis (DEA), efficiency, effectiveness, Mixed Separate DEA (MSDEA), fuzzy data*

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1. Introduction

Data Envelopment Analysis (DEA) is a technique for measuring the relative efficiency of Decision Making Units (DMUs) which produce similar products. Measures of both technical efficiency (a transformation of factors to production) and service effectiveness (consumption of production) for storable commodities are essentially the same because of the commodities, once produced, can be stockpiled until consumed. Nothing will be lost throughout the transformation from production to consumption if one assumes that all the stockpiles are eventually sold, there is no storage cost, and there is no loss incurred. Namely, conventional measures for storable commodities assume perfect sale and no storage cost effectiveness. However, technical efficiency and service effectiveness for non-storable commodities, such as transport services, represent two distinct measurements because one can never store the surplus service during periods of low demand (off peak hours) for use during periods of high demand (peak hours). When such non-storable commodities are produced and a portion of which are not concurrently consumed, the technical effectiveness (a joint effect of both technical efficiency and service effectiveness) would be less than the technical efficiency.

Over the past three decades, various DEA models have been widely used to evaluate the technical efficiency or technical effectiveness of DMUs in different organizations or industries. In transport performance evaluation, numerous applications of DEA have also been found in various fields.

In order to completely and fairly evaluate the relative performance of non-storable transport services, several recent works have employed various DEA approaches to evaluating the efficiency and effectiveness. In general, they can be divided into five categories: separate DEA model (hereinafter, SDEA; e.g. [1,2]), separate two-stage DEA model (hereinafter, STDEA; e.g. [3,4,5]), network DEA model (hereinafter, NDEA; e.g. [6,7,8]), integrated two-stage DEA model (hereinafter, ITDEA; e.g. [9,10,11]), and integrated DEA model (hereinafter, IDEA; e.g. [12]). The SDEA employs independent DEA models to measure technical efficiency, service effectiveness, and technical effectiveness separately. Hence, paradoxical improvement strategies were usually generated based on the results of these independent DEA models. To overcome this shortcoming, the STDEA uses an input-oriented DEA model to evaluate the technical efficiency and an output-oriented DEA model to assess the service effectiveness, holding the output level unchanged. Although the STDEA model will not generate conflicting improvement strategies, it suggests the organization be divided into two independent departments: production and sale, such that the performance of one department is not interrelated with that of the other department. This is of course not exactly true from the organizational perspective. The lack of interrelated performance among different departments may be solved by the NDEA, ITDEA or IDEA modeling. However, due to the complexity of the modeling, the scale economy and slack values for each DMU are hard to compute by the NDEA model, proposed by [6] and [7], which is only applicable to the case of constant returns to scale. The ITDEA model proposed by [11] can be applied to both technologies of constant and variable returns to scale, and the scale economy and slack values can easily be computed as well. However, for the ease of transforming the objective function into a linear form, the ITDEA model sets rather restricted weights proportional to the relative contributions of inputs, outputs and consumption in association with their corresponding virtue multipliers. This would lead to difficulties provided that the organization would value the weights differently across the departments. Strictly speaking, the weights should represent the relative importance of efficiency and effectiveness valued by the evaluator or the decision maker, and they should remain unchanged in evaluating all DMUs. To further rectify this shortcoming, [12] develops Integrated DEA (IDEA) models which jointly evaluate the non-storable commodities' efficiency and effectiveness. They extended the IDEA models to generalized IDEA models. However, the IDEA models are nonlinear and may have multiple optimal solutions. This non-uniqueness of input and output weights would damage ranking of DMUs with the same technical effectiveness whereas their technical efficiency and service effectiveness are different. To resolve this problem, a Mixed Separate DEA (hereinafter, MSDEA) models are suggested that evaluate the technical efficiency, service effectiveness and technical effectiveness separately whereas they are impression mutual.

Furthermore, when some observations are fuzzy, the efficiencies and effectiveness become fuzzy as well. For these cases, the MSDEA approach is extended to analyze the overall performance of non-storable commodities with fuzzy data. The MSDEA approach with fuzzy observations called Fuzzy Mixed Separate Data Envelopment Analysis (FMSDEA) approach.

The rest of the paper is organized as follows. The proposed MSDEA models are presented in Section 2. Section 3 will be described an application example for more illustration. The fuzzy data and the extended models for measuring the technical efficiency, service effectiveness and technical effectiveness [FMSDEA] are introduced in Section 4 and the final section will be the conclusion.

2. A Mixed Separate DEA Models

DEA is a method for measuring the relative efficiency of DMUs that perform similar tasks. A DEA model was developed by [13]. To measure the efficiency and effectiveness for non-storable commodities with avoidance of the above mentioned shortcomings, this paper proposes the MSDEA models under CRS and VRS technologies which are termed as Mixed Separate CCR (MSCCR) and Mixed Separate BCC (MSBCC) models. The formulation of the proposed MSDEA is given as follows.

2.1. Mixed Separated CCR Model for Evaluated the Technical Efficiency

The proposed MSCCR model [MSCCR-TE] aims to maximize the technical efficiency by solving for virtual multipliers corresponding to factor, production and consumption variables. The model is formulated as follow: Where TE_k represents the overall efficiency score of DMU_k. If TE_k quals to one, the DMU is defined

$$
\begin{aligned}
&(MSCCR-TE)\\
&\underset{u,v}{Max}\ \ TE_k=\frac{\sum_{r=1}^{R}u_r y_{kr}}{\sum_{j=1}^{J}v_j x_{kj}}\\
&s.t.\ \ \frac{\sum_{r=1}^{R}u_r y_{ir}}{\sum_{j=1}^{J}v_j x_{ij}}\le 1,\quad i=1,\ldots,I,\\
&\qquad \frac{\sum_{s=1}^{S}w_s z_{is}}{\sum_{r=1}^{R}u_r y_{ir}}\le 1,\quad i=1,\ldots,I,\\
&\qquad u_r\ge 0,\qquad r=1,\ldots,R,\\
&\qquad v_j\ge 0,\qquad j=1,\ldots,J,\\
&\qquad w_s\ge 0,\qquad s=1,\ldots,S.
\end{aligned}
\tag{1}
$$

relatively efficient; otherwise the DMU is relatively inefficient. x_{kj} represents the j^{th} input of DMU_k. y_{kr} denotes the r^{th} output of DMU_k. z_{ks} represents the s^{th} consumption of the DMU_k. The variables v_j, u_r and w_s are corresponding virtual multipliers of the j^{th} input, the r^{th} output and the s^{th} consumption. I, J, R, S are the number of $DMUs$, inputs, outputs and consumption, respectively.

