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# Problem Solving Heuristics on Non-Routine Problems of College Students

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**Abstract** In the K-12 curriculum, problem solving and critical thinking skills have been the center of the framework for Mathematics curriculum in order to develop lifelong learners. With this, problem solving in Mathematics is highly valued. This descriptive comparative study determined the level of problem solving heuristics on non-routine problems of college freshmen at Mindoro State College of Agriculture and Technology. Employing a self-structured problem solving test composed of five non-routine problems, result showed that most of the students were classified as apprentice in heuristics knowledge which means that the skills and strategies used in general have focus but with limited clarity. In procedural knowledge, most of the students were also classified as apprentice which means that generally, most of the students made partial use of appropriate procedures and were not precise in using mathematical terms, principles and procedures. In conceptual knowledge, most of the students were also classified as apprentice which means that the students can extract the essence of the problem but most of them were unable to use this information to solve the problem properly. However, problem solving skills of college students did not differ in terms of heuristics, procedural and conceptual knowledge. Furthermore, the study showed that the most commonly used heuristics are making a model or diagram, using a formula, eliminating, and considering a simpler case. Relevant to this, the study recommended that Mathematics faculty should develop effective measures to enhance problem solving skills of the students such as integration of heuristics in teaching Mathematics problem solving and more exposure to non-routine problems.

***Keywords:*** *heuristics, Mathematics, problem solving*

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## 1. Introduction

Problem solving in Mathematics can be explained as "thinking and working" mathematically [1]. It is an intricate process which calls for a problem solver who is engaged in a mathematical task to organize and deal with specific and general pieces of knowledge. Problem solving expertise is highly valued and is considered as one of the primary skills a student should have [2]. In the K – 12 curriculum, problem solving and critical thinking skills have been the center of the framework for Mathematics curriculum in order to develop lifelong learners. Aside from developing the critical and analytical thinking skills of the students, problem solving promotes conceptual understanding and meaningful learning in mathematics. Candelaria and Limjap [3] reported that when the students are given the opportunity to reflect on their experiences when they confront problem situations, they learn to hypothesize their own ways of reasoning in mathematics. It is further detailed that the students come to recognize their own learning process and are able to face problem situations which facilitate their understanding of the mathematics concepts.

However, problems may be classified as routine if it follows a certain procedure in order to arrive at a solution. Non-routine problems help develop critical and creative thinking among students. Non-routine problems enables solvers to use different solutions and approaches in attacking the problem [4]. Furthermore, application, synthesis and creation are higher order thinking skills in which the students will use when they are provided with a realistic situation.

According to Polya [5], solving a routine problem does not contribute to the mental development of the students. He believed that to provide an opportunity for students to develop higher order thinking skills in the process of understanding, analysis, exploration and application of mathematical concepts, non – routine problems should be employed. However, students generally fear the idea of solving non – routine problems because these problems are usually non – standard, involving unexpected and unfamiliar solutions. Also, students are apprehensive and extremely uncomfortable because they are not able to recall and apply learned procedures in a straightforward. These manifest that students are not much aware in the use of heuristics in solving problems since these are not explicitly taught by the teachers.

Problem solving heuristics are not clear – cut rules in coming up with the correct answers, rather they are possible solutions for certain problems. However, successful problem solving includes four steps: a) understanding the problem; b) choosing strategy; c) solving the problem; and d) looking back. In order to achieve a successful resolution of the problem, students may show their potentials in problem solving with clear and good reasoning [4].

Krulik and Rudnick [2] identified eight strategies (along with their definitions) that are applicable to mathematical problem solving namely: 1. Computing or Simplifying (CS); 2. Using a Formula (F); 3. Making a Model or Diagram (MD); 4. Making a Table, Chart, or List (TCL); 5. Guessing, Checking, and Revising (GCR); 6. Considering a Simpler Case (SC); 7. Eliminating (E); and 8. Looking for Patterns (LP). Although using these strategies have no assurance in finding an optimal solution, they do consistently led the students to a plausible solution and are especially helpful when investigating a totally unfamiliar problem [4].

