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Prospective Teachers' Conceptual and Procedural Knowledge in Mathematics: The Case of Algebra

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Abstract The study investigated prospective mathematics teachers' conceptual and procedural knowledge in algebra. Thirty six prospective teachers participated in the study. The independent variables of conceptual knowledge and procedural knowledge were investigated using quantitative methods. A 20-item instrument was used for collecting data. Descriptive and inferential statistics were used in analyzing the data collected. Specifically, the research questions were answered using the means and standard deviations, while the hypothesis was tested by conducting a paired t-test of difference. One of the findings of the study was the low performance of the respondents on conceptual knowledge test as against their performance on procedural knowledge test. The respondents differed significantly in their performances on conceptual and procedural knowledge, and the difference was in favor of procedural knowledge. It was recommended that teachers should give equal attention to both teaching of concepts and procedures in mathematics.

Keywords: *conceptual knowledge, procedural knowledge, algebra, prospective teachers*

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1. Introduction

The knowledge of concepts and procedures is imperative for competence in mathematics. For a mathematics teacher to be competent and effective in teaching, he must possess the subject matter knowledge. Subject matter knowledge is a combination of the knowledge of both concepts and procedures. Ideally, teachers are expected to demonstrate knowledge of both concepts and procedures, as some researchers have argued that the relation between the two types of knowledge is bi-directional e.g. [30]. [30] Asserted that there is a bi-directional relation between conceptual knowledge and procedural knowledge. In other words, the knowledge of one supports the other and vice versa. Similarly, [34] pointed out that conceptual knowledge and procedural knowledge are similar in many respects, which makes them difficult to differentiate. He said although researchers often discuss them as separate entities, conceptual knowledge and procedural knowledge rely on each other to develop in mathematics.

Prospective mathematics teachers must possess a good knowledge of mathematical concepts and procedures, if they are to succeed in their teaching profession. A good understanding of mathematical concepts and procedures gives the mathematics teacher confidence in the mathematics classroom. This knowledge, combined with the knowledge of pedagogy enhance the competence of the teacher, and help him to address the student's learning difficulties and misconceptions. Hence, the possession of conceptual and procedural knowledge is necessary for effective teaching.

1.1. Literature Review

A concept, according to [40] is, "an idea of something formed by mentally combining all its characteristics or particulars". Conceptual knowledge, which is knowledge of concepts, has to do with abstraction and generalization of particular instances. Knowing definitions and rules in mathematics is not having conceptual knowledge. Students may recall certain definitions, rules and procedures, but it cannot be said they possess conceptual understanding. To demonstrate conceptual understanding, the student must be able to justify why a statement in mathematics is true or where a mathematical rule comes from. For instance, [35] pointed out that "there is a difference between a student who can summon a mnemonic device to expand a product such as (a+b)(x+y), and a student who can explain where a mnemonic comes from". Therefore, the ability to recall into memory either a mathematical rule, definition, or procedure and to apply such is not enough justification of having conceptual knowledge. Being able to explain the rule, definition or procedure involved is required for conceptual knowledge evidence. This is because the knowledge of concepts involves understanding of meaning, and not just ability to recall definitions, rules or procedures.

An individual can commit into memory that two negative numbers can result into a positive number when multiplied or divided, but it is not the same as understanding the reason for the product or quotient to be a positive number. The ability to explain the reason for the answer to be positive is evidence of possessing conceptual knowledge. This knowledge is not restricted to particular problem type [30], it can be explicit or implicit since it is not necessarily expressed in words. [23] Referred to this knowledge as "comprehension of mathematical concepts, operations and relations", (p. 5). [32] Pointed out that conceptual knowledge is "the knowledge that provides an abstract understanding of the principles and relations among bits of knowledge in a certain domain". This definition corroborated the definition given by [22], who defined conceptual knowledge as, "an abstract knowledge addressing the essence of mathematical principles and relations among them", (p. 182).

Furthermore, [19] described conceptual knowledge in mathematics as the knowledge that comprises of symbols and demonstrations. It means, therefore, that this knowledge represents mathematical concepts and relates pieces of mathematical knowledge to each other to provide an understanding of mathematical concepts, rules and propositions. Similarly, [3] and [2] described conceptual knowledge as knowledge of concepts and principles, and their relationship to each other. Therefore, in this study, having conceptual knowledge in mathematics refers to understanding relationships among the concepts, definitions and rules of mathematics, and being able to give satisfactory or justifiable explanations about them. In other words, knowing and explaining the 'why' and 'how' of the procedures which are needed for a logical and correct solution of a mathematical problem is having or possessing a conceptual knowledge in mathematics.

