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Disproof the Birch and Swinnerton-Dyer Conjecture

Leszek W. Guła*

Lublin, Poland
*Corresponding author: yethi@wp.pl

Abstract The Guła's Theorem. Only one in the world the proper proof of Fermat's Last Theorem for $n = 4$. Disproof the Birch and Swinnerton-Dyer Conjecture. The proof of Goldbach's Conjecture.

Keywords: *Birch and Swinnerton-Dyer Conjecture, Common Prime Factor, Diophantine Equations, Diophantine Inequalities, Fermat Equation, Greatest Common Divisor, Newton Binomial Formula*

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1. Introduction

The Guła's Theorem is dated 03-04 June 1997. The Fermat's Last Theorem (FLT) is the famous theorem.

Disproof the Birch and Swinnerton-Dyer Conjecture (2009 Y.) we have on the strength of the Guła's Theorem.

The Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. [4]

2. The Guła's Theorem

Theorem 1 (Guła Theorem). *For each* $g \in \{8,12,16,\ldots\}$ *or for each* $g \in \{3,5,7,\ldots\}$ there exist finitely many pairs (s,t) of positive integers such that:

$$g = \left(\frac{g+d^2}{2d}\right)^2 - \left(\frac{g-d^2}{2d}\right)^2 = s^2 - t^2$$

$$= (s+t)(s-t) = \frac{g}{d}(s-t) = \frac{g}{d}d = g,$$

where $d \mid g$ and $d < \sqrt{g}$ and - $d, \frac{g}{d} \in \{2,4,6,\ldots\}$ with even g or $d \in \{1,3,5,\ldots\}$ with odd g. [2]

Theorem 2. For all $x,u,\upsilon \in \{1,2,3,\ldots\}$ such that $\boldsymbol{gcd}(u,\upsilon) = 1$ and $u - \upsilon \in \{1,3,5,\ldots\}$:

$$\{(u+\upsilon)^x (u-\upsilon)^x$$

$$= \left[\frac{(u+\upsilon)^x (u-\upsilon)^x + (u-\upsilon)^{2x}}{2(u-\upsilon)^x}\right]^2$$

$$- \left[\frac{(u+\upsilon)^x (u-\upsilon)^x - (u-\upsilon)^{2x}}{2(u-\upsilon)^x}\right]^2$$

$$= \left[\frac{(u+\upsilon)^x + (u-\upsilon)^x}{2}\right]^2 - \left[\frac{(u+\upsilon)^x - (u-\upsilon)^x}{2}\right]^2$$

$$\Rightarrow [\left(u^2-\upsilon^2\right)^2 + (2u\upsilon)^2$$

$$= \left(u^2+\upsilon^2\right)^2 \wedge \left(u^2-\upsilon^2\right)^{2+x}$$

$$+(2u\upsilon)^{2+x} < \left(u^2+\upsilon^2\right)^{2+x} \qquad [2]$$

$$= \left(u^2-\upsilon^2\right)^2 \left(u^2+\upsilon^2\right)^x + (2u\upsilon)^2 \left(u^2+\upsilon^2\right)^x]\}$$

$$\Rightarrow \left(u^2-\upsilon^2, 2u\upsilon, u^2+\upsilon^2\right).$$

Theorem 3.

$$\{(2a+b)b : a \in [0,1,2,\ldots] \wedge b \in [3,5,7,\ldots]\}$$

$$= \{9,15,21,25,27,33,35,39,45,49,\ldots\}$$

$$\Rightarrow \{3,5,7,\ldots\} \setminus \{9,15,21,25,27,33,35,39,45,49,\ldots\} \qquad [2]$$

$$= \{3,5,7,11,13,17,19,23,29,31,\ldots\} = \mathbb{P}.$$

3. The Proof of Fermat's Last Theorem For $n = 4$

Theorem 4 (FLT). For all $n \in \{3,4,5,\ldots\}$ and for all $A,B,C \in \{1,2,3,\ldots\}$: $A^n + B^n \neq C^n$.

Remark 1. Sufficient that we prove FLT for $n = 4$ and for $n \in \mathbb{P}$. [3] This is the remark 1.

Remark 2. Fermat did not proved his own theorem for $n = 4$. [3] This is the remark 2.

Remark 3. In [2] we have the proof of FLT for $n \in \mathbb{P}$. This is the remark 3.

