

# Embedding Proof-Writing in Phenomenon-based Learning to Promote Students' Mathematical Creativity

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**Abstract** The study aimed to identify the significant impact of phenomenon-based learning and embedding proof writing to this new teaching strategy on students' mathematical creativity in tertiary education. The study used a quasi-experimental pretest-posttest non-equivalent control group design. There were 2 experimental groups, one exposed to phenomenon-based learning alone and the other was exposed to phenomenon-based learning with classroom proof-writing activities, and one control group exposed to the conventional method. Multiple solution tasks were used to measure students' level of creativity. The data were analyzed using mean, standard deviation, and ANCOVA. The analysis revealed that students exposed to phenomenon-based learning with proof writing had the highest posttest creativity score. Students in this group had a significantly higher level of creativity among the three groups. Further, students exposed to phenomenon-based learning alone may not outperform students exposed to phenomenon-based learning with proof writing, but they still had significantly higher posttest mean scores compared to students exposed to the conventional type of teaching. The researchers recommend to teachers in tertiary education to use phenomenon-based learning with proof writing in teaching mathematics and its related courses since this instructional approach engages students in learning that is more focused to real-life issues, apply skills and knowledge from different subjects, and enhance important skills like creativity, problem-solving, communication, and teamwork.

**Keywords:** *phenomenon-based learning, proof writing, creativity, mathematics*

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## 1. Introduction

The coming anticipated new industrial revolution requires experienced skills needed for the job in the future. Students are required to have stronger mathematical knowledge, skills, and values to pursue higher education, to compete and be part of the technologically oriented workforce, and to be informed citizens. Nevertheless, fact shows that students' ability and creativity toward mathematics are still low and there is a downward trend of them that increases in later years [1,2].

However, previous researches claimed that mathematical talent is frequently measured by speed and accuracy of a student's computational skills with little emphasis on problem-solving and pattern finding and no opportunities for learners to work on rich mathematical tasks that need out-of-the-box thinking. Such an approach limits the use of creativity in the classroom and reduces mathematics to a set of skills to master and rules to memorize [3]. A considerable amount of literature concentrates on mathematical creativity at the primary and secondary levels [4].

However, a thorough study of research at the tertiary or post-secondary level revealed minimal discussion of how students are creative or how creativity can be promoted in undergraduate courses. Given that the students in tertiary courses are the next generation of mathematicians, engineers, or math educators, developing their mathematical creativity is essential [5].

Students who engaged in contextual or real-life mathematical situations can stimulate their mathematical creativity [6]. With the main ideology of incorporating real-life events into school discussions, phenomenon-based learning has received wide media coverage and publicity [7]. Phenomenon-based learning (PhBL) invites students to relate learnings to the real-world context and learn through understanding what they are doing. This focuses on the learners' learning rather than the teacher's teaching [8].

Moreover, in the Philippines, no study has been conducted utilizing phenomenon-based learning which has originated in Finland. In fact, in mathematics education, both the school or university level often fails to equip students with a way of how their learnings in mathematics can be applied and observed on daily living [9].

Further, in proving tasks, which highlights the role of an individual's problem-solving process emphasizes the development of creativity by establishing the foundations for creativity in novel situations. During the process, a student is encouraged to explore concepts, create new ideas, and evaluate those attempts to ultimately create a valid proof. Thus, proving can inform both teachers and students regarding the progression that a student is making in developing his/her mathematical creativity [5].

Thus, these scenarios have prompted the researcher to study the possible factors or variables that may influence the growth of students' creativity in mathematics courses. The researcher highly considered phenomenon-based learning and proof-writing as possible variables that may influence the increase of mathematical creativity and support that phenomena have traditionally been a missing piece in mathematics education [10].

The dependent variable of the study is students' mathematical creativity which has four components, namely fluency, flexibility, originality, and elaboration. Fluency refers to the ability of the students to numerous responses accurately and efficiently. Flexibility refers to students' ability to change thinking paths when encountering a thinking obstruction. Originality refers to students' ability to find a solution path that is especially unique and uncommon among the group. Lastly, elaboration refers to students' ability to create a detailed plan and generalize ideas.

