

Analysis of Mathematical Problem-Solving Abilities Taught Using Problem-Based Learning

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Abstract This study aims to analyze: (1) The level of students' mathematical problem-solving ability, (2) Mistakes made by students in solving mathematical problem-solving problems. This type of research is qualitative research. The subjects of this study were students of class VII-A at Tunas Baru Jin Seung Middle School, Batam City. The results showed that: (1) High-ability students were able to carry out the stages of understanding the problem, devise a plan, carry out a plan and look back. Moderate-ability students are being able to carry out the stages of understanding the problem, devising a plan, and carry out a plan. While low-ability students can carry out the stage of understanding the problem. (2) Students can understand the problem by writing down what is known and asked, but unable to explain in their sentences. Students are not able to devise a plan because of the inability to associate information with one another. Students are not able to carry out a plan, this is due to the inability of students to make plans so they do not understand how to solve problems and calculation errors occur in the completion process. Students are not able to look back obtained and give their arguments.

Keywords: *analysis, problem-solving abilities, problem-based learning*

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1. Introduction

The National Council of Teachers of Mathematics [1], states that school mathematics standards include mathematical content and mathematical processes. According to NCTM, the content standards are (1) numbers and operations; (2) algebra; (3) geometry; (4) measurement; (5) data and probability analysis. The standard process according to NCTM includes (1) mathematical problem solving; (2) mathematical reasoning and proof (mathematical reasoning and proof); (3) mathematical communication (mathematical communication); (4) mathematical connections; (5) mathematical representation. This is consistent with the learning theory proposed [2] that high-level intellectual skills can be developed through problem-solving. Problem-solving is the highest type of learning from the eight types proposed by Gagne, namely: signal learning, stimulus-response learning, chaining, verbal association, discrimination learning, concept learning, rule learning, and problem-solving.

Mathematics can be used to arrange clear, conscientious, precise and consistent principles (through practice) through the practice of solving problems that are pedagogic [3]. Based on the above, one of the mathematical abilities that students must possess in learning is problem-solving ability. Problems occur because of the gap between what is expected and reality, between what is owned by what is needed, or between what is known to what is known. In

learning mathematics, a problem is a challenging problem or question that cannot be solved by routine procedures that are commonly done or are already known. A problem is called a problem for a student, if: (1) the question that is confronted can be understood by students, but the question must be a challenge for him to answer, and (2) these questions cannot be answered by routine procedures that are known to students [4].

There are several reasons [5] why problems solving type problems are given to students, namely: (1) Can lead to curiosity and motivation, foster creative nature; (2) Besides having the knowledge and skills (counting and others), the ability to read and make correct statements is required; (3) Can give rise to answers that are original, new, unique, and diverse, and can add new knowledge; (4) Can improve the application of the knowledge it has acquired; (5) Invite students to have problem-solving procedures, be able to make analysis and synthesis, and be required to make evaluations of their solutions, and (6) Is an important activity for students that involves not only one field of study but (if needed) many fields of study, stimulate students to use all their abilities.

Reference [6] shows the stages of mathematical problem solving abilities include: (1) understand the problem / read the problem; (2) devise a plan / select a strategy; (3) carry out a plan / solve the problem; and (4) look back. Students who can apply the four stages will achieve a good learning process to provide good learning outcomes.

Based on the results of the 2012 PISA [7] study Indonesia ranks 64 out of 65 participating countries, or in

other words ranks second lowest of all PISA participating countries surveyed with an average score of Indonesian students' mathematical abilities of 375, the score is in below the international average score of 494. The factors that caused the low achievement of Indonesian students in PISA were the weak ability to solve non-routine or high-level problem problems. The questions tested in PISA consist of 6 levels (lowest level 1 and highest level 6) and the questions tested are contextual, the problem is taken from the real world. Students in Indonesia are only accustomed to routine questions at level 1 and level 2. It can be concluded that the mathematical problem-solving ability of Indonesian students is low.

One of the teacher's efforts to improve the learning process is to choose appropriate and innovative learning models in mathematics learning, one of which is the Problem-Based Learning (PBL) model. This is consistent with the statement as in [8] stating that PBL is a learning strategy that involves students in solving problems by integrating various concepts and skills from various disciplines. The strategy consists of gathering and uniting information, and present findings.