Remark 4. These hypothesis ($A^4 + B^4 = c^2$ [3]), $A^4 + B^4 = C^4$ **are different** because on the strength of theorem 1 for $c = C^2$: $\frac{(u+\upsilon)^4 + (u-\upsilon)^4}{2} = \left(u^2+\upsilon^2\right)^2$

$$+(2u\upsilon)^2 = C^2 = \left(u^2+\upsilon^2\right)^2 = \left(u^2-\upsilon^2\right)^2 \equiv 0.$$

For some relatively prime $u,\upsilon \in \{1,2,3,\ldots\}$ such that $u - \upsilon$ is positive and odd:

$$\left\{\begin{array}{l}\left(u^2+\upsilon^2\right)^2-(2u\upsilon)^2\\ =A^2\wedge\left[\begin{array}{l}4\left(u^2+\upsilon^2\right)u\upsilon=B^2\\ \Rightarrow X^4+Y^4=z^2<c^2[3]\end{array}\right]\wedge\left(u^2+\upsilon^2\right)^2\\ +(2u\upsilon)^2=c\end{array}\right\}$$

This is the remark 4.

Proof. Suppose that the equation $A^4+B^4=C^4$ has primitive solutions $[A,B,C]$ in $\{1,2,3,\ldots\}$.

Then it must be $(A^5+B^5<C^5\wedge A+B>C\wedge A^2+B^2>C^2\wedge A^3+B^3>C^3)$. [2]

Without loss for this proof we can assume that $\{A,(C-B)\in[1,3,5,\ldots]\wedge 4\nmid B\}$.

For some $A,C,\nu\in\{1,3,5,\ldots\}$ and for some $B\in\{6,10,14,\ldots\}$ such that A,C and B are co-prime:

$$\left\{\begin{array}{l}\left[\begin{array}{l}A-(C-B)=2\nu\wedge A^2+B^2>C^2\wedge C-B+2\nu\\ =A\wedge C-A+2\nu=B\wedge(C-B)+(C-A)+2\nu\\ =C\wedge(C-A+2\nu)^4\\ =(C-A+A)^4-A^4\wedge(C-B+2\nu)^4\\ =(C-B+B)^4-B^4\end{array}\right]\\ \Rightarrow\left[\begin{array}{l}(C-A)^2 2\nu+\frac{3}{2}(C-A)(2\nu)^2+(2\nu)^3+\frac{4\nu^4}{C-A}\\ =(C-A)^2A+\frac{3}{2}(C-A)A^2\\ +A^3\wedge(C-B)^2 2\nu+\frac{3}{2}(C-B)(2\nu)^2\\ +(2\nu)^3+\frac{4\nu^4}{C-B}\\ =(C-B)^2B+\frac{3}{2}(C-B)B^2\\ +B^3\wedge 2\nu^2>(C-A)(C-B)\end{array}\right]\end{array}\right\}$$

Thus

$$\left\{\frac{4\nu^4}{C-A},\frac{4\nu^4}{C-B}\in[1,3,5,\ldots]\wedge 2\nu^2>(C-A)(C-B)\right\}.$$

Hence – For some $c,d,\nu\in\{1,3,5,\ldots\}$ such that c,d are co-prime:

$$\left(\begin{array}{l}c^4=C-B\wedge 4d^4=C-A\wedge c^4+2\nu\\ =A\wedge 4d^4+2\nu=B\wedge\nu^2>2c^4d^4\end{array}\right)\Rightarrow\nu>\sqrt{2}(cd)^2.$$

Therefore – For some $c,d,e\in\{1,3,5,\ldots\}$ such that c,d and e are co-prime: $cde=\nu$.

Further it mus be – For some $c,d,e,A\in\{1,3,5,\ldots\}$ such that c,d and e are co-prime:

$$(2cde+4d^4)^4=[(4d^4+A)^2]^2-(A^2)^2$$
$$=[(4d^4+A)^2+A^2](2d^4+A)8d^4\Rightarrow$$
$$2(ce+2d^3)^4=[(4d^4+A)^2]^2-(A^2)^2$$
$$=[(4d^4+A)^2+A^2](2d^4+A).$$

We assume that for some co-prime $z,w,x\in\{1,3,5,\ldots\}$ and for some $y\in\{6,10,14,\ldots\}$:

$$\left\{\begin{array}{l}zw=ce+2d^3\wedge x+y\\ =2d^4+A+2d^4\wedge x\\ =2d^4+A\wedge y\\ =2d^4\wedge 2(zw)^4\\ =\left[(x+y)^2+(x-y)^2\right]x\\ =2\left(x^2+y^2\right)x\wedge z^4w^4\\ =\left(x^2+y^2\right)x\wedge\left(z^2\right)^2\\ =x^2+y^2\wedge w^4=x\end{array}\right\}\Rightarrow 4\mid y,$$

which is inconsistent with $4\nmid y$. [2] This is the proof.

