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Investigating Students' Physico-mathematical Difficulties in Classical Mechanics and Designing an Instructional Model

Danny Mutambo[1,*], Gurudas T. Baliga[1], Leonard Nkhata[2]

[1]Department of Physics, The Copperbelt University, Kitwe, Zambia
[2]Department of Mathematics and Science Education, The Copperbelt University, Kitwe, Zambia
*Corresponding author: danny.mutambo@cbu.ac.zm

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Abstract The study investigated students' physico-mathematical difficulties in classical mechanics and designed an instructional model. The three objectives that guided the study were namely: to identify physico-mathematical difficulties students have in classical mechanics, to explore the existing instructional models and design a suitable instructional model to address physico-mathematical difficulties in classical mechanics and to verify and evaluate the effectiveness of the designed instructional model. The study was undertaken with a purposive sample of 140 students and 10 instructors learning and teaching classical mechanics respectively from the Copperbelt University and Mukuba University on the Copperbelt Province of Zambia. A descriptive mixed method survey design approach was used. A pilot study was conducted, instruments adjusted and survey implemented, instructional model designed and evaluated for its effectiveness. Four achievement tests comprising physico-mathematical concepts, the questionnaire, interviews and focus group discussions were used to collect data. The Microsoft Excel package was used to generate tables and percentages. Qualitative data from interview and focus group discussions were analysed qualitatively by the MAXQDA (Max Qualitative Data Analysis) software. The results of the study revealed that students had physico-mathematical difficulties in classical mechanics bordering on the students' inabilities to use vectors, graphs, functions and mathematization in classical mechanics. Furthermore, it was found that the strategy of teaching mathematical concepts before introducing concepts in classical mechanics was used at the two institutions. However, these physico-mathematical difficulties still persisted. Therefore, the study recommended that instructors in classical mechanics should have necessary background information about students before teaching them. Information includes the prior knowledge of students. The study also recommended that instructors use suitable learning methods or materials that would engage students or make abstract physico-mathematical concepts more observable.

Keywords: *physico-mathematical, instructional model, instructors, 6L learning model*

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1. Introduction

1.1. Background to the Study

Physics education research indicates that, students' inabilities to identify or combine and apply physico-mathematical concepts to solve physics problems in a variety of situations account for poor achievement in physics courses [1,2]. In a study, Uhden and Pospiech [3] postulated that the problem may be structural where students fail to relate mathematical concepts to physics concepts other than failure to work with numbers. For instance, in classical mechanics, students see equations as relation of entities, as relationships of existing results and physical measurements [4].

In light of this, Nilsen et al [5] from their study concluded that combining meaning in physics with mathematical symbols impacts on the interpretation of physical quantities. Mwangala and Shumba [1] in a study, concluded that these difficulties in understanding physico-mathematical concepts accounts for statistics of failure in secondary school and university physics courses in Zambia. Failure to the understanding of physico-mathematical concepts may be the cause [1]. A number of studies have shown that even students scoring high grade in typical introductory physics courses often cannot apply basic physical principles to realistic situations, solve a real world problem, or organize the ideas of physics hierarchically [6]. What such students do learn, in effective, is how to solve tutorial numerical problems using search-for-equation methods that do not require much physical reasoning. Ho [7] showed in their study that even students worked more than 100 typical homework problems failed to resolve basic conceptual difficulties with classical mechanics.

But in the competitive 21st century, it is important to empower a much more diverse group of students to reason logically about physical problems in a broader set of situations to solve 21st century problems. The 21st century problems are problems that arise from lack of collaboration, communication skills, information literacy, media literacy, social skills, and others [8]. The different situations and different desired outcomes of instructions call for a different approach to teaching the course.

As regards teaching of classical mechanics Shankar [9] expounded his belief that students must first be introduced to the relevant mathematical skills so that later they can give mechanics full attention without struggling with mathematical concepts at the same time. The concept of teaching students the relevant mathematics first is not unique to classical mechanics courses. Similar pedagogical approaches have been used in other fields of physics [10,11,12].

1.2. Statement of the Problem

There is a problem in the students' conceptual understanding of physico-mathematical concepts in classical mechanics [1,13]. Students at secondary and undergraduate levels are not able to translate physico-mathematical concepts to identify, combine and apply physics [14,15,16]. Despite students' high achievement scores in mathematics and the pedagogical strategy of teaching first mathematical concepts before introducing actual classical mechanics, conceptual misunderstandings of physico-mathematical concepts still exist [17,18,19]. This problem of failing to conceptually understand physico-mathematical concepts has academically affected first and second year students because many are scoring low physics marks and therefore avoiding pursuing physics-related courses [20,21]. A possible cause of conceptually failing to understand physico-mathematical concepts is failure to connect vectors, graphs, functions and mathematization to the physical concepts in classical mechanics [22,23]. Perhaps a study, physico-mathematical conceptual barriers to students learning classical mechanics, by descriptive survey method and designing instructional models informed by meaningful-learning theories could solve the problem [24].

1.3. Research Objectives

The research objectives of this study were;

1. To identify physico-mathematical difficulties students have in classical mechanics.
2. To explore the existing instructional models and design a suitable instructional model to address physico-mathematical difficulties in classical mechanics.
3. To verify and evaluate the effectiveness of the designed instructional model.

