

# The Motion in a Central Field under Considering Space-time Expansion---- A Ideal Reclaim of Hoyle's Steady Universe

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**Abstract** In this paper, discusses the motion in a central gravitational field under considering space-time expansion, and realize the unification of structure of big scope space-time and physical phenomena of small scope, and set up a new mechanism of the formation and evolution of galaxies and celestial bodies, which is different from the mechanism given by big bang and is based on the continual generation of matter inside celestial bodies or galaxies. Point out that galaxies and celestial bodies are growing up in the course of spacetime's expansion.

**Keywords:** Newton's gravitational theory, central field, big bang, convex lens effect, continuous generation of matter

**Cite This Article:** Jian liang Yang, "The Motion in a Central Field under Considering Space-time Expansion---- A Ideal Reclaim of Hoyle's Steady Universe." *Frontiers of Astronomy, Astrophysics and Cosmology*, vol. 3, no. 1 (2017): 4-8. doi: 10.12691/faac-3-1-2.

## 1. Introduction

In the early 20th century Hubble found that distant galaxies are going far away from us, and the farther galaxies, the larger speed, this phenomenon is explained as universal expansion or space-time expansion. However, it is obviously not sensible only to understand space-time expansion for galaxies to leave far away from one another, that is to say, inside galaxies or say in small scope there should also be the corresponding response, which is the inevitable requirement of space-time continuity. In fact, with scientific experiments and observations developed deeper and deeper, the effect of space-time expansion has already been found in small scope, for example, the earth is found going away from the sun, and the earth itself is expanding too, the speed of motion of the arms of Milky Way galaxy is increasing, after considering tide reaction the moon has still other movement leaving the earth, as well found a variety of crafts to work to deviate the prediction of classic theory, and so on some problems for current physical theory not to able to explain. The paper begins with generalizing Newton's theory of central gravitational field to expanding space-time, and gradually deepens to the global property of space-time expansion and reveals the connection between large scope space-time structure and small scope physical reality, and systematically understand these new questions and phenomena observed recently, which have great physical significance.

## 2. The Overview of Newton's Theory in a Central Gravitational Field

So-called central motion refers to that a less object (mass  $m$ ) revolves round a larger one (mass  $M$ ), and the larger object may be looked as motionless. For such central motion the track of object is cone curve, in the polar coordinate system ( $r, \theta$ ), the Newton's classical track equation is given by [1]

$$r = \frac{l^2}{GMm^2(1 + e \cos \theta)} \quad (1)$$

Here  $e = \sqrt{1 + \frac{2El^2}{G^2M^2m^3}}$ , refers to centrifugal rate of the cone curve, and  $l$  stands for angle momentum of moving object,  $E$  stands for its mechanical energy, both  $l$  and  $E$  are conserved.

For  $e < 1$ ,  $E = -\frac{GmM}{2a}$ , the curve is ellipse, and  $a$  stands for its half of longer axis. For  $E = 0$ ,  $e = 1$ , corresponding curve is parabola. For  $E = \frac{GmM}{2a}$ ,  $e > 1$ , the curve is hyperbola,  $a$  stands for the half distance between two vertexes. The differential equations of dynamics are [1]

$$\frac{dr}{dt} = \pm \sqrt{\frac{2}{m} \left[ E - V(r) - \frac{l^2}{2mr^2} \right]} \quad (2)$$

$$\frac{d\theta}{dt} = \frac{l}{mr^2} \quad (3)$$

Here  $V(r) = -\frac{GMm}{r}$  is the potential energy of moving object. In Newton theory frame both  $m$  and  $M$  are constant. About equation (1), (2) and (3) readers may refer to any textbooks of theoretical mechanics

### 3. Generalize Newton's Theory to the Expanding Space-time

The paper is a continuation of reference [2] and [3] published previously by present author, and there galaxies and celestial bodies were proven to be bigger and bigger in the course of the expansion of universe, and universal density and pressure are constant all the time, new matter generates continuously inside galaxies or celestial bodies, the mass of any galaxy or celestial body changes with time  $t$  to meet [2,3]

$$\frac{m(t_1)}{m(t_2)} = \frac{R^3(t_1)}{R^3(t_2)} \quad (4)$$

which is the result of the connection of general relativity and cosmological principle. Here  $R(t)$  is the expansion factor of universe, which were already solved in reference [2] or [3]. And equation (4) reflects the connection of locality and whole and describes how the mass of a galaxy or celestial body changes with time, and  $t_1, t_2$  belong to two arbitrary moments.