Besides, the fractional program is not used for actual computation of the efficiency scores due to its non-convex and nonlinear properties. Hence, by using Charnes and Cooper ([14]) transformation, model (1) can be equivalently transformed into the linear program below for solution:

Let $u_r^*(r=1,\ldots,R), v_j^*(j=1,\ldots,J)$ and $w_s^*(s=1,\ldots,S)$ be the optimal solution to the above model. Then,

$$
\begin{aligned}
&(MSCCR-TE)\\
&\underset{u,v}{Max}\ \ TE_k=\sum_{r=1}^{R}u_r y_{kr}\\
&s.t.\ \ \sum_{j=1}^{J}v_j x_{kj}=1,\\
&\qquad \sum_{r=1}^{R}u_r y_{ir}-\sum_{j=1}^{J}v_j x_{ij}\le 0,\quad i=1,\ldots,I,\\
&\qquad \sum_{s=1}^{S}w_s z_{is}-\sum_{r=1}^{R}u_r y_{ir}\le 0,\quad i=1,\ldots,I,\\
&\qquad u_r\ge 0,\qquad r=1,\ldots,R,\\
&\qquad v_j\ge 0,\qquad j=1,\ldots,J,\\
&\qquad w_s\ge 0,\qquad s=1,\ldots,S.
\end{aligned}
\tag{2}
$$

$TE_k^*=\sum_{r=1}^{R}u_r.y_{kr}$ is referred to as a *CCR-Efficiency* of DMU_k.

2.2. Mixed Separated CCR Model for Evaluated the Service Effectiveness

The proposed mixed separate CCR model [MSCCR-SE] aims to maximize the service effectiveness by solving for virtual multipliers corresponding to factor, production, and consumption variables. The model is formulated as follow:

$$
\begin{aligned}
&(MSCCR-SE)\\
&\max_{u,w}\ SE_k=\frac{\sum_{s=1}^{S} w_s z_{ks}}{\sum_{r=1}^{R} u_r y_{kr}}\\
&s.t.\ \frac{\sum_{r=1}^{R} u_r y_{ir}}{\sum_{j=1}^{J} v_j x_{ij}}\le 1,\quad i=1,\dots,I,\\
&\qquad \frac{\sum_{s=1}^{S} w_s z_{is}}{\sum_{r=1}^{R} u_r y_{ir}}\le 1,\quad i=1,\dots,I,\\
&\qquad u_r\ge 0,\quad r=1,\dots,R,\\
&\qquad v_j\ge 0,\quad j=1,\dots,J,\\
&\qquad w_s\ge 0,\quad s=1,\dots,S.
\end{aligned}
\tag{3}
$$

Where SE_k represents the service effectiveness score of DMU_k. x_{kj} represents the j^{th} input of DMU_k. y_{kr} denotes the r^{th} output of DMU_k. z_{ks} represents the s^{th} consumption of the DMU_k. The variables v_j, u_r and w_s are corresponding virtual multipliers of the j^{th} input, the r^{th} output, and the s^{th} consumption, respectively. I, J, R, S are the number of DMUs, inputs, outputs and consumption, respectively. By using Charnes and Cooper ([14]) transformation, model (3) can be equivalently transformed into the linear program below for solution:

$$
\begin{aligned}
&(MSCCR-SE)\\
&\max_{u,w}\ SE_k=\sum_{s=1}^{S} w_s z_{ks}\\
&s.t.\ \sum_{r=1}^{R} u_r y_{kr}=1\\
&\qquad \sum_{r=1}^{R} u_r y_{ir}-\sum_{j=1}^{J} v_j x_{ij}\le 0,\quad i=1,\dots,I,\\
&\qquad \sum_{s=1}^{S} w_s z_{is}-\sum_{r=1}^{R} u_r y_{ir}\le 0,\quad i=1,\dots,I,\\
&\qquad u_r\ge 0,\quad r=1,\dots,R,\\
&\qquad v_j\ge 0,\quad j=1,\dots,J,\\
&\qquad w_s\ge 0,\quad s=1,\dots,S.
\end{aligned}
\tag{4}
$$

Let $u_r^*(r=1,\dots,R), v_j^*(j=1,\dots,J)$ and $w_s^*(s=1,\dots,S)$ be the optimal solution to the above model. Then, $SE_k^*=\sum_{s=1}^{S} w_s . z_{ks}$ is referred to as a *CCR-Service effectiveness* of DMU_k.

2.3. Mixed Separated CCR Model for Evaluated the Technical Effectiveness

The proposed mixed separate CCR model [MSCCR-TET] aims to maximize the technical effectiveness by solving for virtual multipliers corresponding to factor, production, and consumption variables. The model is formulated as follow:

$$
\begin{aligned}
&(MSDEA-TET)\\
&\max_{v,w}\ TET_k=\frac{\sum_{s=1}^{S} w_s z_{ks}}{\sum_{j=1}^{J} v_j x_{kj}}\\
&s.t.\ \frac{\sum_{r=1}^{R} u_r y_{ir}}{\sum_{j=1}^{J} v_j x_{ij}}\le 1,\quad i=1,\dots,I,\\
&\qquad \frac{\sum_{s=1}^{S} w_s z_{is}}{\sum_{r=1}^{R} u_r y_{ir}}\le 1,\quad i=1,\dots,I,\\
&\qquad u_r\ge 0,\quad r=1,\dots,R,\\
&\qquad v_j\ge 0,\quad j=1,\dots,J,\\
&\qquad w_s\ge 0,\quad s=1,\dots,S.
\end{aligned}
\tag{5}
$$

Where TET_k represents the technical effectiveness score of DMU_k. x_{kj} represents the j^{th} input of DMU_k. y_{kr} denotes the r^{th} output of DMU_k. z_{ks} represents the s^{th} consumption of the DMU_k. The variables v_j, u_r and w_s are corresponding to virtual multipliers of the j^{th} input, the r^{th} output and the s^{th} consumption, respectively. I, J, R, S are the number of DMUs, inputs, outputs and consumptions, respectively. By using Charnes and Cooper [14] transformation, model (5) can be equivalently transformed into the linear program below for solution:

$$
\begin{aligned}
&(MSDEA-TET)\\
&\max_{u,w}\ TET_k=\sum_{s=1}^{S} w_s z_{ks}\\
&s.t.\ \sum_{j=1}^{J} v_j x_{kj}=1,\\
&\qquad \sum_{r=1}^{R} u_r y_{ir}-\sum_{j=1}^{J} v_j x_{ij}\le 0,\quad i=1,\dots,I,\\
&\qquad \sum_{s=1}^{S} w_s z_{is}-\sum_{r=1}^{R} u_r y_{ir}\le 0,\quad i=1,\dots,I,\\
&\qquad u_r\ge 0,\quad r=1,\dots,R,\\
&\qquad v_j\ge 0,\quad j=1,\dots,J,\\
&\qquad w_s\ge 0,\quad s=1,\dots,S.
\end{aligned}
\tag{6}
$$

Let $u_r^*(r=1,\dots,R), v_j^*(j=1,\dots,J)$ and $w_s^*(s=1,\dots,S)$ be the optimal solution to the above model. Then, $TET_k^*=\sum_{s=1}^{S} w_s . z_{ks}$ is referred to as a *CCR-Technical effectiveness* of DMU_k.