1.4. Significance of the Study

Addressing the questions of this study is important for understanding physico-mathematical conceptual difficulties to learning classical mechanics. This adds to the valuable knowledge to handling students difficulties with these concepts. Redish [18] analyzed that the physico-mathematical language does not harmonize with the one used in mathematics since the mathematics in physics meaning to physical systems rather than expressing abstract relationships. However, physics is more than just a context for the application of mathematics [25]. This study explored the potential implications of this to wider context of quality physics education at The Copperbelt University and Mukuba University. The study could also lead to the development of a better curriculum development for sustainable development.

This may help students to learn meaningfully. In order to meet the needs of the contemporary 21st century, this study promoted the use of ICT by developing instructional models in the context of 21st century skills.

2. Research Methodology

2.1. Research Design

The research design for this study was a descriptive survey design to identify and describe physico-mathematical difficulties in students' learning classical mechanics.

2.2. Participants

The study involved 140 physics students who were taking classical mechanics at both The Copperbelt University and Mukuba University with 10 instructors. The researcher ensured confidentiality and privacy for research participants. For instance, participants were told about how their responses to the questionnaire and the audio recording and photos in the focus discussion would be used. Before undertaking these procedures consent was secured.

2.3. Data Collection Instruments

Vector probing test: This achievement test was aimed at exploring and probing students' ability to: (1) add vectors in the absence of physical quantities; (2) examine vector nature of forces and kinetic quantities; and (3) relating force to acceleration as per the second Newtonian law of motion. Structurally, vector probing test was organized as: the first 6 questions were multiple choices and were about force and speed; and question 7 to question 13 needed more cognition and was about vector magnitudes, directions and addition.

Graph probing test: This achievement test was meant to explore and assess students' ability to: (1) associate the correct functions with their respective graphs as needed in question 1; (2) extract information mathematically from the speed-time graphs to make physical conclusions as was required from question 1; (3) determine from the position-time graphs the variations in the motion of the particle as was demanded from question 2; (4) Sketch a velocity-time graph from the position-time graph as asked in question 3 and question 5; and (5) determine time from the velocity-time graph if displacement is given as was needed in question 4.

Function probing test: This achievement test was administered so as to find out the students ability to: (1) to interpret the meaning or the function given to find the needed physical quantities as required from question 2 and question 3; (2) represent functions of force in algebraic and tabular form as needed in question 1.

Mathematization probing test: For this achievement test was aimed at investigating the students' capability to: (1) translate and analyse worded-physics problems into mathematical sign and symbols as was required from all the questions.

Questionnaire: The questionnaire was designed with two sections **A** and **B**. Section **A** was aimed at exploring the students' views and perceptions about physico-mathematical concepts that are found in the achievement tests. Students were expected to respond to 20 items on the perceived level of difficulty for concepts in classical mechanics that involve physico-mathematical concepts as presented in the achievement tests. The students were to express their own perceptions of difficulties by responding on the 5-response scale: (1) very difficult (2) difficult (3) not sure (4) not difficult (5) not very difficult. While, in section **B**, students were presented with 20 statements on physico-mathematical concepts to which they were demanded to respond using a 5-response scale: (1) strongly disagree (2) disagree (3) neutral (4) agree (5) strongly agree.

Focus discussion guide: The focus group discussion was done to triangulate (meaning using more two methods of data collection) the findings in the achievement tests and the questionnaire. The researcher facilitated the 60-minute discussion. A focus discussion guide was prepared with 4 questions each demanding about 10 minutes of discussion. The discussion was tape-recorded for the researcher to analyse qualitatively the verbatim.

Interview schedule for instructors: The interview was conducted to get perceptions of instructors about: (1) how could students' physico-mathematical difficulties in classical mechanics be addressed; and (2) what instructional model could address students' physico-mathematical difficulties in classical mechanics.

2.4. Data Analysis

Focus group discussion data and interview data: Focus group data was analyzed by coding (i.e. open, selective and axial) from transcribed data to determine commonalities in the data and draw 'grounded' conclusions about students' difficulties in physico-mathematical concepts. A MAXQDA (Max Qualitative Data Analysis) software package was used to analysed and establish the commonalities in the data.

Students' responses to the questionnaire: The items in the questionnaire used the Likert scale of response. For example, section **A** of the questionnaire had: 1=very difficult, 2=difficult, 3=not sure, 4=not difficult, 5=not very difficult-ordinal level of difficulty and for section **B** of the questionnaire had: 1=strongly disagree, 2=disagree, 3=Neutral, 4=agree, 5=strongly agree ordinal level of perception. To competently analyse questionnaire-generated Likert data, the researcher must appreciate the scale of measurement. The created Likert scale items in the two sections of the questionnaire were analyzed at the interval measurement scale by calculating sum or mean from 20 items in section **A** and **B**. Boone [26] recommends the descriptive statistics for the interval scale questionnaire items including the mean score for the central tendency and for the variations the standard deviations.

Students' responses to achievement tests: Students' responses regarding the physico-mathematical conceptual understanding and scores in achievement tests (i.e. MPT, VPT, GPT and FPT) were analyzed using descriptive statistic [27]. Descriptive statistics were used to describe the meaningful insight into the physico-mathematical difficulties. The students' incorrect responses in relation to physico-mathematical conceptual understanding were analyzed qualitatively by closely examining answer scripts [1].