Several literatures and studies support the key role of problem solving in Mathematics curriculum and the use of heuristics as aids in problem solving process. In order to provide students with high-quality mathematics instruction, the integration of problem solving skills into the mathematics curriculum is necessary [6]. Lee & Chen [7] cited that teaching students how to implement heuristic reasoning and strategies can greatly improve their self-efficacy and level of achievement when they are engaged in problem solving activities. Problem solving, sometimes, requires a combination of strategies. Evidences showed limited range of strategies being taught in the classrooms as well as limited type of problems that are mostly routine problems [4]. In the case study of Laset et al. [8], they investigated on the individual's use of strategies in problem solving in which they found out that junior high school students could employ different problem solving strategies without prior instruction, if given the chance to solve non-routine problems. Another study investigated elementary mathematics teacher candidates' problem solving strategies in routine problems in geometry classes in which he recommended that teachers should be a model to students by using different problem solving strategies in geometry classes and they should encourage students to use different strategies, so it is thought to be useful to give training to teachers and teacher candidates on teaching problem solving strategies [9].

The present study aimed to explore the students' utilization of heuristics in solving non-routine problems to serve as baseline information for possible strategic intervention in enhancing students' problem solving skills especially in non-routine problems. Aside from the assessment of the students' knowledge on problem solving heuristics, this study identified the commonly used problem solving strategies used by students which is vital in the conduct of possible intervention project to enhance their problem solving skills as implication for teaching and learning problem solving.

## 2. Objectives of the Study

Generally, this study determined the level of problem solving skills and problem solving heuristics of college students on non-routine problems. Specifically, it aimed to describe and determine the significant differences in the students' level of problem solving performance of the students on non-routine problems in terms of heuristics knowledge, conceptual knowledge and procedural knowledge. Moreover, it also aimed to identify the commonly used heuristics by the freshmen college students in each non-routine problems. Lastly, to propose a strategic intervention on using heuristics on non-routine problems.

## 3. Hypothesis of the Study

This study tested the hypothesis:

$H_0$: There is no significant difference in the level of problem solving skills of the college students in terms of heuristics knowledge, conceptual knowledge and procedural knowledge.

## 4. Materials and Methods

The study employed descriptive comparative methods of research. Subjects of the study were 30 college students selected from the various programs of non-Mathematics majors in MinSCAT - Calapan City Campus using stratified non-proportional technique. In this study, the level of problem solving skills of the students on non-routine problems specifically in heuristics knowledge, conceptual knowledge and procedural knowledge were described and were compared across indicators.

A self-structured test consisting of five non-routine problems was used as research instrument of this study. This test was validated by five Mathematics professors who are experts in the field. Test-retest method was used in testing the reliability of the test which were given to 15 non-respondents of the study over a ten-day interval. The test was found to be highly reliable with Cronbach's alpha coefficient of 0.822. These five non-routine problems were used in this study to measure the problem solving performance of students on non-routine problems in terms of problem solving strategies, conceptual knowledge and procedural knowledge on heuristics. Oregon Mathematics Problem Solving Rubrics (OMPSR) was also used in evaluating the students' problem solving performance. OMPSR is a rubric for assessing the problem solving skills of the students in terms of heuristic knowledge, conceptual knowledge and procedural knowledge. This was used to classify the performance of the subjects as to novice, apprentice or proficient problem solvers.

The OMPSR is given below:

**Table 1. Classification of the different levels of conceptual understanding**

| Full Conceptual Understanding (Proficient) | Partial Conceptual Understanding (Apprentice) | Lack of Conceptual Understanding (Novice) |
|---|---|---|
| The student uses all relevant information to solve the problem. | The student extracts the "essence" of the problem, but is unable to use this information to solve the problem. | The student's solution is inconsistent or unrelated to the question. |
| The student is able to translate the problem into appropriate mathematical language. | The student is only partially able to make connections between/among the concepts. | The student translates the problem into inappropriate mathematical concepts. |
| The student's answer is consistent with the question/problem. | The student's solution is not fully related to the question. | The student uses incorrect procedures without understanding the concepts related to the task. |

**Table 2. Classification of the use of procedural knowledge**

| Full Use of Appropriate Procedures (Proficient) | Partial Use of Appropriate Procedures (Apprentice) | Lacks Use of Appropriate Procedures (Novice) |
|---|---|---|
| The student uses principles efficiently while justifying the solutions. | The student is not precise in using mathematical terms, principles, or procedures. | The student uses unsuitable methods or simple manipulation of data in his/her attempted solution. |
| The student uses appropriate mathematical terms and strategies. | The student is unable to carry out a procedure completely. | The student fails to eliminate unsuitable methods or solutions. |
| The student solves and verifies the problem. | The process the student uses to verify the solution is incorrect. | The student misuses principles or translates the problem into inappropriate procedures. |
| The student uses mathematical principles and language precisely. | | The student fails to verify the solution. |