The main aim of the study is to determine the significant impact of embedding proof writing in phenomenon-based learning on students' mathematical creativity. Specifically, the study intends to answer the following:

1. What is the pretest and posttest profile of the respondents in terms of the level of mathematical creativity as they are exposed to:

- 1.1 Conventional Method

- 1.2 Phenomenon-based learning

- 1.3 Phenomenon-based learning with proof-writing.

2. How does the level of mathematical creativity of the students compare as influenced by the conventional method, phenomenon-based learning, and phenomenon-based learning with proof-writing?

### 1.1. Hypothesis

There is no significant difference between the mathematical creativity scores of the students as influenced by the conventional method, phenomenon-based learning, and phenomenon-based learning with proof writing.

## 2. Methods

The study used a quasi-experimental design utilizing a pretest-posttest non-equivalent control group. The study was conducted in Amable M. Aguiluz Computer College (AMACC) in Davao City. The study involved freshmen Information Technology and Engineering students who were officially enrolled in the MATH611 - Differential Calculus course for the third trimester of the school year

2019-2020. Due to a small number of population, universal sampling was used. There were 75 Information Technology students and 28 Engineering students. Thus, a total of 103 freshmen students participated in the study.

There were three groups of students, one control group, and two experimental groups. The control group (CG) was taught using the conventional method. The first experimental group (EG1) was exposed to phenomenon-based learning alone and the second experimental group (EG2) was exposed to phenomenon-based learning with proof writing. This particular group was also encouraged to justify every step of their solutions or answers in any assessment or activities as what the teacher had been doing during discussions. Both the control group and experimental group 1 were also taught the proofs of the derivative rules, however, it is by the use of a conventional method unlike with the experimental group 2, who were required to prove the derivative rules on their own. All of these three groups were taught by the researcher to minimize the effect of the teacher factor as an intervening variable that may affect the results of the study. Likewise, the three sections were taught using the same reference book.

The researcher used the Mathematical Creativity Test (MCT) which consists of six open-ended questions in the study that could have multiple solutions. Originally, MCT was developed by Kattou, Christou, & Pitta-Pantazi [11] and translated by Kroesbergen, & Schoevers [12]. Slight revisions were made to fit the current setting of the study.

The maximum administration time of MCT was 45 minutes and students were required to provide multiple solutions. Scores were obtained for fluency, flexibility, originality, and elaboration using the adapted rating scale. To get the fluency score, the number of correct solutions was counted and divided by the maximum number of correct mathematical solutions provided by one of the students from the sample. To calculate the flexibility score, the number of different types or categories of correct solutions was counted per question and divided by the maximum number of different types of solutions provided by a student in the sample of the study. The ratio of the score in fluency and flexibility were then multiplied to 100. Further, originality was calculated by comparing a student's solution with the solutions provided by all students who participated in the study. The originality score ranged between 0 - 100, was given for each question. The rarest correct solution received the highest score: a student was given the score 100 for originality if one or more of his/her answers appeared in less than 1% of the sample's answers. A score of 80, 60, 40, or 20 was given if the frequency of one or more of his/her answers appeared respectively in between 1% and 5%, 6% and 10%, 11%, and 20%, and more than 20% of the sample's answers. Elaboration score ranged from 1 - 5, a student was given the highest score if he can produce a detailed plan and generalized ideas or give in-depth reasoning behind a solution path.

To obtain uniformity, all the scores in fluency, flexibility, and originality were transformed into a 5-point Likert scale. Scores with equivalents from 0% - 20% were

coded as 1, 21% - 40% were coded as 2, 41% - 60% were coded as 3, 61% - 80% were coded as 4 and 81% - 100% were coded as 5. Then, fluency, flexibility, originality, and elaboration scores of each question were added and in this way, the score for mathematical creativity has been obtained. Thus, a perfect score for mathematical creativity is 20 points.

The researcher administered a pretest of MCT to determine the students' level of mathematical creativity upon entering the study. The students were pretested during the first week of classes. To avoid the Hawthorne effect, the students were not informed that they were participants of the study. Right after the pretest, experimental groups were given the treatment.