3. Presentation of Research Findings

3.1. Students' Responses to Physico-mathematical Concept Tests

Vector probing tests: On average, incorrect responses (66.6%, n=130) to the vector probing test indicated that achievement was unsatisfactory. The findings were summarized in Table 1 where the correct responses ranging from 12 correct responses (9.2%, n=130) in question 5 dealing with free body diagrams and net force to 83 correct responses (63.8%, n=130) in question 7 dealing with geometry and vectors. Students gave 71 correct responses (54.6%, n=130) above average in question 1 dealing with vector geometry, force, speed and constant acceleration and they had 71 correct responses (54.6%, n=130) in question 6 dealing with force analysis via free-body diagrams. The worst performance was recorded in question 5 dealing with free-body diagrams, geometry and net force recording 12 correct responses (9.2%, n=130); then in question 2 dealing with vector and scalar addition recording 15 correct responses (11.5%, n=130); followed by question 3 dealing with vector theorems and velocity recording 19 correct responses (14.6%, n=130) and question 10 dealing with vector and geometry, recording 24 correct responses (18.5%, n=130). Overall, a number of students failed the questions in the vector probing questions as can be shown by Table 1.

Graphs probing test: On average, incorrect responses (78.1%, n=129) to the graph probing test indicated that achievement was unsatisfactory. This was summarized in Table 2 where the correct responses ranging from 4.9% in question 4 to 79.8% in question 1 part (a). Students performed very well in only question 1 part (a) (slope or gradient and position-time graph, 79.8%). The worst performance was recorded in question 4 (area and the velocity-time graph, 4.7%); then in question 1 part (b) (slope of lines and position-time graph, 10.1%); question 2 part (b), part (c) and part (d) (heights, slopes and position-time graph, 10.1%). Overall, a number of students failed the questions in the graph probing questions.

Function probing test: On average, incorrect responses (76.9%, n=115) to the function probing test indicated that achievement was unsatisfactory. This was summarized in Table 3 where the correct responses ranged from 11.3% in question 2 part (e) to 67.0% in question 2 part (b).

Students performed fairly well in question 2 part (b) (calculus I, displacement and acceleration, 67.0%); question 2 part (a) (calculus I, displacement and velocity, 33.0%) and questions 2 part (d) (function, velocity, displacement and acceleration, 22.6%) and question 3 (functions, calculus and work, 22.6%). The worst performance was recorded in question 2 part (c) (functions, angles and acceleration 4.3%); followed by questions 1 part (a), and part (e) (tension in the rod and function, 11.3).

Table 1. The table showing the relationship between students' correct and incorrect responses to vector probing test questions (n=130).

QUESTION NUMBERS AND THEIR PHYSICO-MATHEMATICAL CONCEPTS		CORRECT RESPONSES		INCORRECT RESPONSES	
		Frequency (f)	Percentage (%)	Frequency (f)	Percentage (%)
1	Vectors, Geometry, Speed	71	54.6	59	45.4
2	Vector Theorems, Velocity	15	11.5	115	88.5
3	Geometry, Force	19	14.6	111	85.4
4	Geometry, Force	35	26.9	95	73.1
5	Free-body Diagram (FBD), Net Force,	12	9.2	118	90.8
6	Free-body Diagram (FBD), Net Force,	71	54.6	59	45.4
7	Geometry, Vector	83	63.8	47	36.2
8	Geometry, Vector	35	26.9	95	73.1
9	Geometry, Vector	35	26.9	95	73.1
10	Geometry, Vector	24	18.5	106	81.5
11	Geometry, Vector	47	36.2	83	63.8
12	Geometry, Vector	59	45.4	71	54.6
13	Geometry, Vector	59	45.4	71	54.6
	AVERAGE		**33.4**		**66.6**

Table 2. The table showing the relationship between students' correct and incorrect responses to graph probing test questions (n=129)

QUESTION NUMBERS AND THEIR PHYSICO-MATHEMATICAL CONCEPTS		CORRECT RESPONSES		INCORRECT RESPONSES	
		Frequency (f)	Percentage (%)	Frequency (f)	Percentage (%)
1(a)	Slope, Position-Time Graph	103	79.8	26	20.2
1(b)	Gradient, Position-Time Graph	13	10.1	116	89.9
2(a)	Heights, Slopes, Position-Time Graph	39	30.2	90	69.8
2(b)	Heights, Slopes, Position-Time Graph	13	10.1	116	89.9
2(c)	Heights, Slopes, Position-Time Graph	13	10.1	116	89.9
2(d)	Heights, Slopes, Position-Time Graph	13	10.1	116	89.9
3	Position/ Velocity-Time Graph	13	10.1	116	89.9
4	Area, Graph	6	4.7	123	95.3
5	Geometry, Graph	26	20.2	103	79.8
	AVERAGE		**21.9**		**78.1**

Table 3. The table showing the relationship between students' correct and incorrect responses to function probing test questions (n=115)