2.4. Extension to Variable Returns to Scale (Mixed Separate BCC Models)

The above MSCCR models can be easily extended to a Mixed Separated BCC models [MSBCC] by simply adding the convexity constraint, which is expressed as:

$$
\begin{aligned}
&(MSBCC-TE)\\
&\max_{u,v}\ TE_k=\frac{\sum_{r=1}^{R} u_r y_{kr}-u_0}{\sum_{j=1}^{J} v_j x_{kj}}\\
&s.t.\ \frac{\sum_{r=1}^{R} u_r y_{ir}-u_0}{\sum_{j=1}^{J} v_j x_{ij}}\le 1,\quad i=1,\dots,I,
\end{aligned}
$$

$$\frac{\sum_{s=1}^{S} w_s z_{is} - w_0}{\sum_{r=1}^{R} u_r y_{ir} - u_0} \le 1, \quad i=1,\ldots,I,$$
$$u_r \ge 0, \quad r=1,\ldots,R, \tag{7}$$
$$v_j \ge 0, \quad j=1,\ldots,J,$$
$$w_s \ge 0, \quad s=1,\ldots,S.$$

By using Charnes and Cooper [14] transformation, the linear program can be written as follow:

$$(MSBCC-TE)$$
$$\max_{u,v}\ TE_k = \sum_{r=1}^{R} u_r y_{kr} - u_0$$
$$s.t.\ \sum_{j=1}^{J} v_j x_{kj} = 1,$$
$$\sum_{r=1}^{R} u_r y_{ir} - \sum_{j=1}^{J} v_j x_{ij} - u_0 \le 0,\ i=1,\ldots,I,$$
$$\sum_{s=1}^{S} w_s z_{is} - \sum_{r=1}^{R} u_r y_{ir} - w_0 + u_0 \le 0,\ i=1,\ldots,I,$$
$$u_r \ge 0, \quad r=1,\ldots,R,$$
$$v_j \ge 0, \quad j=1,\ldots,J,$$
$$w_s \ge 0, \quad s=1,\ldots,S. \tag{8}$$

Similarly, Model (3) and (4) under VRS technology can be rewritten as model (9) and (10), respectively.

$$(MSBCC-SE)$$
$$\max_{u,w}\ SE_k = \frac{\sum_{s=1}^{S} w_s z_{ks} - w_0}{\sum_{r=1}^{R} u_r y_{kr} - u_0}$$
$$s.t.\ \frac{\sum_{r=1}^{R} u_r y_{ir} - u_0}{\sum_{j=1}^{J} v_j x_{ij}} \le 1, \quad i=1,\ldots,I,$$
$$\frac{\sum_{s=1}^{S} w_s z_{is} - w_0}{\sum_{r=1}^{R} u_r y_{ir} - u_0} \le 1, \quad i=1,\ldots,I, \tag{9}$$
$$u_r \ge 0, \quad r=1,\ldots,R,$$
$$v_j \ge 0, \quad j=1,\ldots,J,$$
$$w_s \ge 0, \quad s=1,\ldots,S.$$

And

$$(MSBCC-SE)$$
$$\max_{u,w}\ SE_k = \sum_{s=1}^{S} w_s z_{ks} - w_0$$
$$s.t.\ \sum_{r=1}^{R} u_r y_{kr} - u_0 = 1$$
$$\sum_{r=1}^{R} u_r y_{ir} - \sum_{j=1}^{J} v_j x_{ij} - u_0 \le 0,$$
$$i=1,\ldots,I, \tag{10}$$
$$\sum_{s=1}^{S} w_s z_{is} - \sum_{r=1}^{R} u_r y_{ir} - w_0 + u_0 \le 0,$$
$$i=1,\ldots,I,$$
$$u_r \ge 0, \quad r=1,\ldots,R,$$
$$v_j \ge 0, \quad j=1,\ldots,J,$$
$$w_s \ge 0, \quad s=1,\ldots,S.$$

Also, the Mixed Separate BCC model [MSBCC-TET] is formulated in (11).

$$(MSBCC-TET)$$
$$\max_{v,w}\ TET_k = \frac{\sum_{s=1}^{S} w_s z_{ks} - w_0}{\sum_{j=1}^{J} v_j x_{kj}}$$
$$s.t.\ \frac{\sum_{r=1}^{R} u_r y_{ir} - u_0}{\sum_{j=1}^{J} v_j x_{ij}} \le 1, \quad i=1,\ldots,I,$$
$$\frac{\sum_{s=1}^{S} w_s z_{is} - w_0}{\sum_{r=1}^{R} u_r y_{ir} - u_0} \le 1, \quad i=1,\ldots,I, \tag{11}$$
$$u_r \ge 0, \quad r=1,\ldots,R,$$
$$v_j \ge 0, \quad j=1,\ldots,J,$$
$$w_s \ge 0, \quad s=1,\ldots,S.$$

The linear form can be reformulated as follow:

$$(MSBCC-TET)$$
$$\max_{u,w}\ TET_k = \sum_{s=1}^{S} w_s z_{ks} - w_0$$
$$s.t.\ \sum_{j=1}^{J} v_j x_{kj} = 1,$$
$$\sum_{r=1}^{R} u_r y_{ir} - \sum_{j=1}^{J} v_j x_{ij} - u_0 \le 0,$$
$$i=1,\ldots,I, \tag{12}$$
$$\sum_{s=1}^{S} w_s z_{is} - \sum_{r=1}^{R} u_r y_{ir} - w_0 + u_0 \le 0,$$
$$i=1,\ldots,I,$$
$$u_r \ge 0, \quad r=1,\ldots,R,$$
$$v_j \ge 0, \quad j=1,\ldots,J,$$
$$w_s \ge 0, \quad s=1,\ldots,S.$$

3. Application

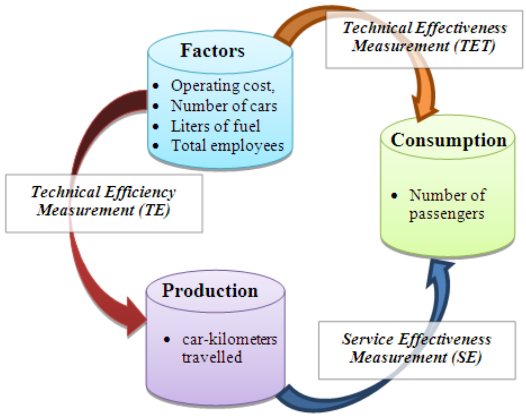

Figure 1. Distinctive performance measurements for car transport service

There are several intercity car companies in Iran. We take 10 of these car companies as our case analysis. Potential variables of four factor variables (Operating cost, Number of cars, Liters of fuel, Total employees), one production variable (car-kilometers travelled) and one consumption variable (Number of passengers) are considered. These variables have been shown in Figure 1 with their relations and the data have been appeared in Table 1.