As earlier noted, expertise in mathematics requires knowledge of both concepts and procedures. Knowledge of procedures is defined by [30] as "knowing how or knowledge of the steps required to attain various goals", (p.3). They pointed out that this knowledge, unlike conceptual knowledge, is limited to a particular problem type. This means its development would be through constant practice of solving problems. [19] Described procedural knowledge as knowing procedures, rules and algorithms required to solve mathematical problems. This description portrays this knowledge as mechanical, as it does not include conceptual understanding. This might be why [18] asserted that procedural knowledge is "meaningful only if it is linked to a conceptual base", (p.8). [22], cited McGehee defined procedural knowledge as "the ability to explain or justify the way one resolves a given problem without knowing the reason behind applying a certain theory, process or law during problem solving process", (p. 182). Procedural knowledge, in this study, can be said to be the ability to commit into memory the rules, procedures, principles and definitions of mathematics, and to recall them when solving problems without necessarily having an understanding of them. That is if procedural knowledge is not linked to or does not have a base of conceptual knowledge, then such knowledge can be termed as 'mechanical'. This is so, as it will be difficult to apply such knowledge to unfamiliar situations or problems.

The relationship between conceptual and procedural knowledge has long been an issue of debate among researchers in mathematics education. [30] Pointed out that researchers in mathematics education have expressed four theoretical perspectives on the relationship between conceptual knowledge and procedural knowledge. The concept-first perspective [10,15] assumes that children from the beginning gain conceptual knowledge, either through parents' explanations or by natural constraints, and then procedural knowledge is achieved through constant practice. On the other hand, the proponents of the procedure-first perspectives, e.g. [20,33] opine that children first learn procedural knowledge through inquisitiveness and later build conceptual knowledge about it. There is the inactivation view point [13,27,28] which says that conceptual knowledge and procedural knowledge develop independently. Even though [14] tried to identify certain relations between conceptual knowledge and procedural knowledge, he stressed that there is no general conclusion concerning the relation between these two types of knowledge. The fourth view point is the iterative perspective [1,29,31], which asserts that the relation between conceptual knowledge and procedural knowledge is bidirectional. This means positive change in conceptual knowledge supports similar change in procedural knowledge and vice versa.

[38] pointed out that knowing mathematics has to do with understanding of certain concepts and procedures. And therefore, there has to be a link between conceptual and procedural knowledge. They said the mathematics curriculum should avail the students the opportunity to discover and make such links. This is imperative, as competence in mathematics is a combination of the knowledge of both concepts and procedures. [39] Investigated conceptual and procedural knowledge of trainee teachers in rational numbers. The study revealed that the prospective teachers' conceptual and procedural knowledge was above average. [8] Examined undergraduate students' performance and confidence in procedural and conceptual mathematics, and the results showed that the performance of the students was better on conceptual knowledge than on procedural. [4] Investigating pre-service mathematics teachers to determine how well they know the subject they will teach, revealed that the majority of the pre-service teachers were unable to justify their answers using conceptual knowledge. In fact, only about a quarter of the prospective teachers were successful in the use of conceptual knowledge to clarify their answers.

[37] Carried out a study of secondary mathematics teachers to determine their knowledge of the concepts of slope. The study compared pre-service and in-service teachers' conceptual knowledge, and the results revealed that pre-service teachers had difficulty in differentiating between linear equations and others; they also were unable to answer questions which relate to rate of change. Similarly, [9] investigated secondary mathematics pre-service teachers conceptual knowledge of rational numbers, and the findings indicated that teachers who were inexperienced provided explanations that were based on knowledge of procedures. The study also revealed that experienced teachers' explanations were based on both knowledge of concepts and procedures.

[22] Conducted a study on elementary school teachers' conceptual and procedural knowledge of rational numbers. The study sample was 57 elementary mathematics

teachers, consisting of 27 inexperienced and 30 experienced teachers. The study revealed that the teachers had average conceptual and procedural knowledge in rational numbers. It was also shown that significant differences existed between the teachers' conceptual and procedural knowledge, and the difference was in favor of procedural knowledge. In other words, the teachers performed better on procedural knowledge than on conceptual knowledge.

[25] Studied prospective secondary teachers' knowledge of the function concept and found that the majority of the teachers have mastered procedural knowledge more clearly than their conceptual knowledge. The teachers' performance on problems that demanded procedural knowledge was better than on the ones that called for conceptual knowledge. [5] Conducted a study on the role of conceptual knowledge in mathematical procedural learning. They found that students gain conceptual understanding before procedural competence with fractions. Hence, they asserted that conceptual knowledge precedes procedural knowledge.