QUESTION NUMBERS AND THEIR PHYSICO-MATHEMATICAL CONCEPTS		CORRECT RESPONSES		INCORRECT RESPONSES	
		Frequency (f)	Percentage (%)	Frequency (f)	Percentage (%)
1(a)	Tension, Function	67	58.3	48	41.7
1(b)	Geometry, Force	51	44.3	64	55.7
1(c)	Geometry, Force	29	25.2	86	74.8
2(a)	Calculus I, Displacement, Acceleration	38	33.0	77	67.0
2(b)	Calculus I, Displacement, Acceleration	77	67.0	38	33.0
2(c)	Functions, Displacement, Acceleration	5	4.3	110	95.7
2(d)	Function, Velocity	26	22.6	89	77.4
2(e)	Function, Acceleration	13	11.3	102	88.7
3	Functions, Angles	26	22.6	89	77.4
	AVERAGE		**32.1**		**67.9**

Table 4. The table showing the relationship between students' correct and incorrect responses to mathematization probing test questions (n=101)

QUESTION NUMBERS AND THEIR PHYSICO-MATHEMATICAL CONCEPTS		CORRECT RESPONSES		INCORRECT RESPONSES	
		Frequency (f)	Percentage (%)	Frequency (f)	Percentage (%)
1	Ratio, Angles, Net Force	88	87.1	13	12.9
2	Ratio, Pulleys, Levers	51	50.5	50	49.5
3	Charge Units, Ratios	15	14.9	86	85.1
4(a)	Units, Conversion, Ratio	25	24.8	76	75.2
4(b)	Dimensions, Units	39	38.6	62	61.4
4(c)	Units, Conversion, Ratio	97	96.0	4	4.0
5	Unit Analysis	25	24.8	76	75.2
	AVERAGE		**48.1**		**51.9**

Mathematization probing test: On average, incorrect responses (51.9%, n=101) to the mathematization probing test indicated that achievement was unsatisfactory. This was summarized in Table 4 where the correct responses ranged from 24.8% in question 4 part (a) to 96.0% in question 4 part (c).

Students performed extremely well in question 4 part (c) (units conversion, ratio, 96.0%); question 1 (ratios, angles and resultant force, 87.1%) and average in questions 2 (ratio, pulleys and levers). The worst performance was recorded in question 4 part (a) and question 5 (unit analysis, 24.8%); then in question 3 (charge units and ratios, 37.6%) and followed by questions 4 part (b) (dimensions and units, 38.6%). Overall, a considerable number of students failed the questions in the mathematization probing questions as shown in Table 4.

3.2. Students' Perceptions to Physico-mathematical Concepts

Difficulties encountered when using vectors in classical mechanics: After analysis using the MAXQDA five common trends emerged from the data. Firstly, 2 (5%) of the participants in the focus discussion indicated that they understood how to combine vectors and the physics concepts. This was in reference to question two of the vector probing test. This implied that 38 (95%) of the respondents had difficulties in combining vector concepts with physics concepts. Secondly, 7 (17.5%) of respondents stated that did acquire the skill to analyse forces and velocity via the free-body diagrams. The worrisome implication was that 33 (82.5%) students had difficulties in analysing forces and other vector quantities using the free-body diagrams. Thirdly, from 13 (32.5%) students were of the view that interpreting vectors in the graphical was generally easy. This meant that 27 (67.5%) respondents struggled with interpreting vectors especially if presented in graphical form. Fourthly, just 4 (10%) students were able to solve and resolve coplanar forces using geometry. On the other hand, 36 (90%) respondents had difficulties in using geometrical concepts when solving coplanar forces. Lastly, 8 (20%) of the respondents were of the view that identifying vectors with identical magnitudes and direction was not difficult. This implied that 32 (80%) students could not competently identify vectors with identical magnitudes and direction.

Difficulties encountered when using graphs in classical mechanics: There were equally five common trends that emerged from the data. Firstly, 5 (12.5%) of the participants in the focus discussion indicated that they understood how to differentiate between the slope and height of the position-time graph or velocity-time graph. This implied that 35 (87.5%) of the respondents had difficulties in differentiating between the slope and height of the position-time graph or velocity-time graph. Secondly, 12 (30%) of respondents stated that did acquire the skill to interpret changes in height and changes in slope of the position-time graph and make sense out of them. The worrisome implication was that 28 (70%) students had difficulties in interpreting changes in height and changes in slope of the position-time graph and make sense out of them. Thirdly, from 14 (35%) of the students were of the view that relating one type of position-time graph to the other was generally easy. This meant that 26 (65%) of the respondents struggled with relating one type of position-time graph to the other. Fourthly, just 3 (7.5%) students were able to give interpretations of area under the position-time graph or velocity-time graph. On the other hand, 37 (92.5%) respondents had difficulties in giving interpretations of area under the position-time graph or velocity-time graph. Lastly, 10 (25%) of the respondents were of the view that representing a negative velocity on a velocity-time graph was not easy. This implied that 30 (75%) students could not competently represent a negative velocity on a velocity-time graph.

Difficulties encountered when using functions in classical mechanics: There were three common trends that emerged from the data under functions. Firstly, 6 (15%) of the participants in the focus discussion indicated that they understood how to represent functions for a central force in algebraic form. This implied that 34 (85%) of the respondents had difficulties in representing functions for a central force in algebraic form. Secondly, 11 (27.5%) of respondents stated that did acquire the skill to represent displacement in graphical form and acceleration in tabular form. The implication was that 29 (72.5%) students had difficulties in representing displacement in graphical form and acceleration in tabular form. Lastly, 15 (37.5%) of the students were of the view that interpreting the meaning of the function of force to find the desired quantities was not easy. This meant that 25 (62.5%) of the respondents struggled with interpreting the meaning of the function of force to find the desired quantities.