**Table 3. Classification of heuristics knowledge**

| Thorough/Insightful Use of Skills/Strategies (Proficient) | Partial Use of Skills/Strategies (Apprentice) | Limited Skills/Strategies (Novice) |
|---|---|---|
| The skills and strategies show some evidence of insightful thinking to explore the problem. | The skills and strategies have some focus, but clarity is limited. | The skills and strategies lack a central focus and the details are sketchy or nor present. |
| The student's work is clear and focused. | The student applies a strategy which is only partially useful. | The procedures are not recorded (i.e., only the solution is present). |
| The skills/strategies are appropriate and demonstrate some insightful thinking. | The student starts the problem appropriately, but changes to an incorrect focus. | Strategies are random. The student does not fully explore the problem and look for concepts, patterns or relationships. |
| The student gives possible extensions or generalizations to the solution or the problem. | The student recognizes the pattern or relationship, but expands it incorrectly. | The student fails to see the alternative solutions that the problem requires. |

Interviews from the subjects were also done to validate their responses on the test provided.

Data gathered from the problem solving test and interview were treated statistically using descriptive statistics namely, frequency, percentage and mean. One way ANOVA was used to determine the magnitude of difference among the variables. Homogeneity of variances was checked prior to the use of analysis of variance in order to ensure that variances of each population is equal as an assumption of ANOVA.

# 5. Results

### 1. Level of problem solving performance of the College Freshmen students on non-routine problems in terms of heuristics knowledge, conceptual knowledge and procedural knowledge.

### 1.1 Level of Problem Solving Performance on Non-routine Problems in terms of Heuristics Knowledge

**Table 4. Level of Problem Solving Performance on Non-routine Problems in terms of Heuristics Knowledge**

| Performance | Frequency | Percentage (%) |
|---|---|---|
| 18 and above (Proficient) | 4 | 13.33 |
| 10 – 17 (Apprentice) | 21 | 70 |
| 9 and below (Novice) | 5 | 16.67 |
| **TOTAL** | **30** | **100** |

Overall Mean: 13.5 Description: Apprentice.

Table 4 presents the mean level of problem solving performance of the respondents in terms of heuristic knowledge.

It can be observed from the table that twenty – one (21) or 70% of the 30 respondents were classified as Apprentice, while five (5) or 16.67% were classified as Novice and Four (4) or 13.33% of the student-respondents were classified as Proficient.

Moreover, it also shows that most of the students were classified as Apprentice in solving non-routine problems in terms of heuristics knowledge as revealed by the overall mean of 13.5. This means that the students are not that proficient in utilizing different heuristics in solving certain problem.

### 1.2 Level of Problem Solving Performance on Non-routine Problems in terms of conceptual knowledge

**Table 5. Level of Problem Solving Performance on Non-routine Problems in terms of conceptual knowledge**

| Performance | Frequency | Percentage (%) |
|---|---|---|
| 18 and above (Proficient) | 2 | 6.67 |
| 10 – 17 (Apprentice) | 20 | 66.67 |
| 9 and below (Novice) | 8 | 26.67 |
| **TOTAL** | **30** | **100** |

Overall Mean: 11.8 Description: Apprentice.

Table 5 presents the respondents' mean level of problem solving performance Non-routine Problems in terms of conceptual knowledge.

As shown in the table, twenty (20) or 66.67% of the respondents got average level of performance classified as Apprentice, eight (8) or 26.67% got below average performance classified as Novice, while only two (2) or 6.67% of the students fall within 18 and above bracket which was categorized as Proficient.

The table also implies that the student – respondents were classified as Apprentice in solving non – routine problems in terms of conceptual knowledge as indicated by the overall mean of 11.8. This can be noted that the students have an average level of problem solving

performance on non-routine Problems in terms of conceptual knowledge.

### 1.3 Level of Problem Solving Performance on Non-routine Problems in terms of Procedural knowledge

**Table 6. Level of Problem Solving Performance on Non-routine Problems in terms of Procedural Knowledge**

| Performance | Frequency | Percentage (%) |
|---|---|---|
| 18 and above (Proficient) | 3 | 10 |
| 10 – 17 (Apprentice) | 16 | 53.33 |
| 9 and below (Novice) | 11 | 36.67 |
| **TOTAL** | **30** | **100** |

Overall Mean: 11.03 Description: Apprentice.

Table 6 presents the respondents' mean level of problem solving performance of the respondents in terms of Procedural knowledge.

There were sixteen (16) or 53.33% of the respondents fall on the bracket of 10-17 which classified as Apprentice, while eleven (11) or 36.67 got a score of 9 and below which can be classified as Novice. On the other hand, only three (3) or 10% fall within the bracket of 18 and above which classified as Proficient. It has an overall mean of 11.03 described as Apprentice.