Students in experimental groups were given a list of phenomena. These phenomena were selected by the researcher and mostly focused on the phenomena observed or experienced in Davao City. The day after their pretest, students in each experimental group were grouped into 6 groups. Each group was asked to choose a phenomenon on the list provided that they want to focus on the whole trimester (3 months). These groups of students were asked to create worded problems or situations related to their chosen phenomenon every week applying the concepts learned for that week's discussion in Differential Calculus. Each group was given 10 minutes to present their work to the class.

At the end of the 9th week of the study, which lasted for 12 weeks, the students were tested again using the same questionnaire used in the pretest. Since the study has been conducted during the COVID19 crisis, the continuation of the experiment and administration of posttest has been conducted online, specifically through Edmodo. This has been made possible because of students' familiarity with these setting since they were already exposed to blended-online learning since the previous 2 trimesters. The two experimental groups were taught, through video conferencing. The researchers sent them lecture notes/videos and weekly activities about the real life applications of calculus via Edmodo, requiring them to submit phenomenon-based outputs every week as well. As for the proof writing treatment, proof writing activities were given to EG2. For the controlled group, video conferences were also conducted and were given the same lecture notes but not the activities requiring them to create/site phenomenon-based outputs.

Students were informed about the task (posttest) that they need to accomplish a day before. Answer sheets were also sent in Edmodo a day ahead for them to reproduce it or copy the format in a bond paper. All students in the three groups were asked to accomplish each task for 45 minutes (same time for the face-to-face pretest), at the same schedule to avoid leakage. A time limit was set in Edmodo. After they accomplished the tasks they were asked to take a picture or a scanned copy of their works and were required to send them immediately thru Edmodo. To avoid bias on giving of scores to the works of the students, both the pretest and posttest results of the students were forwarded and checked by a Mathematics teacher.

Lastly, weighted mean and standard deviation were used to identify the students' level of pretest and posttest mathematical creativity and reasoning skills. In determining the effects of the treatments one-way analysis of covariance (ANCOVA) was used because the samples were intact groups. ANCOVA is a general technique that can increase the precision of the experiment design by adjusting measures in terms of one or more outside covariates. The hypothesis was tested at 0.05 level of significance.

### 3. Results and Discussion

Depicted in [Table 1](#) is the analysis of the mean scores and standard deviation of the students' mathematical fluency as one of the indicators of their mathematical creativity.

Results revealed that before the start of the experiment both the experimental and control groups obtained scores which are described as at the beginning level. This means that before the experiment, the mathematical fluency of the three groups of students has not been manifested, is missing, and is functioning poorly, as evidenced by their pretest scores. In their posttest, though all of the three groups showed an improvement, students in experimental groups showed greater improvement that students in the control group, which indicates that students' exposure to phenomenon-based learning and phenomenon-based learning with proof writing augmented their mathematical fluency scores.

PhBL actively engages students in hands-on exercises aimed at solving problems and answering questions that build their capacity to answer questions [13]. In a study conducted by Borja [14] it has been contended that among the components of mathematical creativity (fluency, flexibility, and originality), students performed best in fluency and least in originality, implying that producing numerous correct solutions is the simplest and coming up with original solutions is the most difficult. Moreover, an increased emphasis on experiential learning and student autonomy in PhBL empowers deeper learning to students [15]. Accordingly, the more in-depth learning the abler the students will become in overcoming the gap between the different subject areas [17].

Also, despite the fact that all of the three groups of students showed an improvement in their posttest fluency score, students exposed to PhBL with proof writing had the most noteworthy mean increase. This corroborates the idea that because of the nature of proof writing, experience in working with it is the key to performance and facilitation the learning, enabling students to think fundamentally and broadly [18]. Students demonstrate proving not only to mirror their understandings and values but also to fulfill the desired expectation by which their work will be judged [19]. However, we should be aware completely grasp the idea of classroom proof-writing activities, much the same that we cannot see the end of a rhizome. Each connection made by the learners, every subject was taken into account, will bring to other territories, which will bring to even different domains which are fluency, and so on [7].