Difficulties encountered when using mathematization in classical mechanics: There were three common trends that emerged from the data under mathematization. Firstly, 9 (22.5%) of the participants in the focus discussion indicated that they understood how to represent a proportional relationship with an algebraic statement. This implied that 31 (77.5%) of the respondents had difficulties in representing a proportional relationship with an algebraic statement. Secondly, 13 (32.5%) of respondents stated that did acquire the skill to quantify physical quantities and use appropriate symbols, variables and constants. The implication was that 27 (67.5%) students had difficulties in quantifying physical quantities and using appropriate symbols, variables and constants. Lastly, 17 (42.5%) of the students were of the view that reasoning with ratio in relation to physical units was not easy. This meant that 23 (57.5%) of the respondents struggled with ratio manipulation in relation to physical units.

3.3. Instructors' Perceptions on How to Address the Students' Physico-mathematical Difficulties

Addressing physico-mathematical difficulties in classical mechanics: There were 7 themes or commonalities that emerged from the data transcribed from the interview with instructors. Firstly, 6 (75%) of the instructors in the interview indicated that learning must be 'fascinating' for students as students work with vectors in classical mechanics. The second commonality was that 5 (62.5%) of the respondents stated that 'retention' of PM concepts in student should never be sacrifice on the altar of covering the syllabus. In addition to retention, 7 (87.5%)

of the instructors felt that employing 'observable' simulation for the student to 'see' vector or other concepts in the physico-mathematical in the physical sense. Fourthly, innovation in teaching physico-mathematical concepts was perceived by 4 (50%) respondents to help address the problem. Fifthly, 8 (100%) of the participants were of the view that the best way to address the physico-mathematical difficulties was to make sure that all learners are 'engaged' in terms of action and thinking. Further, 5 (62.5%) of respondents stated that teaching or learning methods must be holistic in nature to carter for the purpose of learning. As regards instructors knowing the learner, 6 (75%) perceived that having information about students' academic strength and weakness.

4. Discussion

4.1. Research Objective One

Participating students within the study showed misconceptions and having difficulties in physico-mathematical concepts. This is according to the themes that emerged from data where students encountered difficulties involving use of graphs, vectors, functions and mathematization in classical mechanics.

Vectors: From the unsatisfactory performance in the vector probing test, students fail to use vector geometry to solve physical quantities like net force. The dire consequence is failure to analyse the physical situation using the free-body diagrams (McCarthy & Goldfinch, 2012). This is crucial to analyzing and solving many problems in classical mechanics (McCarthy & Goldfinch, 2010). The other students' difficult was failure to add vectors of physical quantities according to the situation. The perceptions as evidenced by the responses that students gave to the questionnaire items indicated that students had difficulties when using vectors in classical mechanics. For example, many students perceived that addition of vectors in the absence of force velocity in 2-D was difficult. In addition, others gave the perceptions that it was a challenge to interpret vector concepts in graphical form. Some other students, the difficult was in identifying vectors with identical magnitudes and directions. The focus group discussion revealed the difficulties that students had in classical mechanics when using vectors were: (1) analyzing force via free-body diagrams; (2) geometry and coplanar forces; (3) interpreting vectors in graphical form; and (4) identifying vectors with identical magnitudes and directions.

Vectors are important to mathematical language of classical mechanics [28]. The implications for these kinds of results could be students' misconceptions. For example, Steinberg [29] found misconceptions of force-and-motion concepts in first year students. Insufficient knowledge of vectors applied to physics leads to many students failing to draw and interpret free-body diagrams and superposition of forces [14]. In a study, students failed to show functional understanding of vectors when interpreting and coding vector equations [30]. Some could not even set up the sign conventions correctly [24]. These difficulties in vectors result in students fail to grasp higher concepts in quantum mechanics and classical mechanics [31]. This leads many students to shun taking careers in physics education or physics itself [32]. This does not help in implementing the part of Zambia vision 2030 which encourages innovation in science and technology [33].

Graphs: From the unsatisfactory performance in the graph probing test, it was clear that students had difficulties in classical mechanics when using graphs. The difficulties fell in two categories. The first, students were failing to relate graphs of motion to the physical concepts. For example, students could not: (1) interpret in a physical sense changes in height and slope of the graph of motion; (2) relate from one type of motion graph to the other graph; (3) interpret area under the position-time or velocity-time graphs; and (4) represent a negative velocity-time graph. This is similar to the findings by other researchers [34, 35]. The second was relating graphs to the real world. For example, students fail to separate the shape of the graph from that of the path of the motion among others [36].

Functions: Overall, the performance in the function probing test was unsatisfactory. It was clear that students had difficulties in classical mechanics when using graphs. One of the difficult was the combinations of angles and functions to determine acceleration. The other was coupling the tension in the rope and the function. The focus discussion and the questionnaire revealed that students had challenges in: (1) representing functions for central force in the algebraic form; (2) representing displacement in graphical form and acceleration in tabular form; and (3) the other were the interpretation of the function of force to find the desired quantities.