2. Methods of Research

This type of research used in this research is descriptive qualitative research. Qualitative research is a complex picture, discussion of words, detailed reports from respondents' views, and comparative studies [9]. The subjects in this study were 38 students of the class VII-A Tunas Baru Jin Seung middle school, as for interview subjects were selected based on their level of mathematical problem-solving ability on Polya's stages. Through answers, students are grouped according to three categories of answers, namely: (1) high ability students; (2) medium-ability students; and (3) low ability students. For the determination of a minimum standard of mathematical problem solving ability students are guided by the Minimum Mastery Criteria (MMC) ≥ 65 [10]. Based on this view the results of tests of students' mathematical problem solving abilities can be seen in the following intervals:

Table 1. Level of Students Mathematical Problem- Solving Ability

LEVEL MPSS	CATEGORY OF ASSESSMENT
$0 \leq MPSS < 65$	Low
$65 \leq MPSS < 80$	Medium
$80 \leq MPSS < 100$	High

3. Result and Discussion

The purpose of this study was to analyze the mathematical problem-solving abilities and errors made by students in solving mathematical problem-solving problems in learning problem-based learning in Tunas Baru Jin Seung Middle School Batam city.

3.1. Level of Mathematical Problem-Solving Ability

Description of student's mathematical problem-solving abilities is obtained based on the scores of each student

based on the results of tests of mathematical problem-solving abilities and interviews. Problem-solving ability test consists of 4 questions in the form of description with material numbers. All student answer sheets from the results of tests of mathematical problem-solving ability are collected to be examined and given a score. Scoring a score for each student's answer is based on a guideline in scoring a mathematical problem-solving ability. The total score of each student obtained is used to categorize the level of students' mathematical problem-solving abilities.

After conducting a mathematical problem-solving ability test, the researcher conducted an interview and then compared the results of the students' answers to the interview transcript for analysis. The analysis was carried out based on the stages of the ability to solve mathematical problems according to Polya and interview guidelines. Quantitatively, the level of students' mathematical problem-solving abilities can be seen in Table 2.

Table 2. Level of Students Mathematical Problem- Solving Ability

LEVEL MPSS	TOTAL STUDENTS	PERCENTAGE	CATEGORY OF ASSESSMENT
$0 \leq MPSS < 65$	10	26 %	Low
$65 \leq MPSS < 80$	20	53 %	Medium
$80 \leq MPSS < 100$	8	21 %	High

MPSS = Mathematical Problem-Solving Score.

Based on Table 2 above, it can be seen that the percentage of mathematical problem-solving abilities of high-ability students is lower than medium abilities and low abilities. The percentage of students with low ability is twice the percentage of students with medium ability. Data on the level of students' mathematical problem-solving abilities in the bar diagram are presented in Figure 1.

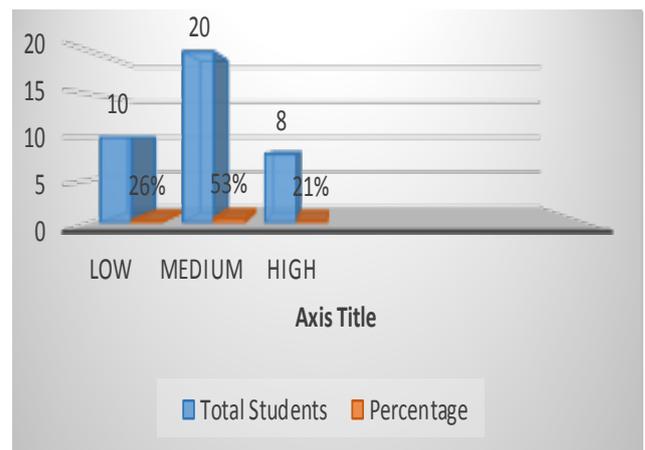


Figure 1. Diagram Levels of Students Mathematical Problem-Solving Ability

Reference [6] show stages of the ability to solve mathematical problems include: (1) understand the problem / read the problem; (2) devise a plan / select a strategy; (3) carry out a plan / solve the problem; and (4) look back. Student answer sheets will be analyzed based on Polya's stages. Quantitatively, the percentage of students' mathematical problem-solving abilities based on Polya's stages can be seen in Table 3.