Table 1. Data set

DMU	Operating cost (X_1)	Number of cars (X_2)	Liters of fuel (X_3)	Total employees (X_4)	Car-kilometers travelled (Y)	Passengers (Z)
1	69257	148	36900	584	11158	5400
2	96201	252	64000	832	22981	6666
3	32769	85	17980	226	6131	2202
4	22450	78	20000	204	6101	1222
5	11170	32	3020	86	2408	696
6	2126	8	1500	17	869	134
7	2537	12	2100	28	858	148
8	732985	30	6040	69	2100	554
9	10995	55	9201	116	1398	518
10	1501	7	1470	11	1200	133

The columns of Table 2 report the CRS results from the MSCCR models (model (2), (4) and (6)) and their ranks (in parentheses). Columns 2 to 4 of Table 2 report the CCR Technical Efficiency ($TE_k^{(CCR)}$), Service Effectiveness ($SE_k^{(CCR)}$) and Technical Effectiveness ($TET_k^{(CCR)}$) of each companies, respectively. It can be clearly seen that only the last DMU (DMU 10) was CCR efficient across all of the DMUs. The lowest technical efficiency belongs to DMU 2 over 0.0001. Similarly, according to columns 4 of Table 2, DMU 10 has been inscribed the best Technical Effectiveness score by seizing the amount of 0.9 for its technical effectiveness score, while, DMU 1 has been recorded the worst technical effectiveness over 0.0012.

Table 2. Technical efficiency, service effectiveness, technical effectiveness and their ranks (in parentheses) of the MSDEA models for the 10 car companies in Iran

DMU	MSCCR		
	$TE_k^{(CCR)}$	**$SE_k^{(CCR)}$**	**$TET_k^{(CCR)}$**
1	0.0002 (9)	0.81 (2)	0.0012 (10)
2	0.0001 (10)	0.21 (7)	0.004 (5)
3	0.0006 (6)	0.54 (3)	0.0030 (7)
4	0.001 (5)	0.16 (9)	0.0018 (9)
5	0.01 (4)	0.49 (4)	0.045 (4)
6	0.19 (2)	0.18 (8)	0.3 (2)
7	0.04 (3)	0.24 (6)	0.1 (3)
8	0.0005 (7)	0.44 (5)	0.0025 (8)
9	0.0003 (8)	1.00 (1)	0.0035 (6)
10	1.00 (1)	0.09 (10)	0.9 (1)

In contrast to these measures, the bottom of the service effectiveness score has been inscribed for DMU 10 and the highest score (= 1.00) recorded by DMU 9 and introduced this DMU as the service effective company.

Table 3. Technical efficiency, service effectiveness, technical effectiveness and their ranks (in parentheses) of the MSDEA models for the 10 car companies in Iran

DMU	**MSBCC**		
	$TE_k^{(BCC)}$	**$SE_k^{(BCC)}$**	**$TET_k^{(BCC)}$**
1	0.13 (5)	1.00 (1)	0.13 (3)
2	1.00 (1)	0.000001 (10)	0.000001 (10)
3	0.08 (6)	0.35 (6)	0.027 (5)
4	0.15 (4)	0.02 (9)	0.003 (7)
5	0.07 (7)	0.24 (7)	0.017 (6)
6	0.2 (3)	0.9 (4)	0.18 (2)
7	0.04 (8)	1.00 (1)	0.04 (4)
8	0.003 (9)	0.17 (8)	0.0005 (9)
9	0.0006 (10)	1.00 (1)	0.0006 (8)
10	1.00 (1)	0.4 (5)	0.4 (1)

Similar pattern was recorded in the technical efficiency and technical effectiveness under VRS technology for DMU 10. Table 3, by solving model (8), (10) and (12), indicates that two DMUs (DMU 2 and 10) were technical efficient and DMU 9 has been recorded the worst inefficiency score over $TE_9^{(BCC)} = 6\times10^{-4}$.

In the case of the service effectiveness, three DMUs (DMU 1, 7 and 9) have been reached the maximum rate of the service effectiveness (= 1.00), whereas, the worst score was recorded for DMU 2 in this category.

DMU 10 hit a peak of over 0.4 on its technical effectiveness which followed by DMU 6 with a decrease of more than 50% on technical effectiveness ($TET_6^{(BCC)} = 0.18$). Finally, 10^{-6} was the lowest score of technical effectiveness which belongs to DMU 2.

To sum up, it can be clearly seen that DMU 10 was the best efficient and technical effective DMU under CRS and VRS technology both. Although its service effectiveness under both technologies was extremely weak ($SE_{10}^{(CCR)} = 0.09 and SE_{10}^{(BCC)} = 0.4$), DMU 10 kept its first place as well.

On the other hand, DMU 9 and DMU 1 were the best two services effective amongst all companies under both technologies, respectively. In the case of variable returns to scale, both of them were services effective by reaching 1.00 in their service effectiveness scores. While, DMU 9 was the only service effective company under constant returns to scale following by DMU 1 which has had an amount of 0.81 on its service effectiveness measurement.