The result implies that most of the students are more precise in using mathematical terms, principles or procedures. They enable to carry out a procedure completely, however the process they used to verify the solution is incorrect.

### 2. Analysis of Variance on the Mean Difference in the Students' Level of Problem Solving Performance in terms of Problem Solving Strategies, Conceptual Knowledge and Procedural Knowledge

Table 7 presents analysis of variance on the mean difference in the students' level of problem solving performance in terms of problem solving strategies, conceptual knowledge and procedural knowledge.

As shown from Table 7, the computed p-value of 0.160333 exceeded the 0.05 level of significance, thus the null hypothesis of no significant difference in the students' level of problem solving performance in terms of problem solving strategies, conceptual knowledge and procedural knowledge is accepted. This indicates that students' level of problem solving performance in terms of conceptual knowledge do not show difference in terms of procedural knowledge. This might be due to the fact that conceptual knowledge is knowledge rich in relationships and understanding of a particular topic which has something to do with procedural knowledge. This means that procedures on how to solve certain math problem could not be learned without conceptual understanding.

### 3. The Commonly Used Heuristics by the College Students in each of the Non-Routine Problems

Table 8 presents the heuristics commonly used by college students in problem number 1.

The Table 8 implies that thirteen (13) or 43.33% of the respondents utilized both using a Formula (F) and Eliminating (E) strategy to solve the problem, while two (2) or 6.67% both used Computing or Simplifying (CS) and Making a Table, Chart or List (TCL) strategy.

This can be a clear indication that the student-respondents have answered Problem 1 using Eliminating (E) and Using A Formula (F) strategy. This further implies that students chose to use procedural representation of the problem in order to arrive with the right solution.

### 3.2 Heuristics Commonly Used by Freshmen College Students in Problem Number 2

Table 9 identifies the heuristics commonly used by college students in problem number 2.

It can be noted that eighteen (18) or 60% of the respondents used Eliminating (E) strategy, six (6) or 20% employed Using a Formula (F), three (3) or 10% applied Completing or Simplifying (CS), two (2) or 6.67% used Making a Table, Chart or List (TLC), while only one (1) or 3.33% utilized Guessing, Checking and Revising (GCR).

The findings imply that majority of the freshmen college students preferred to use Eliminating (E) strategy to solve Problem number 2.

**Table 7. Analysis of Variance**

| Source of Variation | SS | df | MS | F | P-value | Result |
|---|---|---|---|---|---|---|
| Between Groups | 6.688889 | 2 | 3.344444 | 1.869565 | 0.160333 | Not Significant |
| Within Groups | 155.6333 | 87 | 1.788889 | | | |
| Total | 162.3222 | 89 | | | | |

**Table 8. Commonly Used Heuristics by College Students in Problem Number 1**

| Heuristics | Frequency | Percentage (%) |
|---|---|---|
| Computing or Simplifying (CS) | 2 | 6.67 |
| Using a Formula (F) | 13 | 43.33 |
| Making a Table, Chart or List (TCL) | 2 | 6.67 |
| Eliminating (E) | 13 | 43.33 |
| **TOTAL** | **30** | **100** |

**Table 9. Commonly Used Heuristics by College Students in Problem Number 2**

| Heuristics | Frequency | Percentage (%) |
|---|---|---|
| Computing or Simplifying (CS) | 3 | 10 |
| Using a Formula (F) | 6 | 20 |
| Making a Table, Chart or List (TCL) | 2 | 6.67 |
| Guessing, Checking and Revising (GCR) | 1 | 3.33 |
| Eliminating (E) | 18 | 60 |
| **TOTAL** | **30** | **100** |

### 3.3. Commonly Used Heuristics by Freshmen College Students in Problem Number 3

**Table 10. Heuristics Commonly Used by Freshmen College Students in Problem Number 3**

| Heuristics | Frequency | Percentage (%) |
|---|---|---|
| Computing or Simplifying (CS) | 1 | 3.33 |
| Using a Formula (F) | 2 | 6.67 |
| Making a Model or Diagram (MD) | 24 | 80 |
| Eliminating (E) | 3 | 10 |
| **TOTAL** | **30** | **100** |

Table 10 presents the heuristics commonly used by Freshmen college students in problem number 3.

The Table 10 implies that twenty – four (24) or 80% of the respondents have used Making a Model (MD) to solve the problem, three (3) or 10% used Eliminating (E) strategy, two (2) or 6.67% employed Using a Formula (F), while only one (1) or 3.33% among the students applied the Computing or Simplifying (CS) strategy.