Notice that Newton's theory is the approximation of low speed of general relativity, and general saying, celestial bodies' speed are not very high, therefore the generalization of Newton's theory is just the result of general relativity and the practical significance is apparent.

Now set out to generalize Newton gravitation to expansion space-time. And may as well take ellipse motion for example, Newton classical equation reads [1,2,3,4,5].

$$\frac{4\pi^2 a^3}{T^2} = GM \quad (5)$$

$T$  is the moving period of object, namely the time that object revolves round the center for  $2\pi$  angle,  $M$  is the mass of the central body,  $a$  is the half length of ellipse's long axis.

Now consider the expansion of space-time, namely take  $M$  for variable with respect to time  $t$  and meets equation (4), and differentiate equation (5), we attain

$$\frac{da}{dt} = aH + \frac{2a}{3T} \frac{dT}{dt} \quad (6)$$

Here  $H = \frac{dR}{Rdt}$ , which is just Hubble constant and shows the speed of universal expansion and is actually a

variable of time, and saying it for constant is based on the fact any galaxy going far away from us to meet the same Hubble law  $v = Hd$ ,  $d$  is the distance of a galaxy from us.

The later term of the left of equation (6) stands for so-called tide effect, which does not matter with universal expansion, the former stands for the result of universe expansion. In fact, as  $a \rightarrow \infty$ , equation (6) must return to the usual Hubble law, this requires  $dT = 0$ , therefore it is reasonable to explain the last term of (6) for tide reaction.

Equation (6) indicates that only considering space-time expansion the points on the elliptic curve are going far away from the center and meet Hubble law, the ellipse becomes bigger and bigger while the speed of moving object increases, however the period  $T$  is invariable and has no thing to do with space-time expansion.

Further generalize the above result to large scope, the global picture of universal expansion turns up: not only the space between galaxies enlarges all the time but also galaxies themselves enlarge at the same proportion, but the period of revolution or rotation of galaxies or celestial bodies keep invariable. The such situation of universal expansion is similar to we look to the sky at night to use a magnifying glass, all things including the space magnify meanwhile. Thus we say that space-time expansion possess convex lens effect, which is the certain requirement of universal isotropic and is the fundamental mechanism of formation and evolution of galaxies or celestial bodies. Obviously the expansion factor  $R(t)$  amounts to magnification.

The convex lens effect requires the radius of any celestial body or galaxy changes with the expansion of universe to meet

$$\frac{r(t_1)}{r(t_2)} = \frac{R(t_1)}{R(t_2)} \quad (7)$$

which decides, connected with equation (4), the density of any galaxy or celestial body invariable. And why the period of the rotation of galaxy or celestial body is too invariant like revolution might so understand: any galaxy or celestial body can be divided into innumerable small parts and every part might be looked as a object revolving round the center of galaxy or celestial body.

Astronomical observation shows that in early universe large galaxies existed ever, evidently this fact could not be explained by the theory of conventional big bang, and but indeed does not contradict with the present conclusion.

Now take the earth for example to demonstrate the expansion of celestial bodies. Take today's Hubble constant  $H_0 = 75 \text{ km.s}^{-1} . \text{Mpc}^{-1} = 0.76 \times 10^{-10} / \text{yr}$ , the subscript "0", represents today, and space-time expansion requires the radius of the earth to increase and to meet  $r \propto R(t)$ , which means  $dr = Hr dt$ , thus in a year today the radius of the earth will increase by  $\Delta r_0 = H_0 r_0 \Delta t = 0.48 \text{ mm}$ , here  $\Delta t$  for 1 year, and correspondingly according to equation (4) its mass changes up with time to meet  $dm = 3Hm dt$ , so in a year today the mass of the earth increases  $\Delta m_0 = 3H_0 m_0 \Delta t = 13 \times 10^{14} \text{ kg}$ , which means vacuum today contributes energy to the earth for