Table 3. Ability to Solve Students' Mathematical Problems Based on Polya Stages

Stages of Mathematical Problem Solving	Total Students	Percentage
understand the problem	38 orang	100%
devise a plan	21 orang	55%
carry out a plan	16 orang	42%
Look back	5 orang	13%

Based on Table 3 above, it can be seen the percentage of students in class VII-A in solving mathematical problem solving based on Polya's stages. The sequencing of students' mathematical problem-solving stages from highest to low is the stage of understanding the problem, followed by the stage of devising a plan, the stage of carrying out a plan and finally the stage of look back.

In the stage of understanding the problem, students are able to understand the problem well that is writing what is known and asked of the problem correctly. At the stage of devise a plan, students are able to make a complete plan by making plans according to procedures and directing the correct solution. At the stage of carrying out a plan, students are able to carry out the completion plan by doing the right process and getting the right results. In the look back, students are able to interpret the results obtained and provide their arguments correctly.

3.2. Mistakes Made by Students in Solving Mathematical Problem Solving Problems

To find out the mistakes made by students in solving mathematical problem-solving problems, a more in-depth analysis of the test results is performed. Of the 38 people, there were 8 highly skilled students. High ability student answer sheets will be analyzed based on Polya stages. Quantitatively, the classification of mathematical problem-solving abilities of high-ability students based on the Polya stage can be seen in Table 4.

Table 4. Percentage of Errors Made by Students in Solving Mathematical Problem Solving Problems

Stages of Mathematical Problem Solving	Total Students (Percentage)		
	Correct Answer	Wrong Answer	No Answer
understand the problem	38 People (100%)	-	-
devise a plan	21 People (55%)	12 People (32%)	5 People (13%)
carry out a plan	16 People (42%)	16 People (42%)	6 People (16%)
Look back	5 People (13%)	19 People (50%)	14 People (37%)

Based on Table 4 above, it can be seen that there are two errors made by students in solving mathematical problem-solving abilities, namely answers and not answers. The most mistakes made are in the re-checking stage, followed by the stage of carrying out the plan and then the stage of making the plan.

S-2 coded students are one of the low-ability students who were selected as research subjects to examine the mistakes made by students in solving mathematical problem-solving abilities. During the learning process using the PBL model, students feel happy during the

learning process but the written test results of students' mathematical problem solving is still low. The results of the S-2 subject's problem-solving ability are still many wrong because the subject cannot relate the information obtained so that he is unable to make a solution plan that impacts the inability to carry out the plan and look back at the problem. Figure 2 below is the result of a written test of a research subject with students' mistakes in solving a mathematical problem-solving ability of an S-2 subject.

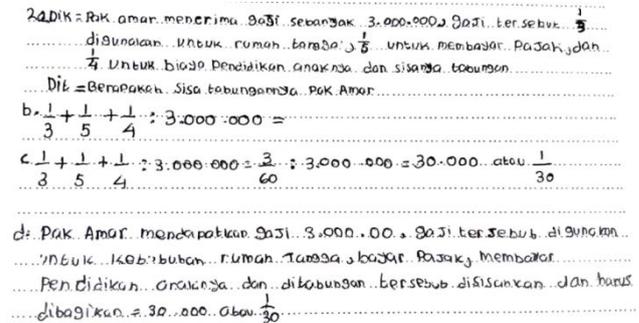


Figure 2. Written Test Results of Low Ability Students Subject S-2

After comparing the students' answer sheets to the interview transcript it is described that 1) In the stage of understanding the subject of the subject S-2 is able to carry out by writing known and asked as seen on the answer sheet, but not able to recite using sentences themselves. 2) At the stage of making a plan of completion, the subject of S-2 is not able to make a plan of completion because it is unable to link information to the problem. 3) At the stage of implementing the plan to settle the subject, S-2 is not able to carry out the plan because there are no plans made, and can not do calculations correctly and do not know the concept of Numbers. 4) At the stage of re-examining the subject S-2 can not check back correctly because it does not understand the question part d. Subject S-2 cannot translate the results obtained from previous answers into the context of the problem.