This can be a clear indication that the student – respondents have answered Problem number 3 by using the strategy of Making a Model or Diagram (MD). This means that students opt to have a visual representation of the problem in order to arrive with the right solution.

### 3.4 Heuristics Commonly Used by Freshmen College Students in Problem Number 4

**Table 11. Heuristics Commonly Used by Freshmen College Students in Problem Number 4**

| Heuristics | Frequency | Percentage (%) |
|---|---|---|
| Computing or Simplifying (CS) | 5 | 16.67 |
| Using a Formula (F) | 1 | 3.33 |
| Making a Model or Diagram (MD) | 2 | 6.67 |
| Making a Table, Chart or List (TCL) | 10 | 33.33 |
| Eliminating (E) | 1 | 3.33 |
| Looking for Patterns | 11 | 36.67 |
| **TOTAL** | **30** | **100** |

Table 11 shows the heuristics commonly used by freshmen college students in problem number 4.

It was gleaned from the table that eleven (11) or 36.67% of the respondents employed the strategy of Looking for Patterns to answer the problem, ten (10) or 33.33% applied Making a Table, Chart or List (TCL), five (5) or 16.67% used Computing or Simplifying (CS), two (2) or 6.67% utilized Making a Model or Diagram (MD), while only one (1) or 3.33% used both Using a Formula (F) and Eliminating (E).

The findings revealed that most of students used the strategy of Looking for Patterns to solve Problem number 5. This implies that students determined the common characteristics of the given problem to generalize appropriate information needed in choosing proper solution. They could have based it on some obvious facts clearly indicated in the problem that seemed to show them the right path of obtaining the correct answer.

### 3.5 Heuristics Commonly Used by Freshmen College Students in Problem Number 5

Table 12 presents the heuristics commonly used by freshmen college students in problem number 5.

**Table 12. Heuristics Commonly Used by Freshmen College Students in Problem Number 5**

| Heuristics | Frequency | Percentage (%) |
|---|---|---|
| Computing or Simplifying (CS) | 7 | 23.33 |
| Using a Formula (F) | 3 | 10 |
| Making a Model or Diagram (MD) | 6 | 20 |
| Considering a Simple Case (SC) | 14 | 46.67 |
| **TOTAL** | **30** | **100** |

It revealed that fourteen (14) or 46.67% of the respondents applied Considering Simple Case (SC) to solve the given problem, seven (7) or 23.33% used Computing or Simplifying (CS) strategy, six (6) or 20% employed Making a Model or Diagram (MD), while three (3) or 10% of students utilized Using a Formula (F).

## 6. Conclusion and Recommendation

Nowadays, heuristics teaching method is one of the most important teaching methods that can be utilized in teaching and learning problem solving in Mathematics. It is imperative that when teaching specifically in problem solving in mathematics, new information must be embedded meaningfully in relevant, previously existing knowledge to ensure that it will be retrievable when necessary. A wide variety of variables may directly and indirectly affect the capacity to collect and retrieve information from memory, including patterns, formula, models or diagrams and the context and manner in which information is learned, and relevant practice in retrieval. This is whether students “possession” of knowledge is being evaluated/tested based on their “use” of knowledge.

Problem solving is, however, more than a vehicle for teaching and reinforcing mathematical knowledge and helping to meet everyday challenges. It is also a skill which can enhance logical reasoning. A cited by NCTM [10], individuals can no longer function optimally n society by just knowing the rules to follow to obtain a correct answer. They also need to be able to decide through a process of logical deduction what algorithm, if any, a situation requires, and sometimes need to be able to develop their own rules in a situation where an algorithm cannot be directly applied. For these reasons, problem solving can be developed as a valuable skill in itself, a way of thinking rather than just as the means to an end of finding the correct answer.

Teaching and learning strategies for problem solving may therefore, be concentrated at in enhanced problem solving skills especially on non-routine problems, to reduce dependence on context, and to provide repeated relevant practice in retrieving information. Much of the development of student expertise involves the transition from using general problem – solving routines to using heuristics for non – routine problems that reduces the need for archetypal problem solving. This is supported by the findings of Laset [8] that exposing the students to non-routine problems, can develop students’ mathematical reasoning power and foster their understanding that mathematics is a creative endeavor. Also non-routine problems provide students a realistic situation where they are challenged to use higher order thinking skills including their critical and creative thinking.

Thus, teaching of problem solving heuristics even in the elementary and high school years should be encouraged in mathematics classrooms to create proficient problem solvers.

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