4. Fuzzy Data and Fuzzy Mixed Separated DEA Approaches

4.1. Fuzzy Mixed Separated CCR Models

In a set of DMUs, suppose that the factors $\tilde{x}_{ji}$, productions $\tilde{y}_{jr}$ and consumptions $\tilde{z}_{js}$ are approximately known and can be represented by convex fuzzy numbers with membership functions $\mu_{\tilde{x}_{ji}}, \mu_{\tilde{y}_{jr}}$ and $\mu_{\tilde{z}_{js}}$, respectively. In the fuzzy environment, Model (1) becomes:

$$\begin{aligned}
&\underset{u,v}{Max}\ \tilde{TE}_k = \frac{\sum_{r=1}^{R} u_r \tilde{y}_{kr}}{\sum_{i=1}^{I} v_i \tilde{x}_{ki}} \\
&s.t. \\
&\sum_{r=1}^{R} u_r \tilde{y}_{jr} \le \sum_{i=1}^{I} v_i \tilde{x}_{ji}, \qquad j = 1,\ldots,J, \\
&\sum_{s=1}^{S} w_s \tilde{z}_{js} \le \sum_{r=1}^{R} u_r \tilde{y}_{jr}, \qquad j = 1,\ldots,J, \\
&u_r \ge 0, \qquad r = 1,\ldots,R, \\
&v_i \ge 0, \qquad i = 1,\ldots,I, \\
&w_s \ge 0, \qquad s = 1,\ldots,S.
\end{aligned} \tag{13}$$

Let $S(\tilde{X}_{ji})$, $S(\tilde{Y}_{jr})$ and $S(\tilde{Z}_{js})$ denote the support of $\tilde{X}_{ji}$, $\tilde{Y}_{jr}$ and $\tilde{Z}_{js}$; respectively. The α-cuts, also known as the α-level sets of $\tilde{X}_{ji}$, $\tilde{Y}_{jr}$ and $\tilde{Z}_{js}$ are defined as:

$$\forall j,i;\ \ (X_{ji})_\alpha = \left\{ x_{ji} \in S(\tilde{X}_{ji}) \middle| \mu_{\tilde{X}_{ji}}(x_{ji}) \ge \alpha \right\}, \tag{14a}$$

$$\forall j,r;\ \ (Y_{jr})_\alpha = \left\{ y_{jr} \in S(\tilde{Y}_{jr}) \middle| \mu_{\tilde{Y}_{jr}}(y_{jr}) \ge \alpha \right\}, \tag{14b}$$

$$\forall j,s;\ \ (Z_{js})_\alpha = \left\{ z_{js} \in S(\tilde{Z}_{js}) \middle| \mu_{\tilde{Z}_{js}}(z_{js}) \ge \alpha \right\}, \tag{14c}$$

They can also be expressed in another form:

$$\begin{aligned}
(X_{ji})_\alpha &= [(X_{ji})_\alpha^L, (X_{ji})_\alpha^U] \\
&= [\min_{x_{ji}} \left\{ x_{ji} \in X_{ji} \middle| \mu_{\tilde{X}_{ji}}(x_{ji}) \ge \alpha \right\}, \\
&\quad \max_{x_{ji}} \left\{ x_{ji} \in X_{ji} \middle| \mu_{\tilde{X}_{ji}}(x_{ji}) \ge \alpha \right\}],
\end{aligned} \tag{15a}$$

$$\begin{aligned}
(Y_{jr})_\alpha &= [(Y_{jr})_\alpha^L, (Y_{jr})_\alpha^U] \\
&= [\min_{y_{jr}} \left\{ y_{jr} \in Y_{jr} \middle| \mu_{\tilde{Y}_{jr}}(y_{jr}) \ge \alpha \right\}, \\
&\quad \max_{y_{jr}} \left\{ y_{jr} \in Y_{jr} \middle| \mu_{\tilde{Y}_{jr}}(y_{jr}) \ge \alpha \right\}],
\end{aligned} \tag{15b}$$

$$\begin{aligned}
(Z_{js})_\alpha &= [(Z_{js})_\alpha^L, (Z_{js})_\alpha^U] \\
&= [\min_{z_{js}} \left\{ z_{js} \in Z_{js} \middle| \mu_{\tilde{Z}_{js}}(z_{js}) \ge \alpha \right\}, \\
&\quad \max_{z_{js}} \left\{ z_{js} \in Z_{js} \middle| \mu_{\tilde{Z}_{js}}(z_{js}) \ge \alpha \right\}],
\end{aligned} \tag{15c}$$

Based on Zadeh's extension principle ([15,16,17]), the membership functions of $\tilde{TE}_k$, $\tilde{SE}_k$ and $\tilde{TET}_k$ can be defined as:

$$\begin{aligned}
\mu_{\tilde{TE}_k}(t) = \sup_{x,y}\ \min\{&\mu_{\tilde{X}_{ji}}(x_{ji}), \mu_{\tilde{Y}_{jr}}(y_{jr}), \\
&\forall i,j,r \,|\, t = TE_k(x,y)\},
\end{aligned} \tag{16}$$

$$\begin{aligned}
\mu_{\tilde{SE}_k}(t) = \sup_{y,z}\ \min\{&\mu_{\tilde{Y}_{jr}}(y_{jr}), \mu_{\tilde{Z}_{js}}(z_{js}), \\
&\forall s,j,r \,|\, t = SE_k(y,z)\},
\end{aligned} \tag{17}$$

$$\begin{aligned}
\mu_{\tilde{TET}_k}(t) = \sup_{x,z}\ \min\{&\mu_{\tilde{X}_{ji}}(x_{ji}), \mu_{\tilde{Z}_{js}}(z_{js}), \\
&\forall s,j,r \,|\, t = TET_k(x,z)\},
\end{aligned} \tag{18}$$

Where $TE_k(x,y)$, $SE_k(y,z)$ and $TET_k(x,z)$ represent the technical efficiency, service effectiveness and technical effectiveness scores of DMU_k. To find the membership function $\mu_{\tilde{TE}_k}$, it suffices to find the lower and upper bounds of the α-cut of $\tilde{TE}_k$, which based on equation (16) and [18], can be solved as:

$$\begin{aligned}
&\underset{u,v}{Max}\ (\tilde{TE}_k{}^{(CCR)})_\alpha^L = \frac{\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L}{\sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U} \\
&s.t. \\
&\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U,
\end{aligned}$$

$$
\begin{aligned}
&\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U,\\
&\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^L,\\
&j=1,\dots,J,\ j\ne k,\\
&\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^L \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L,\\
&j=1,\dots,J,\ j\ne k,\\
&u_r \ge \varepsilon > 0, \quad r=1,\dots,R,\\
&v_i \ge \varepsilon > 0, \quad i=1,\dots,I,\\
&w_s \ge \varepsilon > 0, \quad s=1,\dots,S.
\end{aligned} \tag{19a}
$$

And

$$
\begin{aligned}
&\underset{u,v}{Max}\ \ (T\tilde{E}_k{}^{(CCR)})_\alpha^U = \frac{\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U}{\sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L}\\
&s.t.\\
&\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L,\\
&\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L,\\
&\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^U,\\
&j=1,\dots,J,\ j\ne k,\\
&\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^L \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U,\\
&j=1,\dots,J,\ j\ne k,\\
&u_r \ge \varepsilon > 0, \quad r=1,\dots,R,\\
&v_i \ge \varepsilon > 0, \quad i=1,\dots,I,\\
&w_s \ge \varepsilon > 0, \quad s=1,\dots,S.
\end{aligned} \tag{19b}
$$

Model (19) is the conventional DEA model, which can be solved by transforming to linear programs ([14]) and utilizing any linear programming solver. The α-level set of $T\tilde{E}_k(x,y)$ is constructed from (19) as $T\tilde{E}_k = [(TE_k)_\alpha^L, (TE_k)_\alpha^U]$ and the membership function of $T\tilde{E}_k(x,y)$ is constructed from $(TE_k)_\alpha$ at different α values $(\alpha \in [0,1])$.

