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Comparing the Effectiveness of Four Interventions for the Support of Students with Learning Disabilities in Acquiring Arithmetic Combinations of Multiplication and Division

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Abstract The present study examined the effectiveness of four instructional interventions for teaching arithmetic combinations of multiplication and division to students with arithmetic combinations of multiplication and division to students with Learning Disabilities. The basic intervention, through which the control group was instructed, was based on six fundamental principles of effective instruction. A combination of this basic intervention and an alternative grouping and presentation form of arithmetic combinations constituted the intervention for the 1st experimental group. The addition of fact finding strategies to the intervention for the 1st experimental group yielded the intervention used for the 2nd experimental group. Finally, the intervention through which the 3rd experimental group was supported consisted of the 2nd experimental group's intervention plus components of number sense. The results showed that the majority of students of all groups presented significant improvement in learning the arithmetic combinations, while no significant differences were found among groups. However, in some groups differentiation was observed regarding the final performance in multiplication or division arithmetic combinations, probably due to the nature of the interaction of certain instructional techniques and the specific characteristics of the two arithmetic operations.

Keywords: *learning disabilities, arithmetic combinations, effective instruction, fact strategies, number sense*

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1. Introduction

One of the most important structural elements of mathematical knowledge is the immediate recall of the results of operations with single-digit numbers, called "arithmetic combinations" (ACs) [1]. For example, in multiplication the ACs are the results from 0x0 to 9x9 (e.g. 5x9 = 45). In reference to the division, ACs are the quotients obtained when dividing the products of multiplication ACs by each of the factors (e.g. 45: 5 = 9) [2].

It has been estimated that about 50% of general student population has difficulty in learning multiplication and division ACs, while the failure rate among students with serious learning problems, such as students with Learning Disabilities (LD), can reach 90% [3]. The importance of knowledge of ACs for mathematical progress has led to the search of ways and processes of memorizing and directly recalling them. One such way is to present ACs as groups based on a common characteristic rather than according to the magnitude of their result [4]. For example, among the multiplications from 0x0 up to 9x9, the multiplications of "twin numbers' grouping can be distinguished and taught separately (e.g. 3x3, 4x4). In fact, due to its distinctive feature and ease in memorization, this group is one of the first taught in relevant programs. This means that in an instructional program for ACs which uses this kind of alternative grouping of ACs, the 7x7 = 49, for example, will be taught before the 4x7 = 28, because the group to which the 4x7 is integrated, is considered more difficult to remember than the group of "twin numbers." Results of studies including alternative grouping in order to facilitate the learning of ACs are very positive [5,6,7].

Another means through which the acquisition of ACs can be improved is to use already known ACs in order to learn unknown ones, as it is shown in the example: 6x7=?, 6x7= (5x7)+(1x7)=35+7=42. The need for using this and other similar fact -finding strategies results from the well established difficulty of students with low performance in the acquisition of ACs to develop alternative ways of finding the results when they cannot retrieve them directly from the memory [8].

In the context of the efforts for improving interventions for ACs central role has the instruction of components of number sense [9]. For example, the *part- whole* relationship, (the knowledge that a number of objects may be shared in equal or unequal *parts* but the *whole* remains unchanged), is a prerequisite knowledge of decomposition and recomposition of the factors of an AC during implementation of a strategy that uses known ACs for answering unknown ones [e.g. 6 x 7=?, thinking 6 x7= (5x7) + (1x7)=35+7=42, but also 6 x 7 = (3x7) + (3x7) = 21+21 =42. Number sense facilitates considerably the understanding of the commutative property in addition and multiplication, whereby if a + b = c, then b + a = c, and if a x b = d then b x a = d [8,10]. The commutative property in turn greatly facilitates the immediate recall of ACs from memory. For example, a student learning 3x5=15 simultaneously learns 5x3=15 just by reversing the order of the factors. It is worth noting, however, that the results of studies that examine the role of number sense on learning ACs are not clear. It seems that the characteristics of arithmetic operations affect the results of the student's effort. For example, reference [9] argue that number sense promotes the learning of subtraction ACs more than the learning of addition ACs.

The arrangements and techniques to facilitate memorization and recalling of ACs are of particular importance in the case of students with Learning Disabilities (LD). One of the characteristics of students with LD is that they tend to face each AC as separated mathematical information because they have difficulty in using the properties and characteristics of ACs in order to connect and correlate them [11]. For example, although students with LD may know that the product of 3 x7 is 21, they do not necessarily know the commutative property (3x7 = 7x3). So, for 7 x 3 =.... they may use repeated additions of 3 (3 + 3 + 3 + 3 + 3 + 3 + 3 = 21), which is a time- consuming and error prone strategy [8]. Students with LD need special teaching of properties and generally fact strategies, because they do not discover this knowledge spontaneously and they are in a difficult position in the general classroom, as the general school's curriculum does not usually emphasize the learning of fact strategies [4].