**Table 1. Level of Students' Fluency in MCT**

Group of Learners	Pretest		Descriptive Level	Posttest		Descriptive Level
	Mean	SD		Mean	SD	
CG	1.43	0.63	Beginning	2.14	0.59	Developing
EG1	1.75	0.89	Beginning	2.60	0.66	Approaching Proficiency
EG2	1.58	0.81	Beginning	3.65	0.61	Proficient

Perfect Mean is 5.00  
 Legend:  
 Mean Intervals Description  
 4.20 - 5.00 Excellent  
 3.40 - 4.19 Proficient  
 2.60 - 3.39 Approaching Proficiency  
 1.80 - 2.59 Developing  
 1.00 - 1.79 Beginning.

**Table 2. Level of Students' Flexibility in MCT**

Group of Learners	Pretest		Descriptive Level	Posttest		Descriptive Level
	Mean	SD		Mean	SD	
CG	2.29	0.66	Developing	2.07	0.26	Developing
EG1	2.16	0.75	Developing	2.73	0.62	Approaching Proficiency
EG2	2.16	0.82	Developing	3.74	0.63	Proficient

Perfect Mean is 5.00.

Reflected in [Table 2](#) is the analysis of the mean scores and standard deviation of the students' mathematical flexibility as one of the indicators of their mathematical creativity. It can be observed that before the start of the experiment both the experimental and control groups obtained scores that are described in the developing level. This means that before the experiment, the mathematical flexibility of the three groups of students has been manifested but not at a profound level, tending to function limitedly as evidenced by their pretest scores. In their posttest, only the experimental groups showed an improvement, and students in the control group a mean score decrease, which indicates that students' exposure to phenomenon-based learning and phenomenon-based learning with proof writing augmented their mathematical flexibility scores.

This result is strongly influenced by phenomenon-based learning. Since the beginning stage of phenomenon-based learning is the reality and real-world observation of learners into a phenomenon from different points of view, then they are empowered to attempt various strategies to solve the problem, perhaps by themselves or in collaboration with peers [\[20\]](#). By working on real-life problems from various angles and trying different strategies to solve them cooperatively, students are then well-prepared to answer and give solutions at various categories [\[21\]](#).

Further, in PhBL, students are given the opportunity to upgrade their flexibility in investigating mathematical ideas and trying to discover alternative methods for solving problems [\[22\]](#). The students' capacity in solving their self-made problems related to their chosen phenomenon has exercised their mathematical abilities widely. Also, experimentation and utilization of online resources show the ability of students in detailing ideas, developing from different angles, and finding their way of solving problems [\[23\]](#).

Inevitably, proofs aren't just ways to show that statements are valid or substantial. They help to confirm a

student's understanding of givens, axioms, theorems, rules, and hypotheses. And they affirm how and why math helps explain our world and how it works [\[24\]](#). This is why students exposed to PhBL with proof-writing had the most improved flexibility posttest score. To lay it out plainly, teaching in phenomenon-based learning likewise works in agreement with constructivism and sociocultural learning theories in the manner that doesn't exclusively think about students as active knowledge builders and information but flexibly supplies an ideal framework for students to move beyond what they currently know towards higher and further levels [\[7,25\]](#).

**Table 3. Level of Students' Originality in MCT**

Group of Learners	Pretest		Descriptive Level	Posttest		Descriptive Level
	Mean	SD		Mean	SD	
CG	1.25	0.63	Beginning	1.53	0.52	Beginning
EG1	1.30	0.58	Beginning	3.56	0.57	Proficient
EG2	1.23	0.53	Beginning	3.74	0.59	Proficient

Perfect Mean is 5.00

[Table 3](#) above depicts the pretest and posttest mean scores and standard deviation of the students' mathematical originality as one of the indicators of their mathematical creativity. It can be gleaned from the table that before the start of the experiment both the experimental and control groups obtained scores which are described as a beginning. This means that before the experiment, students' level of mathematical originality has not been manifested, is missing, and is functioning poorly. Surprisingly, in their posttest, the experimental groups showed greater improvement than the students in the control group, which indicates that students' exposure to phenomenon-based learning with proof-writing and phenomenon-based learning augmented their level of mathematical originality or novelty.