$\Delta E_0 = \Delta m_0 c^2 = 11.7 \times 10^{31} J$  a year, obviously it is the remarkable energy that makes the earth inside engender a variety of changes and motions including earthquake and volcano burst, and in reality the break of the crust caused by earthquake is just the performance of the expansion of the earth but not the effect of plate collision, as a whole after the earth evolved long time there should be no relative motion in the interior of the earth, it is merely a imagination for the plates to impact. The expansion of every part of the earth in the same time makes seas enlarge and continents also enlarge, look as if continents are drifting far away. Ever, by means of comparing ancient geographic map with today's one physicists concluded that the earth's radius increases by 0.5mm a year, which is amazingly in accordance with the above result. At present, there are already a lot of observed evidences about the expansion of the earth [6-13]. Besides, the increase of the distance between the earth and the sun is calculated as 11m, and the radius of the sun increases by 0.5m a year.

Because any celestial body's radius changes to meet  $r \propto R(t)$ , referring to equation (4) the density (average) of celestial body is easily proved to be invariable in the course of its expansion. And therefore the gravity acceleration  $g$  of surface of celestial body gradually increases with their expansion. In fact, the surface's  $g = GM/r^2 = 4\pi G\rho r/3$ ,  $\rho = const$ , and thus  $dg = Hgdt$ , namely

$$\frac{g(t_1)}{g(t_2)} = \frac{R(t_1)}{R(t_2)} \quad (8)$$

Equation (8) verify the fact that modern geological researches found that ancient most animals were larger than today's ones, for example then dragonflies were as large as today's hawks, that is to say gravity acceleration then was less than today's that or they would be collapse by their own weight. And about the mechanism of generation of interior matter inside celestial bodies refer to [3]. The following discussion neglects the tide's effect for the moment, only take account of space-time expansion

The convex lens effect of space-time expansion requires the angle speed  $\omega = \omega(\theta, t)$  of revolving object round a central body to meet

$$\omega(\theta, t_1) = \omega(\theta, t_2) \quad (9)$$

which makes sure its moving period invariable, namely  $dT = 0$ . Referring to equation (1) after considering the effect of space-time expansion the track equation of moving object becomes

$$r = r(\theta, t) = \frac{l^2}{GMm^2(1 + e \cos \theta)} \quad (10)$$

Notice that now  $l$ ,  $m$ ,  $M$  are variables with respect to time  $t$ , that is to say,  $m$ ,  $M$  meet equation (4), and  $l = m\omega r^2$ . Easily prove the centrifugal rate

$$e = \frac{c}{a} = \sqrt{1 + \frac{2El^2}{G^2 M^2 m^3}}$$
 is invariable ye. In fact, write the

half long axis  $a = k_1 R(t)$ , the half short axis  $b = k_2 R(t)$ ,  $k_1, k_2$  are constants [5], then we have

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{k_1^2 - k_2^2}}{k_1} = \text{constant, which indicates}$$

the ellipse's figure not to change though it is expanding. However the difference of long axis and short axis is increasing with time because of  $a - b = (k_1 - k_2)R(t)$ .

And if time  $t$  takes different parameter values, equation (10) obviously represents a series of concentric ellipses. And from equation (10) we have

$$\frac{r(\theta, t_1)}{r(\theta, t_2)} = \frac{R(t_1)}{R(t_2)} \quad (11)$$

correspondingly the speed  $v = v(\theta, t)$  of moving object meets

$$\frac{v(\theta, t_1)}{v(\theta, t_2)} = \frac{R(t_1)}{R(t_2)} \quad (12)$$

And we see  $\frac{\partial v(\theta, t)}{\partial t} = v(\theta, t)H(t)$ , which indicates the moving object has the tangential acceleration, this is completely the effect of space-time expansion rather than exist real force. This fact demonstrates the recent observation that the speed of the Milky Way arm revolving is becoming faster and faster, which could not be understood by conventional conception. It is easy to prove equation (12), as follows. Since  $E = -\frac{GMm}{2a} = -\frac{GMm}{r(\theta, t)} + \frac{mv^2(\theta, t)}{2}$ , we have