3.3. Discussion

3.3.1. Level of Mathematical Problem Solving Ability

Mathematical problem solving is often seen as one of a number of skills taught in the school curriculum. Reference [11] shows places mathematical problem-solving in a hierarchy of skills to be obtained by students leading to certain consequences for the role of mathematical problem-solving in the curriculum.

Problem-solving skills through PBL learning can be described and discussed in accordance with research data for each level of ability and the level of mathematical problem-solving stages. Based on the written results that have been done, it can be concluded that the percentage of mathematical problem-solving abilities of high-ability students is lower than medium abilities and low abilities. The percentage of students with low ability is twice the percentage of students with medium ability.

Based on the results of the study obtained an overview of students' mathematical problem-solving abilities based on Polya's speech: Students with high abilities are able to understand the problem. It is characterized by being able

to write what is known and asked about the problem, being able to explain the problem by using own sentences during the interview, being able to simplify the problem and being able to find sub-goals and being able to sort the information in the problem. At the stage of devising a plan, students are able to understand the appropriateness between the known and asked, can make plans or strategies in solving problems, and can determine the mathematical operations used in solving problems. At the stage of carrying out a plan, students are able to carry out the completion plan that has been made with the correct calculation. In the look backstage, students are able to interpret the final results obtained from the previous answers into the context of the problem and provide their arguments.

Students with the ability to understand the problem, the students are able to write what is known and asked about the problem, able to explain the problem using their own sentences during the interview, able to find sub-goals, and able to sort the information in the problem. In the devise a plan stage, students are able to understand the appropriateness between being known and asked, and can make plans or strategies in solving problems. At the carry out a planning stage, students are able to carry out the completion plan that has been made with the correct calculation. At the look backstage, students have not been able to interpret the final results obtained into the context of the problem and provide arguments. Students are only able to make conclusions from the solutions that he made.

Students with low ability at the stage of understanding the problem are able to write what is known and asked to the problem but are not able to explain the problem in their own sentences at the interview. At the stage of devising a plan, students can understand the details between the known and asked but can not make plans or strategies in solving problems, and are not able to sort the information in the problem. At the carry out a planning stage, students are not able to make the completion stage. At the look backstage, students are not able to interpret the final results obtained into the context of the problem and provide arguments. Students are only able to draw conclusions without being re-associated with the problem.

Based on the research carried out, the percentage of students' ability to understand the problem is 100%, the devise a plan stage is 55%, carry out a plan phase is 42%, and the look backstage is 13%. This shows the ability of students at the stage of understanding better than at the other Polya stage. The results of the study are relevant to the research as an [12] showing the percentage of students' ability to understand problems reaches 87.10% and in the excellent category, the percentage of students' problem solving abilities to plan 40.32% in the unfavorable category, the percentage of students' ability to solve problems according to the plan 21.19% is classified in very less, the percentage of students' ability to re-examine the results obtained 48.39% in the unfavorable category. Reference [13] also shows that the percentage of students' problem-solving abilities in the first stage of understanding the problem is 75.08%, the second stage devising a plan to solve the problem is 66.12%, the third stage implementing the plan is 29.03%, and the fourth stage of reflection on the problem is 24.19%.

Based on the results of research and opinions above, it can be concluded that high-ability students are able to carry out the stages of problem-solving properly and appropriately, while capable students are able to understand problems, make plans and implement plans well. Low-ability students are able to understand problems but are less able to make plans, carry out plans, and check again.