Phenomenon-based learning has created its teaching framework and provided a potentially positive learning environment for learners in which they have plenty of

opportunities to produce their bits of knowledge and authentic experiences in connection with academic theories, principles, and concepts to effectively solve the imitated contexts and ultimately accomplish genuine meanings of their learning goals [26].

PhBL creates opportunities for students to ponder creatively about their thoughts in finding new ideas or information from the learning process. Additionally, it has been declared that through PhBL, students regularly transmit outside knowledge into their discussions critically, although they often repeat information and post statements requiring further thought [27]. In this manner, an individual exhibits originality when one makes an answer that is unusual, uncommon, and novel, in a mathematical situation, in a numerical circumstance.

Meanwhile, proof writing activities in the classroom is a key to mathematical comprehension [28], and the measure of a student's mathematical understanding can be found inside each student's construction of proof and application to solving problems they encountered. Since proof-writing varies from one person to another, originality of one's understanding and techniques to communicate what he/she understood is therefore exercised.

Shown in Table 4 is the students' pretest and posttest mean scores and standard deviation in mathematical elaboration as the last indicator of their mathematical creativity. Still, before the start of the experiment both the experimental and control groups obtained scores which are described as a beginning. This means that before the experiment, students' level of elaboration has not been manifested, is missing, and is functioning poorly. Surprisingly, in their posttest, only the experimental groups showed improvement which indicates that students' exposure to phenomenon-based learning with proof-writing and phenomenon-based learning augmented their level elaboration.

The improved posttest elaboration scores of the participants in the experimental groups have been upheld that the enhanced elaboration and thinking skills in PhBL results in reinforcing students' comprehensive understanding of the phenomena occurring in their own lives. In PhBL students are not passive recipients of lessons, but proactive participants, adding and gaining from the topic, giving them profound understanding [29].

Eventually, PhBL welcomes the innovative utilization of technology, and exploiting learning environments outside the school assumes a significant role in engaging and activating students in learning [30]. PhBL approach inspires students to figure out how topics are related to

each other, empowering them to exercise the way they explain things around them [29]. Consequently, students create causal explanations for the connections between objects observed in phenomena [31]. Students learn by phenomena-based learning through demonstration activities about natural phenomena related to optic materials, evaluation, discussions, experimental activities, and reflection on investigation activities. Such learning helps students to practice explaining [32].

Thus, students exposed to PhBL with proof writing still had the highest posttest score in elaboration among the three groups. This is supported by the study of [33] when they interviewed mathematicians. One of the mathematicians they interviewed expressed that if one becomes acquainted with study proofs one gets rehearsed with mathematical reasoning, something one can draw extraordinary advantages of problem-solving because problem-solving is an art of formulation. Since, phenomenon-based teaching is embedded in a problem-solving environment, where the teacher begins by posing questions or problems and the students build answers together to questions or problems posed concerning a phenomenon that interests them [7,25], the consolidation of classroom proof-writing activities in this strategy can bring impact to students elaboration skills [32].

Table 5 below shows the students' level of creativity which is from the sum of the students' level of fluency, flexibility, originality, and elaboration. It can be gleaned from the table that before the start of the experiment both the experimental and control groups obtained scores which they are described as productive. This means that before the experiment, students' level of mathematical creativity is still in technical creativity, which means that students used rules and physical laws to constrain their thinking, with little expressive spontaneity. Things that emerge to them may be new, but may already be known to the world allowing them to create expressions in a myriad of ways. Impressively, the students exposed to phenomenon-based learning with proof writing managed to show mathematical creativity scores in the level of innovative creativity which indicates that their creativity level involves departing from existing thinking patterns and making the leap to "out of the box" thinking, creating some results in something that seemingly has not been done before. Eventually, students exposed to phenomenon-based learning alone managed to improve to inventive creativity which means that they were able to develop the ability to creatively combine existing technical concepts using prior design solutions to create new designs.

**Table 4. Level of Students' Elaboration in MCT**

Group of Learners	Pretest			Posttest		
	Mean	SD	Descriptive Level	Mean	SD	Descriptive Level
CG	1.78	0.82	Beginning	1.46	0.76	Beginning
EG1	1.77	0.60	Beginning	2.62	0.42	Approaching Proficiency
EG2	1.76	0.81	Beginning	2.92	0.57	Approaching Proficiency

Perfect Mean is 5.00.