4. Disproof the Birch and Swinnerton-Dyer Conjecture

Mathematicians have always been fascinated by the problem of describing all solutions in whole numbers x,y,z to algebraic equations like

$$x^2+y^2=z^2.$$

Euclid gave the complete solution for that equation, but for more complicated equations this becomes extremely difficult. Indeed, in 1970 Yu. V. Matiyasevich showed that Hilbert's tenth problem is unsolvable, i.e., there is no general method for determining when such equations have a solution in whole numbers. But in special cases one can hope to say something. When the solutions are the points of an abelian variety, the Birch and Swinnerton-Dyer conjecture asserts that the size of the group of rational points is related to the behavior of an associated zeta function $\zeta(s)$ near the point $s=1$. In particular this amazing conjecture asserts that if $\zeta(1)$ is equal to 0, then there are an infinite number of rational points (solutions), and conversely, $\zeta(1)$ is not equal to 0, then there is only a finite number of such points [1].

Conjecture 1 (Brich and Swinnerton-Dyer Conjecture) If $\zeta(1)$ is equal to 0:

$$\zeta(1)=\left(\frac{x}{z}\right)^2+\left(\frac{y}{z}\right)^2-1=0,$$

then there are an infinite number of rational points $\left(\frac{x}{z},\frac{y}{z}\right)$.

Proof. For all relatively prime $u,\upsilon\in\mathbb{Z}\setminus[0]$ and for some $x,y,z\in\mathbb{Z}$ such that x,y,z are co-prime:

$$\left(\frac{u^2-\upsilon^2}{u^2+\upsilon^2},\frac{2u\upsilon}{u^2+\upsilon^2}\right)=\left(\frac{x}{z},\frac{y}{z}\right).\clubsuit$$

Disproof. On the strength of the Theorems 1 and 2 – If $\zeta(1)$ is not equal to 0, then – For all $a\in\mathbb{Q}$ and for all $u\in\mathbb{Z}\setminus[0]$ and for all $\upsilon\in\mathbb{Z}\setminus[-1,0,1]$ and for some $b\in\mathbb{Q}$ such that $\mathbf{gcd}(u,\upsilon)=1$, these two equations

$$\left\{\begin{array}{l}\left[\left(\frac{u}{\upsilon}\right)^2=\left(\pm\frac{u^2-\upsilon^2}{2\upsilon^2}\right)^3+a\left(\pm\frac{u^2-\upsilon^2}{2\upsilon^2}\right)\right.\\ \left.+b\vee\left(\pm\frac{u^2-\upsilon^2}{2\upsilon^2}\right)^2=\left(\frac{u}{\upsilon}\right)^3+a\frac{u}{\upsilon}+b\right]\\ \Leftarrow\left[\begin{array}{l}\left(\frac{u}{\upsilon}\right)^2+\left(\pm\frac{\frac{u^2}{\upsilon^2}-1}{2}\right)^2=\left(\frac{u^2+\upsilon^2}{2\upsilon^2}\right)^2\\ \Leftrightarrow\left(\frac{u^2-\upsilon^2}{u^2+\upsilon^2}\right)^2+\left(\frac{2u\upsilon}{u^2+\upsilon^2}\right)^2=1\end{array}\right]\end{array}\right\}$$

have infinite numbers of such points in $(\mathbb{Q}\setminus\mathbb{Z})^2$, namely $\left(\pm\frac{u^2-\upsilon^2}{2\upsilon^2},\frac{u}{\upsilon}\right)$ or $\left(\frac{u}{\upsilon},\pm\frac{u^2-\upsilon^2}{2\upsilon^2}\right)$.

If $\zeta(1)$ is not equal to 0, then – For all $a\in\mathbb{Q}$ and for all relatively prime $u,v\in\mathbb{Z}\setminus[0]$ and for some $b\in\mathbb{Q}$ such that $(u-v)^2>4$, these two equations

$$\left\{\begin{array}{l}\left[\begin{array}{l}\left(\pm\frac{u+\upsilon}{u-\upsilon}\right)^2\\ =\left(\frac{2u\upsilon}{(u-\upsilon)^2}\right)^3+a\frac{2u\upsilon}{(u-\upsilon)^2}+b\vee\left(\frac{2u\upsilon}{(u-\upsilon)^2}\right)^2\\ =\left(\pm\frac{u+\upsilon}{u-\upsilon}\right)^3+a\left(\pm\frac{u+\upsilon}{u-\upsilon}\right)+b\end{array}\right]\\ \Leftarrow\left[\begin{array}{l}\left(\pm\frac{u+\upsilon}{u-\upsilon}\right)^2+\left(\frac{2u\upsilon}{(u-\upsilon)^2}\right)^2=\left(\frac{u^2+\upsilon^2}{(u-\upsilon)^2}\right)^2\\ \Leftrightarrow\left(\frac{u^2-\upsilon^2}{u^2+\upsilon^2}\right)^2+\left(\frac{2u\upsilon}{u^2+\upsilon^2}\right)^2=1\end{array}\right]\end{array}\right\}$$