3.3.2. Mistakes made by Students in Solving Mathematical Problem Solving Problems

Based on the findings of the research, interviews conducted on the subject, and triangulation of data, it was found some student errors in solving problems based on the numbers in accordance with the steps of problem-solving. The majority of mistakes made by students in solving mathematical problem-solving problems are at the stage of re-checking that as many as 19 people answered incorrectly and 14 people did not answer. This stage clearly shows the students' lack of interpretation in applying the knowledge they have and giving arguments.

a. Understand the Problem

The stage of understanding the problem is the stage where it must be able to understand the language or terms used in the problem, formulate what is known, what is asked, whether the information obtained is sufficient, what conditions/conditions must be met, write the problem in a more operational form so make it easy to solve. Reference [14] shows that in understanding the problem, one must understand the meaning of a sentence, identify the known, unknown and the relationship between information, and know the concepts that have been previously learned that are needed to solve the problem. Student errors in solving mathematical problem-solving questions at the stage of understanding the problem by 0%, meaning that there are no student errors at the stage of understanding the problem. The stage of understanding the problem, students are able to relate the problem with the material that has been studied, this makes students be careful in exploring the information in the problem. The ability to connect problems with material results in students being able to formulate what is known and asked of the problem so that it is easy to solve problems.

b. Devise a Plan

The stage of devise a plan of completion is the stage were looking for possibilities that can occur and then compiling a procedure for completion. Reference [15] argues the use of appropriate problem strategies is very important in the success of problem solving. Student errors in solving problems solving mathematical problems at the stage of understanding the problem by 57.9%. At the stage of making a settlement plan, students cannot determine what must be done to solve the problem so that they make a mathematical operation error in preparing the completion steps. This is because students do not like to read long and non-routine problems.

c. Carry out a Plan

The stage of carrying out the settlement plan that is carried out is to carry out the strategies that have been

made with diligence and accuracy to obtain a solution. Student errors in solving problems solving mathematical problems at the stage of understanding the problem by 60.5%. At the stage of implementing the completion plan, students make operational errors in the calculations.

d. Looking Back

Looking back phase is done in interpreting the results obtained into the context of the problem. Student errors in solving mathematical problem-solving questions at the looking backstage was 86.8%. The mistake made by students is the inability of students to interpret the results obtained in the context of the problem and give their arguments.

4. Conclusion

Based on the results of the analysis and discussion as well as findings during learning with the problem-based learning model, several conclusions are obtained which are answers to the questions raised in the formulation of the problem. These conclusions are:

1. For the level of mathematical problem-solving abilities obtained several conclusions as follows:
 - a. Based on the research, from 38 students of class VII-A obtained a level of mathematical problem-solving ability, the number of students who obtained grades with intervals of $0 \leq \text{SPMM} < 65$ low category as many as 10 students with a percentage of 26%. The number of students who scored with intervals of $65 \leq \text{SPMM} < 80$ in the medium category was 20 students with a percentage of 53%. The number of students who scored at intervals of $80 \leq \text{SPMM} < 100$ in the high category was 8 students with a percentage of 21%.
 - b. Mathematical problem-solving abilities of students who are highly capable of the first stage, namely understanding the problem, students are able to understand the problem by writing what is known and asked correctly and are able to explain the problem in their own sentences. Make a settlement plan by sorting the information in the problem and being able to link the information to one another. Carry out procedures or plans that have been made appropriately as well as good and correct calculations. For the re-checking stage, students are able to interpret the results of the answers into the context of the problem by giving their arguments.
 - c. Mathematical problem-solving abilities of students who are able to the first stage is to understand the problem, students are able to understand the problem by writing what is known and asked correctly but cannot explain in their own sentences. Make devise a plan by sorting the information in the problem and being able to mention the mathematical operations used in solving the problem. Carry out procedures or plans that have been made appropriately as well as good and correct calculations. For look backstage, students are not able to interpret the results of the answers into the context of the problem and give their arguments.

- d. Mathematical problem-solving ability of students who are low-ability for the first stage is understanding the problem, students are able to understand the problem by writing what is known and asked correctly but cannot explain in their own sentences. Students are not able to sort and relate the information contained in the problem so they are unable to make devise a plan. Because they are not able to devise a plan, students are not able to carry out a plan. For the look backstage, students are not able to interpret the results of the answers into the context of the problem and give their arguments.

2. Errors experienced by students in solving problem-solving problems are:

- a. Students are able to understand problems by writing down what they know and ask but are unable to explain in their own sentences.
- b. Students are not able to make devise a plan because of the inability to relate information to one another
- c. Students are not able to carry out the completion plan, this is due to the inability of students to make plans so they do not understand how to solve problems and calculation errors occur in the completion process.
- d. Students are not able to interpret the results obtained.

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