Similarly, by using equation (17) the similar models can be written for measuring $S\tilde{E}_k(y,z)$ as follows:

$$
\begin{aligned}
&\underset{u,w}{Max}\ \ (S\tilde{E}_k{}^{(CCR)})_\alpha^L = \frac{\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L}{\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U}\\
&s.t.\\
&\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U,\\
&\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U,\\
&\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^L,\\
&j=1,\dots,J,\ j\ne k,\\
&\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^U \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L,\\
&j=1,\dots,J,\ j\ne k,\\
&u_r \ge \varepsilon > 0, \quad r=1,\dots,R,\\
&v_i \ge \varepsilon > 0, \quad i=1,\dots,I,\\
&w_s \ge \varepsilon > 0, \quad s=1,\dots,S.
\end{aligned} \tag{20a}
$$

And

$$
\begin{aligned}
&\underset{u,w}{Max}\ \ (S\tilde{E}_k{}^{(CCR)})_\alpha^U = \frac{\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U}{\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L}\\
&s.t.\\
&\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L,\\
&\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L,\\
&\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^U,\\
&j=1,\dots,J,\ j\ne k,\\
&\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^L \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U,\\
&j=1,\dots,J,\ j\ne k,\\
&u_r \ge \varepsilon > 0, \quad r=1,\dots,R,\\
&v_i \ge \varepsilon > 0, \quad i=1,\dots,I,\\
&w_s \ge \varepsilon > 0, \quad s=1,\dots,S.
\end{aligned} \tag{20b}
$$

Also by using equation (18) the following models can be written for measuring technical effectiveness:

$$
\begin{aligned}
&\underset{v,w}{Max}\ \ (T\tilde{E}T_k{}^{(CCR)})_\alpha^L = \frac{\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L}{\sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U}\\
&s.t.\\
&\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U,\\
&\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U,\\
&\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^L,\\
&j=1,\dots,J,\ j\ne k,\\
&\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^U \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L,\\
&j=1,\dots,J,\ j\ne k,\\
&u_r \ge \varepsilon > 0, \quad r=1,\dots,R,\\
&v_i \ge \varepsilon > 0, \quad i=1,\dots,I,\\
&w_s \ge \varepsilon > 0, \quad s=1,\dots,S.
\end{aligned} \tag{21a}
$$

And

$$
\begin{aligned}
&\underset{v,w}{Max}\ \ (T\tilde{E}T_k{}^{(CCR)})_\alpha^U = \frac{\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U}{\sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L}\\
&s.t.\\
&\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L,\\
&\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L,\\
&\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^U,\\
&j=1,\dots,J,\ j\ne k,
\end{aligned}
$$

$$\begin{aligned}
&\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^L \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U,\\
&j=1,\ldots,J,\ j\ne k,\\
&u_r \ge \varepsilon > 0, \qquad r=1,\ldots,R,\\
&v_i \ge \varepsilon > 0, \qquad i=1,\ldots,I,\\
&w_s \ge \varepsilon > 0, \qquad s=1,\ldots,S.
\end{aligned} \tag{21b}$$

To illustrate how the proposed method is applied to find fuzzy Technical Efficiency, fuzzy Service Effectiveness and fuzzy Technical Effectiveness measures, consider three DMUs with one factor variable (X), one production variable (Y) and one consumption variable (Z). All data have been shown in Table 4. At a specific possibility level α, the lower bound of the α-cut of the membership function of the Technical Efficiency, according to model (19a), can be solved as the following model.

$$\begin{aligned}
&\underset{v,w}{Max}\ \ (T\tilde{E}_A{}^{(CCR)})_\alpha^L = \frac{23u}{(7-2\alpha)v}\\
&s.t.\\
&23u \le (7-2\alpha)v,\\
&7w \le 23u,\\
&(26-2\alpha)u \le 5v,\\
&(28-3\alpha)u \le (1+2\alpha)v,\\
&(6-2\alpha)w \le (22+\alpha)u\\
&(9-2\alpha)w \le (15+7\alpha)u\\
&u,v,w \ge \varepsilon > 0,
\end{aligned} \tag{22}$$

With the same constraints of model (22), the lower bound of the membership functions of Service Effectiveness and Technical Effectiveness can be obtained by solving the similar models which their objective's functions can be reformulated as $Max(S\tilde{E}_A)_\alpha^L = \frac{7w}{23u}$ and $Max(T\tilde{E}T_A)_\alpha^L = \frac{7w}{(7-2\alpha)v}$, respectively.

Table 4. Factor variable, production variable and consumption variable of three DMUs

DMU	A	B	C
X	(2,5,7)	5	(1,3,6,11)
α-cut	[2+3α, 7-2α]	[5, 5]	[1+2α, 11-5α]
Y	23	(22,23,25,26)	(15,22,25,28)
α-cut	[23,23]	[22+α,26-2α]	[15+7α, 28-3α]
Z	7	(3,4,6)	(4,5,7,9)
α-cut	[7,7]	[3+α, 6-2α]	[4+α, 9-2α]

And the upper bound of $\mu_{T\tilde{E}_A}$, can be obtained by solving the following model:

$$\begin{aligned}
&\underset{u,v}{Max}\ \ (T\tilde{E}_A{}^{(CCR)})_\alpha^U = \frac{23u}{(2+3\alpha)v}\\
&s.t.\\
&23u \le (2+3\alpha)v,\\
&7w \le 23u,\\
&(22+\alpha)u \le 5v,\\
&(15+7\alpha)u \le (11-5\alpha)v,\\
&(3+\alpha)w \le (26-2\alpha)u\\
&(4+\alpha)w \le (28-3\alpha)u\\
&u,v,w \ge \varepsilon > 0,
\end{aligned} \tag{23}$$

With the same constraints of model (23), the upper bound of the membership functions of Service Effectiveness and Technical Effectiveness can be reached by solving the similar models with differences in their objective functions which they can be reformulated as $Max(S\tilde{E}_A)_\alpha^U = \frac{7w}{23u}$ and $Max(T\tilde{E}T_A)_\alpha^U = \frac{7w}{(2+3\alpha)v}$, respectively.