Special teaching methods and techniques have certainly a prominent place in the context of any intervention for students with disabilities, but, as a literature review conducted by the present authors showed, highly effective instructional programs for teaching ACs to students with LD are based not only on special teaching techniques (such as alternative grouping or fact strategies), but also on principles of effective instruction e.g. [12,13,14,15]. Principles utilized in most of the reviewed studies are: direct instruction, use of the three modes of knowledge representation (concrete, representational, abstract), guided and independent practice, systematic progress monitoring, and regular feedback of students. Direct instruction supplies students with clear and well-organized information, and prevents gaps in learning by leading step by step to success through a review of prerequisite knowledge [13,16]. The use of the three modes of knowledge representation promotes understanding of math concepts [17]. The two levels of practice, guided and independent, promote errorless learning [18], and provide opportunities both for discovering relationships between ACs and developing strategies for finding them. The systematic monitoring of students' progress ensures the adequate achievement of a learning goal before moving on to the next one. Finally, the precise feedback to students regarding the results of their efforts contributes to their concentration on the learning goal and enhances their learning motives [19,20].

Taking into consideration the above delineated theoretical framework, it can be concluded that supporting students with LD in their attempt to learn ACs should be based both on the general principles of effective instruction and on the special teaching arrangements that can assist students in managing their cognitive difficulties. The exact syntheses of principles, methods and techniques that make up the best options for teaching ACs effectively to students with LD is an important issue that needs systematic investigation. The purpose of this research, then, is to compare the efficacy of four interventions for teaching multiplication and division ACs to students with LD. Multiplication and division ACs were selected because, after a literature review by the present authors revealed that they have not been adequately studied, especially ACs of division [21,22,23].

The questions which this study sought to answer are:

Can an intervention grounded on the principles of effective instruction, without using special techniques, help students with LD to significantly improve their performance on fluency, maintenance and generalization of multiplication and division ACs?

Is there any difference in effectiveness between the four interventions resulting from the gradual enrichment of the initial intervention (grounded on principles of effective instruction), firstly with an alternative grouping of ACs based on a common characteristic, then by teaching certain fact strategies, and ultimately by teaching certain components of number sense?

2. Methodology

2.1. Participants and Context

The initial number of participants was 96 pupils aged 9-12 years (M = 10.05, SD = 1.25), who attended 27 different schools situated in an urban region of Northern Greece. The selection of schools was made based on the positive response of teachers to the study proposal, which was addressed by the researchers to a total of 50 schools chosen randomly from a list of schools with Resource Rooms. All students participating in the survey had been diagnosed as students with LD from medical-educational state agencies. After the initial selection and the written permission of parents', students were randomly assigned to one of the groups of the study. The final number of participants per group was: control group 20 students, 1st experimental group 19 students, 2nd experimental group 26 students, 3rd experimental group 24 students, making a total of 89 students. The interventions took place in the Resource Rooms in which the students were enrolled, and were implemented by the special education teachers, who had received two-week-training courses by the researchers.

2.2. Procedures and Tools

The interventions included the following stages: initial pre- intervention assessment, implementation of interventions, control of fluency and generalization of ACs' use after the intervention and, finally, control of maintenance three weeks after the end of interventions.

The basic intervention (control group) was grounded on the fundamental principles of effective teaching: direct instruction, the use of 3 modes of knowledge representation (concrete, representational, abstract), guided practice, independent practice, systematic progress monitoring, and regular student feedback. The intervention for the 1st experimental group was a combination of the instruction of the control group and an alternative grouping of ACs based on a common characteristic. Examples of groups for the division ACs are: divisions by 1 (9 ACs, e.g. 7: 1), divisions by the number itself (8 ACs, e.g. 8: 8), divisions by 2 (8 ACs, e.g., 10: 2), divisions by 5 (8 ACs, e.g. 15: 5), divisions by 9 (8 ACs, e.g. 18: 9), divisions as successive subtractions (5 ACs, e.g. 14: 7), divisions using relevant multiplication ACs (35 ACs, for example 42: 6, 46: 7 → 6x7=42, 7x6=42).