have infinite numbers of such points in $(\mathbb{Q}\setminus\mathbb{Z})^2$, namely $\left(\frac{2u\upsilon}{(u-\upsilon)^2},\pm\frac{u+\upsilon}{u-\upsilon}\right)$ or $\left(\pm\frac{u+\upsilon}{u-\upsilon},\frac{2u\upsilon}{(u-\upsilon)^2}\right)$.

If $\zeta(1)$ is not equal to 0, then – For all $a\in\mathbb{Q}$ and for all relatively prime $u,v\in\mathbb{Z}\setminus[0]$ and for some $b\in\mathbb{Q}$ such that $u^2-v^2\neq 0$, these two equations

$$\left\{\left[\left[\left(\pm\frac{u^2-\upsilon^2}{\left(u^2+\upsilon^2\right)^2}\right)^2\right.\right.\right.$$

$$=\left(\frac{2u\upsilon}{\left(u^2+\upsilon^2\right)^2}\right)^3+a\frac{2u\upsilon}{\left(u^2+\upsilon^2\right)^2}+b\vee\left(\frac{2u\upsilon}{\left(u^2+\upsilon^2\right)^2}\right)^2$$

$$\left.=\left(\pm\frac{u^2-\upsilon^2}{\left(u^2+\upsilon^2\right)^2}\right)^3+a\left(\pm\frac{u^2-\upsilon^2}{\left(u^2+\upsilon^2\right)^2}\right)+b\right]$$

$$\Leftarrow\left[\left(\pm\frac{u^2-\upsilon^2}{\left(u^2+\upsilon^2\right)^2}\right)^2+\left(\frac{\frac{\left(u^2-\upsilon^2\right)^2}{\left(u^2+\upsilon^2\right)^4}-\frac{(u-\upsilon)^4}{\left(u^2+\upsilon^2\right)^4}}{2\frac{(u-\upsilon)^2}{\left(u^2+\upsilon^2\right)^2}}\right)^2\right.$$

$$=\left(\pm\frac{u^2-\upsilon^2}{\left(u^2+\upsilon^2\right)^2}\right)^2+\left(\frac{2u\upsilon}{\left(u^2+\upsilon^2\right)^2}\right)^2=\frac{1}{\left(u^2+\upsilon^2\right)^2}$$

$$\left.\left.\Leftrightarrow\left(\frac{u^2-\upsilon^2}{u^2+\upsilon^2}\right)^2+\left(\frac{2u\upsilon}{u^2+\upsilon^2}\right)^2=1\right]\right\}$$

have infinite numbers of such points in $(\mathbb{Q}\setminus\mathbb{Z})^2$, namely $\left(\frac{2u\upsilon}{\left(u^2+\upsilon^2\right)^2},\pm\frac{u^2-\upsilon^2}{\left(u^2+\upsilon^2\right)^2}\right)$ or $\left(\pm\frac{u^2-\upsilon^2}{\left(u^2+\upsilon^2\right)^2},\frac{2u\upsilon}{\left(u^2+\upsilon^2\right)^2}\right)$.

This is the disproof.

5. The Proof of Goldbach's Conjecture

Conjecture 2 (Goldbach Conjecture). For all $Z\in\{6,8,10,\ldots\}$ and for some $X,Y\in\mathbb{P}$: $Z=X+Y$. [4]

Proof. The key of this proof are two common prime factors: 2 and 3.

$$\left\{\begin{array}{l}18,24,30,36,42,48,54,60,66,72,\\ 78,84,90,96,102,108,114,\ldots\end{array}\right\}$$

$$=\left\{\begin{array}{l}(3Z):(3Z)=(3X)+(3Y)\wedge 3X\le 3Y\wedge X,Y\\ \in\mathbb{P}\wedge(3X),(3Y)\\ \in[9,15,21,27,33,39,45,51,\ldots]\end{array}\right\}\ [2]$$

$$\Rightarrow\{6,8,10,12,14,16,18,20,22,\ldots\}$$

$$=\{Z:Z=X+Y\wedge X\le Y\wedge X,Y\in\mathbb{P}\}.$$

This is the proof.

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