$$\begin{aligned} \frac{v^2(\theta, t)}{2} &= -GM\left(\frac{1}{2a} - \frac{1}{r(\theta, t)}\right) \\ &= -Gk_3 R^3(t)\left(\frac{1}{2k_1 R(t)} - \frac{1}{k_2 R(t)}\right), \end{aligned}$$

as a result equation (12) holds. Here  $k_1, k_2, k_3$  are three constants. Also we see that some conservation laws including energy conserved law no longer strictly exist, they are merely the approximation in small time or under no considering space-time expansion. So far we say (7) ~ (12) are qualified enough to describe the motion in a central gravitational field and not only are used to deal with elliptic motion. As for the mechanism of new matter generation inside celestial body, readers might refer to reference [3].

Below, through several specific example that decide the speed and position of moving object at arbitrary time interpret their meanings and applications in practice.

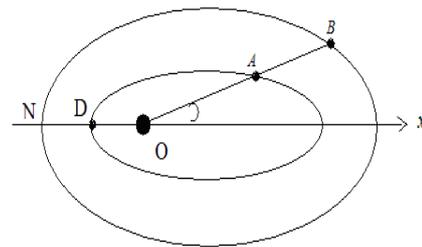


Figure 1. sketch map of a moving particle on different ellipses at different time

**Example 1:** refer to Figure 1. Assume time  $t_1$  object is at point D, speed is  $v_D$ , and if not considering space-time expansion its track is the interior small ellipse, now decide the position and speed of time  $t_2$  under considering the expansion.

Solving: at time  $t_2$  point D arrives at point N to follow Hubble law, the real position of moving object time  $t_2$  is sure on the ellipse that passes point N, may as well think at point B, and join point O (center body position) to point B, then point A is the position that object should arrive at according to classical theory at time  $t_2$ , the speed  $v_A$  is easily solved using classical theory

$$\text{Since } ON = OD \frac{R(t_2)}{R(t_1)}, \quad OB = OA \frac{R(t_2)}{R(t_1)}, \quad \text{here } OD,$$

$v_D$  already known.

From equation (2) we have

$$t_2 - t_1 = \int_{OD}^{OA} \frac{dr}{\sqrt{\frac{2}{m} \left[ E + \frac{GMm}{r} - \frac{l^2}{2mr^2} \right]}}$$

so OA can be solved, notice that here  $M, m, l$  take the values of time  $t_1$ , and are treated as constant in the course of integral. And according to

$$\frac{1}{2}mv_D^2 - \frac{GMm}{r_D} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A},$$

we can solve  $v_A$ , here  $r_D = OD$ . Finally using

$$v_B = v_A \frac{R(t_2)}{R(t_1)}, \quad \text{we can decide the real speed } v_B \text{ of time } t_2.$$

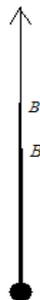
**Example 2.** Vertically upward projectile motion. Refer to figure 2, assume a object is thrown up straightly at time  $t_1$  and speed  $v_1$  from the ground, try to decide the speed and position at time  $t_2$ .

Solving: once considering space-time expansion, the real position that object arrive at is point  $B'$  but not point B which the object should arrive at according to classical theory at time  $t_2$ .

According to classical method we have  $v_B = v_1 - gt_2$ ,

$$r_B = v_1 t_2 - \frac{1}{2}gt_2^2, \quad \text{so the real position } r_2 \text{ and speed } v_2$$

$$\text{are given by } r_2 = \frac{r_B R(t_2)}{R(t_1)}, \quad v_2 = v_B \frac{R(t_2)}{R(t_1)}$$



**Figure 2.** Sketch map of motion orbit of a particle in expanding space-time

This results show that the real position and speed are higher than that of the prediction of classical theory, and the real speed to return to the ground is bigger than the its initial thrown speed, this is also beyond the prediction of classical theory.