Table 1. Demografic characteristics of participants

		Gender		Age				Grade		Years	
		Boys	Girls	Boys		Girls		3rd- 4th	5th– 6th	in integration	
Groups	*N*	*N*	*N*	*M*	*TA*	*M*	*TA*	*N*	*N*	*M*	*TA*
Control group	20	12	8	10.3	1.2	10.2	1.3	12	8	2.32	0.82
Exp. G. 1	19	14	5	9.66	1.6	9.25	0.8	10	9	2.67	1.23
Exp. G.2	26	14	12	10.1	1.3	9.99	1.3	14	12	2.02	0.94
Exp. G.3	24	14	10	9.34	1.2	11	1.2	14	10	1.96	0.97
Tolal	89	54	35	9.94	1.33	10.15	1.18	50	39	2.21	1

The intervention for the 2nd experimental group included a combination of the intervention used for the 1st experimental group and some strategic instruction for ACs, such as the commutative property, the use of known division ACs for finding unknown ones (e.g. 14:2=7 → 28:2=14 because 28=2x14), and the complementary relationship between multiplication and division ACs (fact families, e.g. 5x6=30 → 30:6=5). Finally, the intervention for 3rd experimental group consisted of the 2nd experimental group's intervention plus components of number sense, such as the *part-part whole relationship*, the connection between quantitative - symbolic representations of numbers, and skip counting skills.

The order of integrating each specific technique in the initial teaching intervention was based on the following rationale: Since most students with LD have difficulties in retaining and recalling ACs due to memory limitations, it is reasonable to add initially alternative grouping to effective instruction, so that students acquire as many ACs as possible. For example, knowing the rule that in all multiplications with 1 (a x 1) the result is always the other factor (e.g. 8x1 = 8, 1x9 = 9), students can effectively learn all ACs *x1*, without additional effort and practice for every AC separately [24]. However, despite the effectiveness of alternative grouping, not all students with LD can be expected to overcome their difficulties in recalling ACs only through this technique, as their composite weaknesses will probably not allow direct memory access at all times. In these cases, the use of fact finding strategies is appropriate, especially using known ACs for finding unknown ones. Obviously, if students do not possess an adequate number of ACs, the strategic instruction should be ineffective. That is why the alternative grouping preceded the instruction of strategies. On the other hand, of course, the use of strategies by students with LD might be limited by the fact that they do not understand or are insufficiently aware of interconnections, analysis and synthesis of numbers and correlations between known and unknown ACs [25]. Therefore, strategic knowledge is important to be combined with components of number sense. This thought led to the formation of the 4th instructional synthesis of this research.

As for the sequence of teaching activities, there had initially been an "*activation*" of known ACs, that were already possessed by students, as it is known that students with LD do not spontaneously transfer knowledge even if they do have it [1,26]. Activation of pre-existing knowledge was followed by direct instruction of ACs, with the use of the three modes of knowledge representation, which promotes the conceptual understanding and the gradual increase of student's involvement in learning process [12]. Next stage was guided practice, where students with their teacher's help were brought in the accuracy and speed of recalling ACs with time-limited activities (guided practice criteria: accuracy - at least 80% correct answers, speed - 3" at the most for each answer [7]. Guided practice was followed by the independent practice, in which each student worked on the accuracy and fluency of ACs on his/her own and the teacher controlled over the final results. The next stage involved the final assessment of teaching, which was done by measuring the fluency of new ACs taught in each lesson [13]. Finally, students were given detailed information on their progress using a scoreboard, where the raw score of the final assessment for fluency, maintenance and generalization of ACs which had been taught was noted. The teacher praised students for their effort and progress and the student had the opportunity to check his/her learning progress and evaluate his/her efforts [20].

2.2.1. Teaching Content

The teaching content was identical for all groups to ensure the same level of difficulty and it included 120 of the totally 190 multiplication and division ACs (60 from each operation).

2.2.2. Teaching Duration

The total number of teaching hours was 12 sessions (teaching hours) of 35-40 minutes each, spread over 4 - 6 weeks (2-3 times per week). The second experimental group received additional four hours of teaching (total 16) for the learning of strategies for finding ACs. The third experimental group, besides the above 16 hours, received additional 6 hours for teaching number sense (total 22). It should be stressed that the additional time in both experimental groups did not directly aim at teaching ACs, but at the prerequisite skills and concepts.

2.2.3. Measurements

The measurements made to determine the effectiveness of interventions were:

Fluency of ACs: Fluency was measured by having students answer in 1 minute as many ACs as they could, from a total of 20 ACs for each operation (20 ACs x 3'' seconds for each =60'' for 20 ACs). The raw score was the sum of all the correct answers for each operation separately [26,27]. The reliability of the measurements was satisfactory (a = 0.85).