Wittmann [37] revealed that the sources of these difficulties were failure by students to manipulate the mathematics formalism often indicating the inability to relate and interpret mathematics in physics. The result, for example, students fail to comprehend the general ideas of functions [38]. Additionally, they fail to recognize the connection between the physical situations and the associated functional expression [14,39].

Mathematization: While in some questions students performed extremely well, in many questions overall the performance was not satisfactory. When mathematizing concepts in physics concepts students had challenge in analyzing units, ratios and dimensions. The questionnaire and focus discussion findings indicated that students had: (1) reasoning with ratios in the physical sense; (2) represent proportional relationship with an algebraic statement; (3) quantifying physical quantities; and explain and use appropriate symbols variable and constants. For example, students fail to understand the general ideas of mathematization of physics concepts [40].

4.2. Research Objective Two

Addressing physico-mathematical difficulties in classical mechanics: There were 7 themes or commonalities that emerged from the data transcribed from the interview with instructors. Firstly, 6 (75%) of the instructors in the interview indicated that learning must be 'fascinating' for students as students work with vectors in classical mechanics. The second commonality was that 5 (62.5%) of the respondents stated that 'retention' of PM concepts in student should never be sacrifice on the altar of covering the syllabus. In addition to retention, 7 (87.5%)

of the instructors felt that employing 'observable' simulation for the student to 'see' vector or other concepts in the physico-mathematical in the physical sense. Fourthly, innovation in teaching physico-mathematical concepts was perceived by 4 (50%) respondents to help address the problem. Fifthly, 8 (100%) of the participants were of the view that the best way to address the physico-mathematical difficulties was to make sure that all learners are 'engaged' in terms of action and thinking. Further, 5 (62.5%) of respondents stated that teaching or learning methods must be holistic in nature to carter for the purpose of learning. As regards instructors knowing the learner, 6 (75%) perceived that having information about students' academic strength and weakness.

4.3. Research Objective Three

The research objective three was addressed using three 'dimensions'. The first dimension was the physico-mathematical difficulties that students had in classical mechanics. The second dimension helps to establish how the already designed instructional models or strategies addressed physics pedagogical challenges as suggested by instructors. The third dimension was the conceptual framework. The third dimension helped to ensure that the designed instructional model was relevant to Zambian students, encouraged ICTs (Information, Communication Technologies), 21st century skills and encouraged meaningful learning theories. Addressing the research question three in these dimensions resulted in designing the instructional model.

4.3.1. The 6L Instructional Model and Suggested Application

Based on the physico-mathematical difficulties, the already existing instructional models with their 'gaps' and the conceptual framework of the study, an instructional model was designed by the researcher. This was called *6L instructional model*. This was the 6-phased instructional model as shown in Figure 1.