**Example 3.** Investigate the change of distance of perigee or apogee of moon owe to space-time expansion, and by the way estimate afresh the change rate of rotation period of the earth.

Solving: today, the distance of perigee of the moon is  $d_1 = 36.3 \times 10^4 km$ , and according to Hubble law in a year the distance increases by  $\Delta d_1 = H_0 d_1 \Delta t = 27.588 mm$ , here  $\Delta t$  takes 1 year, and on the other hand, the distance of apogee is  $d_2 = 40.6 \times 10^4 km$ , for alike reason the distance of apogee increases in a year by  $\Delta d_2 = H_0 d_2 \Delta t = 30.85 mm$ , thus the increase of the difference of distances of apogee and perigee owe to space-time expansion is  $\Delta d_2 - \Delta d_1 = 3.27 mm$  a year, correspondingly the half long axis increases by  $\frac{\Delta d_2 + \Delta d_1}{2} = 29.21 mm$ , and the current observational

result is 3.8cm [14], which means tide make the half long axis increase only 0.88cm a year today, this conclusion is reasonable. If attribute entirely the change of 3.8cm to tide, through careful calculation James Williams found the change of eccentricity of lunar orbit 3 times bigger than observation [15]. Notice that space-time expansion does not change the eccentricity.

Now calculate the change of rotation period of the earth. First remember that space-time does not change rotation or revolution periods of celestial bodies or galaxies and only tide make the change.

The half long axis is  $a = \frac{d_2 + d_1}{2} = 38.4 \times 10^4 km$ . the moon recedes from the earth a year  $\Delta a = 3.8 cm$  observed, the period of the moon revolution  $T = 27 days$ , and using equation (6) the decrease of the period a year today is

$$\Delta T = \frac{3T(\Delta a - H_0 a \Delta t)}{2a} = 0.000081 s. \quad \text{On the other hand,}$$

tide makes the moon go far away from the earth  $\Delta a' = \Delta a - H_0 a \Delta t = 0.88 cm$  a year, so the increase of the angle momentum of the moon's revolution, caused by the

tide, is given by  $\Delta l = \frac{1}{2} \sqrt{\frac{GM_e m^2}{a}} \Delta a'$ , and the decrease of angle momentum of the earth's rotation owe to tide for

$$\Delta l' = \frac{0.66 \pi M_e r_e^2}{T^2} \Delta T', \quad \text{here } T' \text{ is the period of the earth's}$$

rotation today,  $r_e$  stands for the radius of the earth today for 6400km, according to angle momentum conserved law we have  $\Delta l' = \Delta l$ , so work out  $\Delta T' = 5.4 \times 10^{-11} s$ .

The above result show that the influence of the tide is quite small and may be neglected generally saying, space-time expansion is the main dynamic for universe to evolve. Obviously it is a big mistake for people to think such common movement far away from a center caused by the tide, in reality only when the scale of celestial body is not

too small compared with the distance between them the tide is worthy to be thought.

## 4. Conclusions

While space enlarges celestial bodies and galaxies themselves also enlarge at the same proportion, new matter continuously generates inside celestial bodies or galaxies. Some conserved laws, such as energy conserved law, angular momentum conserved law and moment conserved law, are the approximation of small time and small scope, and in big time and big scope they are no longer strictly valid. The earthquakes and the drift phenomenon of continents are exactly the performance of the earth is growing up, and new matter namely energy constantly generates and accumulates within celestial bodies including the earth all the time, as a result of which the earth's radium and mass increase  $0.48\text{mm}$  and  $13 \times 10^{14}\text{kg}$  respectively a year today. And the distances between the earth and the sun is increasing for  $11\text{m}$  a year today.

## Acknowledges

The paper was finished in He Nan Province Huai Yang Xian Lu Tai, October 24 2015, author is grateful to friends, Niu Buo Wen, Wang Chao Qing, who gave many helps and encouragements to author. Besides, author is too very grateful to the anonymous reviewers for giving some beneficial advice.

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