Similarly, The lower and upper bounds of the α-cuts of $\mu_{T\tilde{E}_B}, \mu_{T\tilde{E}_C}, \mu_{S\tilde{E}_B}, \mu_{S\tilde{E}_C}, \mu_{T\tilde{E}T_B}$ and $\mu_{T\tilde{E}T_C}$ can be reached. Table 5-Table 7 present the α-cuts of technical efficiency, service effectiveness and technical effectiveness at five values of α: 0, 0.25, 0.5, 0.75 and 1.

Figure 2 indicates the membership functions of $\mu_{T\tilde{E}_A}, \mu_{T\tilde{E}_B}, \mu_{T\tilde{E}_C}, \mu_{S\tilde{E}_A}, \mu_{S\tilde{E}_B}$, and $\mu_{S\tilde{E}_C}$. The membership functions of the technical effectiveness of A, B and C were shown in Figure 3.

It can be clearly seen that, there is no direct corresponds between the membership functions of the efficiency and effectiveness measures and the observations. For instant, the production variable (Y) and the consumption variable (Z) of DMU A and the factor variable (X) of DMU B were crisp while the rest of variables were fuzzy.

Table 5. The α-cuts of technical efficiency at five α values

α	0.0	0.25	0.50	0.75	1.0
$(T\tilde{E}_A)_\alpha^L$	0.11784	0.1956	0.2897	0.406	0.552
$(T\tilde{E}_A)_\alpha^U$	1	1	1	1	1
$(T\tilde{E}_B)_\alpha^L$	0.1562	0.245	0.3396	0.4417	0.552
$(T\tilde{E}_B)_\alpha^U$	1	1	1	1	1
$(T\tilde{E}_C)_\alpha^L$	0.1186	0.2054	0.3312	0.5161	0.7639
$(T\tilde{E}_C)_\alpha^U$	1	1	1	1	1

Table 6. The α-cuts of service effectiveness at five α values

α	0.0	0.25	0.50	0.75	1.0
$(S\tilde{E}_A)_\alpha^L$	0.5075	0.6	0.7031	0.8217	1
$(S\tilde{E}_A)_\alpha^U$	1	1	1	1	1
$(S\tilde{E}_B)_\alpha^L$	0.1923	0.2512	0.3238	0.4133	0.5238
$(S\tilde{E}_B)_\alpha^U$	1.3442	1.2552	1.1683	1.0832	1
$(S\tilde{E}_C)_\alpha^L$	0.4694	0.5125	0.558	0.6061	0.6571
$(S\tilde{E}_C)_\alpha^U$	1	1	1	1	1

Table 7. The α-cuts of technical effectiveness at five α values

α	0.0	0.25	0.50	0.75	1.0
$(T\tilde{E}T_A)_\alpha^L$	0.06	0.1168	0.2	0.33	0.5278
$(T\tilde{E}T_A)_\alpha^U$	1	1	1	1	1
$(T\tilde{E}T_B)_\alpha^L$	0.0357	0.0705	0.1222	0.1966	0.2263
$(T\tilde{E}T_B)_\alpha^U$	1.1374	1.0952	1.0514	1.0058	0.9583
$(T\tilde{E}T_C)_\alpha^L$	0.1039	0.1712	0.2647	0.3978	0.5704
$(T\tilde{E}T_C)_\alpha^U$	0.5357	0.6147	0.6981	0.7864	0-88

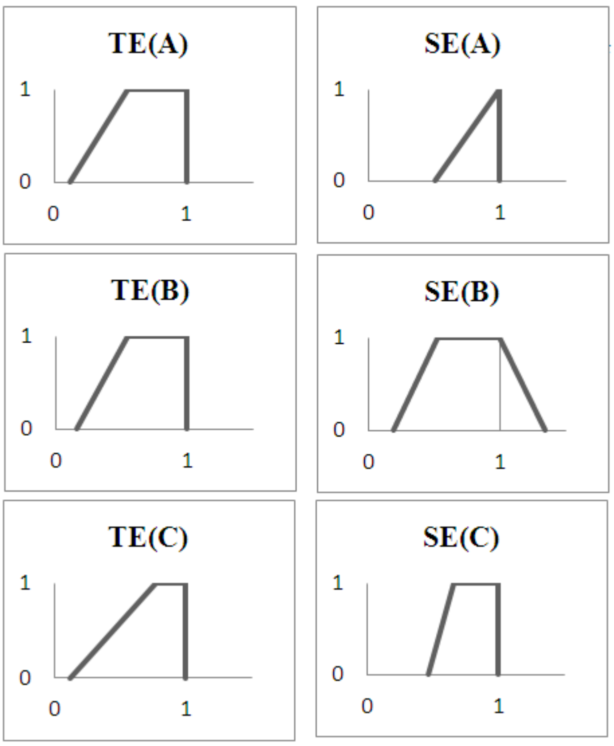

Figure 2. Membership functions of technical efficiency and service effectiveness of DMU A, B and C ($\mu_{T\tilde{E}_B}$, $\mu_{T\tilde{E}_C}$, $\mu_{S\tilde{E}_B}$, $\mu_{S\tilde{E}_C}$, $\mu_{T\tilde{E}T_B}$ and $\mu_{T\tilde{E}T_C}$)

As shown in Figure 2 and Figure 3, all measurements of the membership functions of the technical efficiency, service effectiveness and technical effectiveness of these three DMUs were trapezoidal fuzzy number with the exception of the membership function of the service effectiveness of DMU A which it was a triangular fuzzy number.

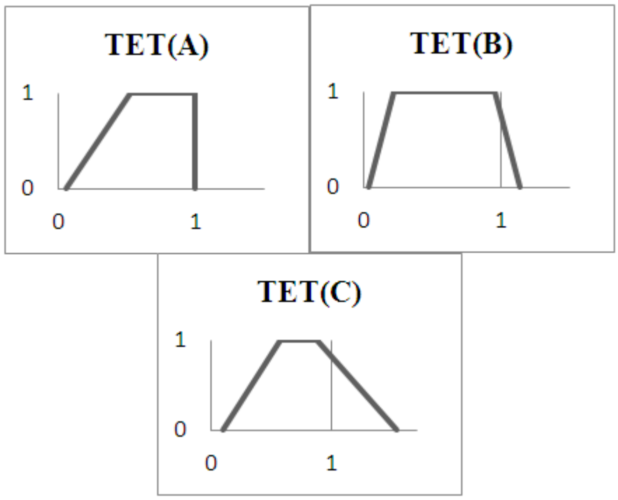

Figure 3. The membership function of technical effectiveness of DMU A, B and C ($\mu_{T\tilde{E}T_A}$, $\mu_{T\tilde{E}T_B}$ and $\mu_{T\tilde{E}T_C}$)