1.2. The Present Study

The present study focused on prospective mathematics teachers' conceptual and procedural knowledge of algebra. [36] Asserted that most studies on conceptual and procedural knowledge have been on topics such as counting, single-digit and multi-digit addition and subtraction and also on fractions. Not much has been done regarding concepts/procedural knowledge on topic areas such as algebra. In fact, [36] said, "notably absent are the studies of the development of procedural and conceptual knowledge in algebra, geometry, and calculus" (p. 2). Many studies have been conducted on conceptual/procedural knowledge in fractions, e.g. [16,17,21,26]. Similarly, there are many studies of the development of conceptual and procedural knowledge in single-digit and multi-digit addition and subtraction e.g. [6,7,11,12,24].

Considering the importance of algebra in the learning of mathematics, there is the need to investigate students' conceptual and procedural knowledge of it. Also, given the fact that students have difficulty in transit from arithmetic to algebra, which leads to misconceptions in algebra, this study is timely and relevant. Competence in mathematics is a combination of the knowledge of both concepts and skills. For a mathematics teacher to be effective in teaching, he has to have conceptual and procedural knowledge. Though this knowledge is not a sufficient condition for teacher's effectiveness, but it is a necessary condition. This is a key component of the teacher's knowledge. It is the teacher's subject matter knowledge, without which the teacher cannot teach. Unfortunately, studies have shown that prospective teachers performed better on tasks requiring procedural knowledge than on those demanding conceptual knowledge (e.g. [25]; [22]. Also, literature reviewed indicated that most of the studies on conceptual and procedural knowledge have been on topics such as fractions, single and multi-digits addition, subtraction and multiplication.

This study was proposed to investigate prospective teachers' conceptual and procedural knowledge of algebra. The problem for the study, therefore, was to inquire into the prospective teachers' knowledge of concepts and skills in the topic area of algebra.

1.3. Purpose of the Study

The main purpose of the study was to investigate prospective teachers' knowledge of concepts and skills in algebra. Specifically, the study was:

1. To find out prospective teachers' performance on tasks requiring conceptual and procedural knowledge in algebra
2. To find out whether prospective teachers' perform differently on tasks demanding conceptual and procedural knowledge in algebra.

1.4. Research Questions

The following research questions were formulated to guide the study:

1. What is the performance of prospective teachers on tasks demanding conceptual knowledge in algebra?
2. What is the performance of prospective teachers on tasks demanding procedural knowledge in algebra?
3. Do prospective teachers differ in performance on tasks requiring conceptual and procedural knowledge?

1.5. Hypothesis

The following hypothesis was tested at the 0.05 alpha level of significance:

H0: There is no significant difference between the mean scores of prospective teachers in conceptual and procedural knowledge of algebra.

2. Methodology

2.1. Research Design

The survey research design was implemented in this study. The independent variables of conceptual knowledge and procedural knowledge were addressed using quantitative methods.

2.2. Participants

The participants in this study were undergraduate students undergoing training to become secondary school mathematics teachers. They were 54 in number, and in their final year. They have had courses in both Mathematics and Education, and had also participated twice in the mandatory teaching practice exercise.

2.3. Instrument for Data Collection

A 20-item instrument was used to collect data in this study. Of the 20 items, 10 were on tasks requiring knowledge of concepts in algebra, while the other 10 were on tasks requiring knowledge of procedures. All the 20 items are open ended questions. Some of the items were adapted from reviews of literature, while others were constructed by the researcher. The items were pilot tested and had a reliability coefficient of 0.87, using the method of split half. Table 1 shows sample items. Items 1 and 2 are on conceptual knowledge and items 3 and 4 are on procedural knowledge.

Table 1. Sample of Conceptual and Procedural Knowledge Test Items

1. The first step in the procedures for adding, subtracting, multiplying, and dividing algebraic expressions (like the examples shown below) is always to factor the numerators and denominators

1. $\frac{x^2-6x-16}{x^2-9}\cdot\frac{x+3}{x^2-2x-8}$ 2. $\frac{4}{x^2+6x+9}-\frac{x-7}{x^2-9}$

What is the advantage of factoring the expressions first, before attempting to add, subtract, multiply, or divide?

Please be as complete as possible in your response.

2. Here is a typical "polynomial long division" problem: $(x^3+2x^2-3x-6)\div(x+2)$

Describe some ways you can verify that you have done the long division procedure correctly.

Don't just work the problem provided. Instead, try to list any ideas you could use to verify that your answer is correct.

Please be as complete as possible in your response.