Table 5. Level of Students' Creativity in MCT

Group of Learners	Pretest		Descriptive Level	Posttest		Descriptive Level
	Mean	SD		Mean	SD	
CG	6.75	2.21	Technically Creative	7.20	1.30	Technically Creative
EG1	6.98	2.39	Technically Creative	11.51	1.53	Inventive Creativity
EG2	6.73	2.28	Technically Creative	14.05	1.50	Innovatively Creative

Perfect Mean is 20

Legend:

Mean Intervals Description

16.20 - 20.00 Emergent Creativity

12.40 - 16.19 Innovatively Creative

8.60 - 12.39 Inventive Creativity

4.80 - 8.59 Technically Creative

1.00 - 4.79 Expressive Creativity.

Mathematical creativity is the capacity of the students to generate their creative ideas or plans to solve the open-ended mathematical problems with the utilization of creative problem-solving skills [34]. Creativity in mathematics requires the activation of multiple ideas, and sources of information simultaneously, to think of alternative solutions for mathematical problems.

Accordingly, it's significant to note that the starting point of phenomenon-based teaching is constructivism, in which students are viewed as active knowledge builders, and information is seen as being constructed as a result of problem-solving. Ideas are constructed out of "little pieces" into a whole that suits the situation in which it is used at the time. When phenomenon-based learning occurs in a collaborative setting, it supports the sociocultural and socio-constructivist learning theories, in which information is not seen only as an internal element of an individual; instead, information is seen as being formed in a social setting [25]. Students, who are viewed to be active actors in their lifelong learning cycles, are aware of their own learning and acquire a joyful, creative, and reflective activity and strategies of how to learn, to ensure a good life [35]. Positive emotional encounters the joy of learning and creative activities promote learning and motivate the students to build their creativity.

PhBL is a strategy that takes advantage of the phenomenon as a learning asset [31,36]. Students can make a portrayal of the phenomenon investigated to be involved in cognitive processes [36]. The phenomenon that is presented through simulation can build up students' ability to organize and analyze the question, then select and develop solutions to the problems being studied [37] that hereby result in creativity. Further, phenomenon-based learning assists students' understanding of physics and other sciences through problems presented and solving these problems based on phenomena [32].

Moreover, students' exposure to classroom proof-writing activities encourages them to think beyond the four corners of the classroom. Expanded information about different elements of proof conveys something about the significance of proof in mathematical practice, and their awareness should therefore be essential for how newcomers experience the practice. The similarity between proving and problem-solving has been studied and discussed. This is an interesting perspective as we can likewise think the other way around, i.e., students can learn techniques and concepts in problem-solving that they can use in proving tasks [33]. This suggests that the ability of reasoning plays an important role in the success of students. Those who

have good reasoning skills are expected to have a good learning achievement that includes creativity.

Furthermore, to determine if there is a significant effect of the treatment on the mathematical creativity profile of the participants, analysis of covariance (ANCOVA) was used.

Reflected in Table 6 is the result of the analysis of the covariance of the participants' level of mathematical creativity. It can be gleaned from the table that, after adjustment for students' pre-test in mathematical creativity test, there was a statistically significant difference in post-test mathematical creativity scores between three groups,  $F(2, 99) = 642.30$ ,  $p < .0005$ , which led to the rejection of the null hypothesis of the study. Thus, there is a significant difference in the students' mathematical creativity scores in favor of the experimental group. This further implies that the experimental groups who were exposed to phenomenon-based learning with proof-writing and phenomenon-based learning alone performed better than the control group who were exposed to a conventional type of learning. Though the control group showed an increase in their posttest mathematical creativity score, as shown in the previous table, this increase is not comparable to the mean increase of the participants in the experimental groups.

Consequently, post hoc analysis was performed with a Bonferroni adjustment. Post-test mathematical creativity scores were statistically significantly greater in students exposed to phenomenon-based learning with proof writing than the other two groups of students. Further, students exposed to phenomenon-based learning alone may not outperform students exposed to phenomenon-based learning with proof writing, but they still had significantly higher posttest mean scores compared to students exposed to the conventional type of teaching. Thus, students in the two experimental groups performed way better than the students in the control group as what can be seen in Table 6, phenomenon-based learning and proof-writing could be a perfect combination in promoting students' mathematical creativity.