4.2. Fuzzy Mixed Separated BBC Models

The above FMSCCR models can be easily extended to a Fuzzy Mixed Separated BCC models [FMSBCC] by simply adding the convexity constraint, which is expressed as:

$$\underset{u,v}{Max}\ (T\tilde{E}_k{}^{(BCC)})_\alpha^L = \frac{\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L - u_0}{\sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U}$$

s.t.

$$\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U,$$

$$\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L - w_0 \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U - u_0,$$

$$\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^L,$$
$$j = 1,\ldots,J,\ j \ne k,$$
$$\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^U - w_0 \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L - u_0,$$
$$j = 1,\ldots,J,\ j \ne k, \qquad (24a)$$
$$u_r \ge \varepsilon > 0, \quad r = 1,\ldots,R,$$
$$v_i \ge \varepsilon > 0, \quad i = 1,\ldots,I,$$
$$w_s \ge \varepsilon > 0, \quad s = 1,\ldots,S.$$

And

$$\underset{u,v}{Max}\ (T\tilde{E}_k{}^{(BCC)})_\alpha^U = \frac{\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U - u_0}{\sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L}$$

$s.t.$

$$\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L,$$
$$\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U - w_0 \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L - u_0,$$
$$\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^U,$$
$$j = 1,\ldots,J,\ j \ne k, \qquad (24b)$$
$$\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^L - w_0 \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U - u_0,$$
$$j = 1,\ldots,J,\ j \ne k,$$
$$u_r \ge \varepsilon > 0, \quad r = 1,\ldots,R,$$
$$v_i \ge \varepsilon > 0, \quad i = 1,\ldots,I,$$
$$w_s \ge \varepsilon > 0, \quad s = 1,\ldots,S.$$

Similarly, Model (20) under VRS technology can be rewritten as model (25), respectively.

$$\underset{u,w}{Max}\ (S\tilde{E}_k{}^{(BCC)})_\alpha^L = \frac{\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L - w_0}{\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U - u_0}$$

$s.t.$

$$\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U,$$
$$\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L - w_0 \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U - u_0,$$
$$\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^L,$$
$$j = 1,\ldots,J,\ j \ne k, \qquad (25a)$$
$$\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^U - w_0 \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L - u_0,$$
$$j = 1,\ldots,J,\ j \ne k,$$
$$u_r \ge \varepsilon > 0, \quad r = 1,\ldots,R,$$
$$v_i \ge \varepsilon > 0, \quad i = 1,\ldots,I,$$
$$w_s \ge \varepsilon > 0, \quad s = 1,\ldots,S.$$

And

$$\underset{u,w}{Max}\ (S\tilde{E}_k{}^{(BCC)})_\alpha^U = \frac{\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U - w_0}{\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L - u_0}$$

$s.t.$

$$\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L,$$
$$\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U - w_0 \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L - u_0,$$
$$\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^U,$$
$$j = 1,\ldots,J,\ j \ne k, \qquad (25b)$$
$$\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^L - w_0 \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U - u_0,$$
$$j = 1,\ldots,J,\ j \ne k,$$
$$u_r \ge \varepsilon > 0, \quad r = 1,\ldots,R,$$
$$v_i \ge \varepsilon > 0, \quad i = 1,\ldots,I,$$
$$w_s \ge \varepsilon > 0, \quad s = 1,\ldots,S.$$

Also, the Fuzzy Mixed Separate BCC model [FMSBCC-TET] is formulated in (26).

$$\underset{v,w}{Max}\ (T\tilde{E}T_k{}^{(BCC)})_\alpha^L = \frac{\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L - w_0}{\sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U - u_0}$$

$s.t.$

$$\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^U,$$
$$\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^L - w_0 \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U - u_0,$$
$$\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^L,$$
$$j = 1,\ldots,J,\ j \ne k, \qquad (26a)$$
$$\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^U - w_0 \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L - u_0,$$
$$j = 1,\ldots,J,\ j \ne k,$$
$$u_r \ge \varepsilon > 0, \quad r = 1,\ldots,R,$$
$$v_i \ge \varepsilon > 0, \quad i = 1,\ldots,I,$$
$$w_s \ge \varepsilon > 0, \quad s = 1,\ldots,S.$$

And

$$\underset{v,w}{Max}\ (T\tilde{E}T_k{}^{(BCC)})_\alpha^U = \frac{\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U - w_0}{\sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L - u_0}$$

$s.t.$

$$\sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^U - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ki}\right)_\alpha^L,$$
$$\sum_{s=1}^{S} w_s \left(z_{ks}\right)_\alpha^U - w_0 \le \sum_{r=1}^{R} u_r \left(y_{kr}\right)_\alpha^L - u_0,$$
$$\sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^L - u_0 \le \sum_{i=1}^{I} v_i \left(x_{ji}\right)_\alpha^U,$$
$$j = 1,\ldots,J,\ j \ne k, \qquad (26b)$$
$$\sum_{s=1}^{S} w_s \left(z_{js}\right)_\alpha^L - w_0 \le \sum_{r=1}^{R} u_r \left(y_{jr}\right)_\alpha^U - u_0,$$
$$j = 1,\ldots,J,\ j \ne k,$$
$$u_r \ge \varepsilon > 0, \quad r = 1,\ldots,R,$$
$$v_i \ge \varepsilon > 0, \quad i = 1,\ldots,I,$$
$$w_s \ge \varepsilon > 0, \quad s = 1,\ldots,S.$$

5. Conclusion

This paper proposes a Mixed Separate Data Envelopment Analysis (MSDEA) approach, to measure the overall performance for non-storable commodities under both constant and variable returns to scale technologies from two aspects: technical efficiency and service effectiveness. The linearization problem from previous papers will be solved by using the proposed approach. An application of ten intercity car companies provided more illustrations. Furthermore, In transportation cases, when some observations are fuzzy, the efficiencies and effectiveness become fuzzy as well. For these cases, the MSDEA approach is extended to analyze the overall performance of non-storable commodities with fuzzy data. The MSDEA approach with fuzzy observations called Fuzzy Mixed Separate Data Envelopment (FMSDEA) approach which explanation with an example.

Other types of inaccuracy like as ordinal, qualitative or probabilistic data could be identified as future studies. Moreover, the measurement of the overall efficiency with two units, production (technical efficiency) and sale (service effectiveness) were indicated in this paper. The extension of MSDEA models to evaluate the overall performance of systems with more than two series and/or parallel units (Network systems) is a challenging issue (e.g., the supply chain systems, the railway and subway train transport and hubs, airlines etc.).

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