3. Solve the equation $x^2-6x-7=0$

4. Simplify $\frac{x^2-6x-16}{x^2-9}\cdot\frac{x+3}{x^2-2x-8}$

3. Results and Discussion

3.1. Results

To respond to research, question one: What is the performance of prospective mathematics teachers on tasks demanding conceptual knowledge of algebra? The mean and standard deviation of the scores of the prospective teachers were calculated. Table 2 shows this information. It can be seen from this table that the teachers performed below average on tasks that require the knowledge of concepts. The standard deviation indicates that the teachers' scores did not vary much from one another. The maximum score for the test is 30, and only one respondent got 15 out of 30. Respondents scored as low as 2 on these tasks.

Table 2. Means and Standard Deviations for CKT and PKT

	Mean	N	Standard deviation
CKT	7.25	36	3.22
PKT	18.25	36	7.35

Key
CKT- Conceptual Knowledge Test
PKT- Procedural Knowledge Test

Research question two was on the performance of teachers on tasks requiring knowledge of procedures in algebra. To respond to this, the mean and standard deviation of the scores of the prospective teachers were calculated. This is shown in Table 2. The values in the table indicate that the prospective teachers performed above average on tasks demanding procedural knowledge in algebra, and their performance did not vary much from one another. The maximum score on the test is 30, the mean performance of the teachers was about 18, and 20 (66.7%) out of 36 respondents scored above the mean score.

The third research question is: Do prospective mathematics teachers differ in performance on tasks requiring conceptual and procedural knowledge of algebra? To respond to this question, the mean scores of the respondents in conceptual and procedural tests were compared, and the results showed that the respondents' performance on procedural tasks is higher than their performance on conceptual tasks. As can be seen in Table 2, the mean score in conceptual knowledge tasks is 7.25, while that of procedural knowledge tasks is 18.25. This indicates a difference in performance, however, to confirm whether the difference was significant or not a paired t-test was conducted. Hence, the hypothesis tested was: There is no significant difference between the mean scores of prospective mathematics teachers in conceptual knowledge and procedural knowledge. The summary of the paired t-test is as shown in Table 3. The t-test indicates a statistically significant difference in the performance of the prospective mathematics teachers between conceptual knowledge and procedural knowledge.

Table 3. Summary of Paired t-test of Differences

	Mean	Standard Deviation	Standard Error	t	df	Sig. (2 tailed)
CKT-PKT	-11	5.8162	0.96937	-11.3	35	0.000

3.2. Discussion

One of the findings of this study is that the prospective mathematics teachers performed below average in tasks requiring knowledge of concepts in algebra. In fact, the performance of the respondents was generally very low compared with their performance on items demanding the knowledge of procedures. This finding is in line with the findings of [4,22,25]. These studies found that prospective mathematics teachers performed low on tasks demanding conceptual knowledge of rational numbers and the function concept. In this study the respondents were required to either explain certain concepts in algebra, list any ideas that could be used to verify whether certain answers are correct, or suggest better ways of solving certain algebraic problems.

On tasks requiring knowledge of procedures, the study revealed that prospective mathematics teachers performed above average. This finding is similar to the findings of [4,22,25,39], which revealed that prospective teachers performed above average on procedural knowledge in rational numbers and the concept of slope.

The answer to the question whether prospective mathematics teachers performed differently on conceptual and procedural knowledge was evident. The mean scores in conceptual and procedural knowledge indicated a difference in performance; and the hypothesis test revealed the significance of the difference. This agrees with the finding of [22] which revealed significant differences between conceptual knowledge and procedural knowledge of pre-service mathematics teachers in rational numbers, and the difference was in favor of procedural knowledge. However, other researchers such as [39] found that pre-service teachers performed above average on both conceptual and procedural knowledge, while [8] found that performance was better on conceptual knowledge than on procedural knowledge.

3.3. Conclusion

The study investigated prospective mathematics teachers' conceptual knowledge and procedural knowledge in algebra. The literature review indicated that most studies on conceptual and procedural knowledge have been on rational numbers. And findings from these studies have revealed either better performance on procedural knowledge than conceptual knowledge or vice versa; in some cases average performance on both. In this study, the performance was better on procedural knowledge than on conceptual knowledge. The respondents were in most cases unable to either explain or suggest better ways of solving algebraic problems. This has implications for the teaching and learning of mathematics. Ideally, knowledge of concepts and knowledge of procedures should be related, and therefore, performance on both in any aspect of mathematics is expected to show no marked or significant difference. This is more so that some researchers in mathematics education have shown that there is a bidirectional relationship between conceptual knowledge and procedural knowledge e.g. [30].

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