It has been recommended that pragmatic tasks and real-life mathematical problems which are based on the procedure can be used to help students to foster their mathematical creativity [38]. Instruments used to assess mathematical creativity and creative problem solving on mathematics can be created dependent on the meaning of mathematical creativity to determine the parameters of fluency, flexibility, and originality.

**Table 6. One-Way ANCOVA Unequal n Summary of Students' Mathematical Creativity in MCT**

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Squares	F	p-value
Treatment Between Groups	2	642.30	321.15	225.755	0.000*
Error	99	140.83	1.423		
Total	103	13098.98			

\*Significant at 0.05 level.

The results above are likewise upheld by the study of Kozłowski, Chamberlin, & Mann, [39]. In their study, they inferred that it is simpler for the students exposed to phenomenon-based learning to come up with a variety of possible solution paths than it is for others. Their study outlined the relationship between students' interaction with multiple solutions tasks and their ability to create original, flexible, and fluent solutions.

Further, phenomenon-based learning has several benefits for students. New experiences about phenomena can prompt questions in the minds of students. Through phenomenon-based explanations, students may gain preferred memories than the teacher's explanation [40]. Some activities in phenomenon-based learning among students observe phenomena, conduct an inquiry, compile temporary explanations, the final explanation, and provide reasons to support the explanation.

Indeed, phenomenon-based learning can accomplish concept acquisition. During phenomenon observation, students may have intellectual conflicts [41]. Phenomenon observation and explanation assists students to confront and improve their concepts [42]. With a phenomenon, students work on planning evidence-based scientific explanations by collecting, discussing, and interpreting data [36]. By assessing scientific ideas, which are principles, concepts, or theories, and comparing them with the data, it will give students a profound comprehension of the ideas. Based on theoretical studies, experiential learning and phenomenon-based learning process gives a positive outcome on concepts acquisition and coordinated with the idea of the application of sciences and mathematics just like physics [43].

Additionally, proofs are the core of mathematics and they assume an intricate role in generating mathematical knowledge and understanding [44]. Students' exposure to classroom proof-writing activities in the study, has likewise enabled them to outperform the other two groups of students. This is strengthened by the words of Salleh & Zakaria [45] that the Thai middle school students perform a significant relationship between the ability of reasoning with their math learning outcomes. This suggests that the ability of reasoning plays an essential role in the success of students. Those who have good reasoning skills are expected to have good learning achievement. Experienced proof-writing solvers show various phases in their solution process [46], and achieve an understanding that more than one approach can prompt equivalent results and the ability to solve problems in different ways, which are the portrayal of mathematical creativity [47].

Further, in proving tasks, which emphasizes the role of an individual's problem-solving process highlights the development of creativity by establishing the foundations for creativity in novel situations. During the proving process, a student should be encouraged to create new

ideas, explore various concepts, and evaluate those attempts to ultimately create a valid proof [5]. Creativity can also include creating tools or tricks in the proving process. One can be very creative about how he approaches the solving process of the question, either with new tools or with a really good idea for a partial result. Using these tricks or tools can be original to the student or the course, thus leading to relative creativity in their proving. Thus, proving can inform both students and teachers about the progression that a student is making in building up his/her mathematical creativity [5].

## 4. Conclusions and Recommendations

Based on the foregoing findings that undergone in-depth analysis, the researcher concludes that Phenomenon-based learning with proof writing is an effective combination in improving students' mathematical creativity. The phenomenon-based approach to teaching and learning invites us to break the boundaries of traditional subject teaching and move toward interdisciplinary explorations of phenomena. Phenomena can drive the lesson, learning, and reflection/monitoring throughout. Using phenomena in these ways leads to deeper learning and helps students build more usable and generative knowledge. The researchers hereby recommend that phenomenon-based learning be used in teaching mathematics in tertiary education. Future research can consider student perspectives when choosing phenomena. This study can also be used to know if there will be an improvement to students' reasoning skills. This project can be replicated in any country and any level.

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