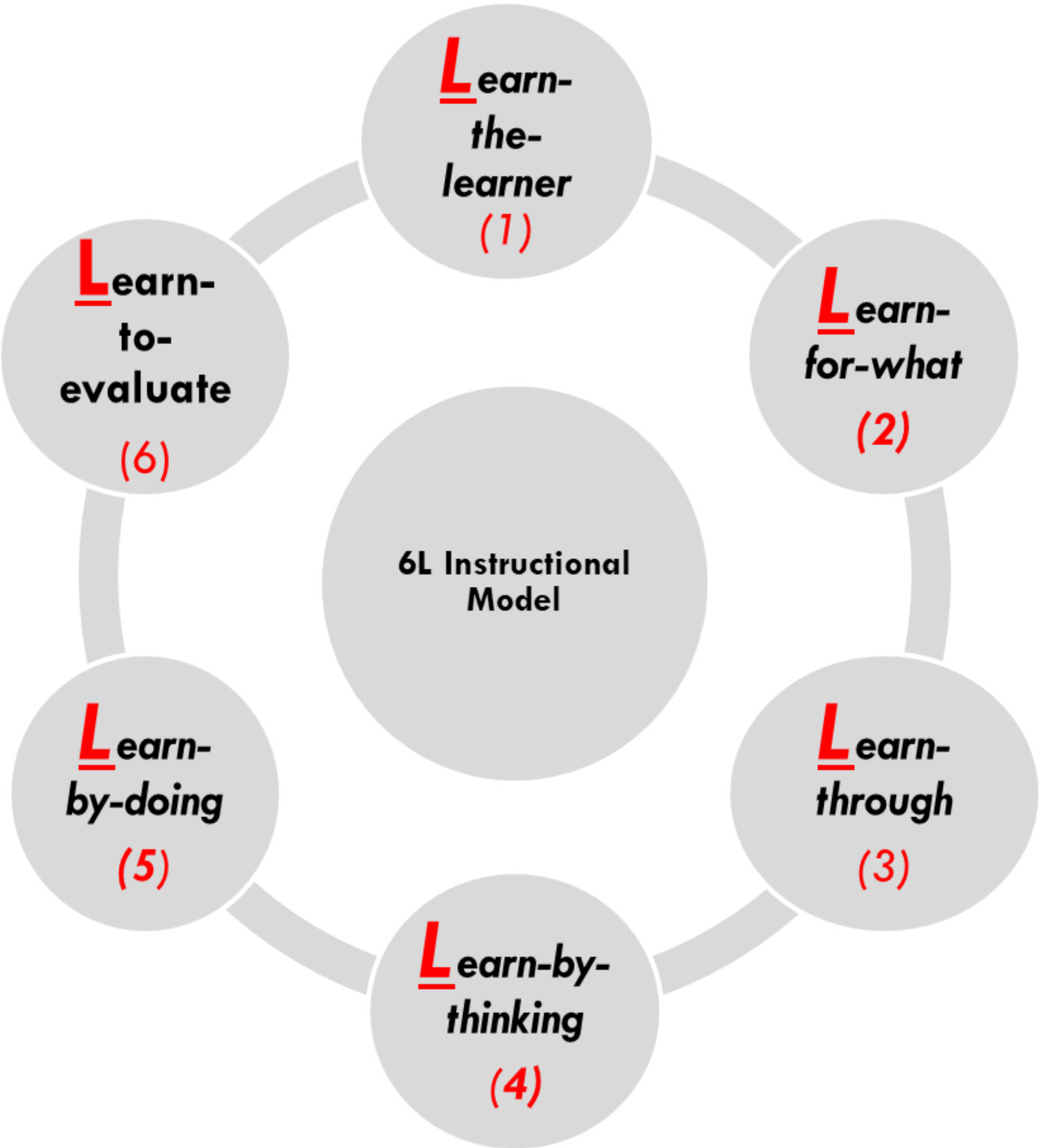

Figure 1. The 6L instructional model to address physico-mathematical

The 6 phases were: (1) *Learn-the-learner;* (2) *Learn-for-what;* (3) *Learn-through;* (4) *Learn-by-thinking;* (5) *Learn-by-doing;* and (6) Learn-to-evaluate [41,42,43]. The detailed description and suggested application of the 6L instructional model were discussed under:

Learn-the-learner: In this phase the instructor learned students in 3-D (dimensions): (1) Career awareness which may help to know the relevant skills to pursue the given career. Boone [44] argued that career development should inform pedagogical discourse. For example, when teaching use of graphs in classical mechanics to medical students, the instructor may emphasize *interpreting* of changes in height or slope in graphs of motion. (2) Age/level to adapt the presentation of physico-mathematical concepts in classical mechanics. For example, when analyzing forces via free-body diagrams different concepts can be taught at different levels. (3) Prior/knowledge/Connections/experiences: Breslow [45] found that prior knowledge had a strong influence on meaningful learning PM concepts. For example, if the instructor wanted to teach how to add coplanar forces, he or she would have to assess the geometrical understanding.

Learn-for-what: The objective of learning should be for: (1) Skills and concept necessary for a successful life. Skills that shape learners' identities guide their responses to failure or success and prepare them to tackle the difficult twenty-first century problems [46]. The document added that within the context of core knowledge instruction, students must also learn the essential skills for success in today's world, such as critical thinking, problem solving, communication and collaboration. Therefore, learning should be *active* and *adaptive*. For example, when the students were learning the application of function to forces, they could be demanded to *interpret* the function of force to find the desired quantities. To make learning *adaptive* they should be able to interpret similar real situations. (2) Behaviour: Learning should go beyond skills and concepts in classical mechanics. For example, when student interpreted graphs of motion in classical mechanics in group discussions, it would encourage flexibility, communication, social and cross-cultural skills.

Learn-through: The instructors selects: (1) method; (2) media; and (3) materials. For example, when students are learning *sketching* v(t), and a(t) from the position of the particle, the instructor could select cooperative learning as a *method*, computer-assisted simulations as the *medium* and education game as *materials.*

Learn-by-thinking: Drawing on literature in cognitive psychology and neuroscience, Evans and Stanovich [47] proposed that one of the components of learning was reflection or intentionally attempting to synthesize, abstract or articulate the key lessons taught by experience. Minds-on activities should part of the learning process. Therefore, provide direct learning experiences that can engage students. For example, students can play and manipulate the motion of the car to interpret the graphs of motion physically and mathematically.

***Learn-by-doing*:** Hands-on activities should be a part of the learning process. Therefore, an instructor provides student engagement strategies with the methods, media and materials fittingly chosen. Learning by doing and learning by thinking as complementary [48]. For example, an instructor teaching sketching velocity-time, displacement-time, and acceleration-time graphs could select cooperative learning and Genius Maker software to engage students.

Learn-to-evaluate: Evaluate student performance using a rubric, instructor performance using students and media effectiveness.

Table 5 illustrates how the model can be in a class pedagogical discourse.

4.3.2. Verifying and Evaluating the Effectivenessof the 6L Instructional Model

The purpose of this study was not only to design the 6L instructional model but also to verify and evaluate its effectiveness in addressing the physico-mathematical difficulties. Simons [49] suggested a case study for an in-depth exploration from multiple perspectives of a particular project, plan of action, institution, program or system in a real context. Since one of the research objectives was to verify the effectiveness of the model, a case study was considered an appropriate method. Mixed methods included the qualitative data from researcher's interview with students after implementing the 6L instructional model [49,50]. The 6L instructional model was applied in the class taking classical mechanics at The Copperbelt University in which students were from diverse majors. The total of 96 students was interviewed after classroom session.

From the post-6L-interview it was evident that about 80(83.3%) were of the opinion that when learning teachers should know their career, their prior knowledge and experiences in order to cater for the skills relevant to them [49-54]. For example, at implementation of phase (I) of the model, students were asked their careers of interest and 72(75%) in class were interested in medicine, therefore the discussion focused on relating the interpretation of slope or height of the velocity-time to interpretation of medical graphs [6,35,54]. The study revealed that 74(77.1%) that the strategy from phase (II) of the model of clearly explaining graphs of motion in relation to their careers made them focus and see the relevance of the graphs of motion.

Table 5. The table shows the 6L Instructional Model class activities for each phase of the model when learning graphs of motion

PHASES	LEARNER-INSTRUCTOR CLASS ACTIVITIES
(1) Learn-the-learner	Instructor asked learners prior information (career, etc.) and many gave medicine, engineering. The discussion focused on relating graphs of motion to real examples in those fields.
(2) Learn-for-what	The explanation and discussion of graphs of motion by student in relation to their careers made them focus and see the relevance of interpretation (skill) of the slope or height of graphs of motion.
(3) Learn-through	This made the instructor to use appropriate or suitable instructional materials closest to real students' life experience like the Genius Maker software.
(4) Learn-by-thinking (5) Learn-by-doing	As students worked with the Genius Maker simulating graphs of motion they imagined and manipulated the values of velocity and time while relating to the real-world.
(6) Learn-to-evaluate	On evaluating their learning outcomes, rubric was made available to students which clearly explained the demands of each of the question; namely: derive, explain relevant theories the, units.

As regards phases (III), (IV) and (V) of the model, the study showed that 81(84.4%) students enjoyed the experience learning sketching of the velocity-time graphs in groups, using simulations from the Genius Maker software [55,56]. Three-in-four of those that enjoyed this experience said that helped to thinking and act and relate manipulation of velocity-time graph in the real-world [57]. Pertaining to phase (VI) of the model, 90(93.8%) suggested that having the rubric helped them answer question according to the demands of an instructor [57]. For example, when students asked to derive the third equation of motion, the rubric demanded explaining clearly each step and theories relevant to the derivation.

5. Conclusion

The study revealed that students taking classical mechanics experience difficulties when using vectors, graphs, functions and mathematization. Firstly, in using vectors these difficulties included failing to: (1) combine vector concepts with physics concepts; (2) analyse forces and other vector quantities using free-body diagrams; (3) interpret vectors especially those presented in graphical form; (4) use geometrical concepts resolving coplanar forces; and (5) identify vectors with identical magnitudes and direction. Secondly, in using graphs these difficulties included failing to: (1) differentiate between the slope and height of the position-time graph; (2) interpret changes in height and changes in height and changes in slope of the position-time graph; (3) relate one type of position-time graph to the other; (4) interpret the area under the position-time or velocity-time graphs; and (5) represent a negative velocity on a velocity-time graph. Thirdly, in using functions these difficulties included failing to: (1) represent functions for a central force in algebraic form; (2) represent displacement in form and acceleration in tabular from; and (3) interpret the meaning of the function of force. Lastly, in using mathematization these difficulties included failing to: (1) represent a proportional relationship with an algebraic statement; (2) quantify physical quantities and using appropriate symbols variables and constants; and (3) manipulate ratio and physical units.

The study also revealed that high failure rates in classical mechanics were due to misconceptions of physico-mathematical concepts. Physico-mathematical concepts were essential to meaningfully learn classical and quantum mechanics. Thus, the need for the high quality physics education.

In addition it was evident from the study that to address physico-mathematical difficulties in classical mechanics was by: (1) making pedagogical discourse fascinating to students; (2) prioritizing retention of concepts; (3) making concepts observable by simulating them; (4) using innovative learning methods and materials; and (5) knowing the academic strength or weakness of students.

Further, this study introduced a new idea of the designed instructional model. The designed instructional model should be investigated further within physics education research, perhaps an experimental kind of study. In addition, this study provided further insight into the other instructional models designed in physics education.

While the study revealed the physico-mathematical difficulties in classical mechanics experience by students, it also designed an instructional model to address the difficulties.

Based on the findings of the study the following are recommended:

1. Before undertaking the learning process of the physico-mathematical concepts in classical mechanics, instructors should have necessary information about students like:

- Career pathway to focus on relevant skills.
- Prior knowledge or information that students have.

2. Instructors should make sure that their students know the purpose of learning the particular physico-mathematical concepts in classical mechanics.

3. Instructors should choose suitable instructing methods or materials.

Instructors should encourage students to learn-by-thinking or learn-by-doing to engage them in the learning process.

The study focused on identifying the students' physico-mathematical difficulties in classical mechanics. It also focused on designing an instructional model to address the difficulties. This study ought to be seen as a preliminary effort in this area of physics education research (PER).

There is need for further research which would focus specific issues such as:

1. An experimental study, to determine the effect or impact of the designed instructional model, 6L Learning Model, on meaningful learning of physico-mathematical concepts in classical mechanics.
2. A comparative study in the physico-mathematical difficulties that students experience at the two learning institutions.
3. A comparative study in the physico-mathematical difficulties between male students and female students.

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