Generalization of ACs: Generalization was controlled by having students finding the answers to 10 exercises for each operation with 3 single-digit numbers, e.g. (3 x 5) + 2 =..... and (12 : 4) - 2 = ... [28]. The raw score was the sum of all the correct answers. The reliability of the measurements was satisfactory (a = 0.76).

Maintenance of ACs: A test similar to the one testing fluency was used for testing maintenance, with 10 ACs for each operation, three weeks after the end of interventions. The reliability of the measurement was satisfactory (a = 0.74).

2.2.4. Fidelity of Implementation

The fidelity of the interventions was examined by the two researchers with systematic observation and recording of at least 4 sessions for each intervention group. Quality of implementation included the degree to which the special education teachers followed scripted procedures throughout the lesson. The implementation was rated on a 4-point scale (1 = poor, 4 = excellent). In all cases the results showed full implementation according to the criteria of reference [6].

2.2.5. Statistical Analysis

Initially, there was a checking of dispersion and normal distribution of the data using Shapiro-Wilk and Levene's test, respectively, so as to determine whether requirements for using parametric tests are fulfilled. The results showed that some variables do not follow standard normal distribution. Therefore non-parametric tests were selected. Wilcoxon Signed-rank test was used to evaluate intra-group comparisons between initial and final performance in each research group separately, while Kruskall- Wallis test was used for inter -group comparisons in final fluency, generalization and maintenance of ACs. Furthermore, the effect size was used to examine differences in the impact of the interventions [29]. The examination was based on the formula

$$r = Z \;/\; N \qquad (1)$$

where Z is the conversion of the individual data in a standardized format and *N* is the number of participants' observations. For example, in the control group the number of participants was 20 students and the data were 20 before and 20 after the intervention (pre- and post-intervention assessment), so *N* = 20 + 20 = 40 data [30,31]. Use of the r statistic is recommended when the research population is small and analyzes are made with non-parametric tests, as was done in this research. According to reference [31] correlations between values of r and the effect of an intervention is: r = 0.10-0.30 small effect, r = 0.31-0.50 modest effect and r> 0.51 large effect (Fritz et al., 2012). Due to the design of the research, where only one new element is added to the intervention of each group, the effect size allows us to make separate estimations about the effect of each additional element (alternative grouping, teaching strategies and number sense), as well as about the combinations arising between them.

3. Results

The results of appropriate controls (Kruskall-Wallis) showed no significant differences between the four groups of participants in: gender (χ^2 = 1.893, df = 3, p = 0.595), age (χ^2 = 2.080, df = 3, p = 0,556), grade (χ^2 = 0.897, df = 3, p = 0.826), and years of attending the Resource Room (χ^2 = 4.825, df = 3, p = 0.185) (Table 1). Also, in terms of their initial performance on fluency of multiplication ACs (χ^2= 1.492, df = 3, p = 0.68) and division ACs (χ^2 = 3.440, df = 3, p = 0.33) there were no statistically significant differences between groups that could affect the results (Table 2).

The results concerning fluency of ACs before and after the interventions and the effect size of the interventions in the final performance of each group are presented in Table 2. The results concerning the maintenance and the generalization of ACs are presented in Table 3. Evidently, all groups, regardless of the intervention, improved their levels of fluency of ACs significantly for both operations.

Table 2. Intra-groiup comparisons in fluency pre and post interventions

		Fluency pre		Fluency post		
Multiplication	N	M	SD	M	SD	r
Control group	20	6.5	3.98	12.25	5.1	-0.59**
Exp. Group 1	19	5.89	3.23	12.74**	3,84	-0.58
Exp. Group 2	26	7.12	3.81	13,58**	3.75	0.62
Exp. Group 3	24	6.63	4.07	14.3**	4.25	0.61
Division						
Control group	20	5.5	4.76	10.06*	6.07	0.42
Exp. Group 1	19	4.37	4	11.00**	4.09	0.60
Exp. Group 2	26	3.15	3.49	11.12**	4.68	0.60
Exp. Group 3	24	4.5	3.98	10.87**	6.06	0.54

*p<0.05, **p<0.001 in comparisons between pre and post intervention.

Specifically, as for the fluency in using multiplication ACs, all groups appeared to have achieved significant improvements and the effect of the intervention proved to be significant too: control group Z = -3.793, p = 0.00, r = -0.59, 1st experimental group Z = -3.585, p = 0.00, r = -0.58, 2nd experimental group Z = -4.476, p = 0.00, r = -0.62, and 3rd experimental group Z = -4.200, p = 0.00, r = -0.61

Moreover, all groups showed significant improvement on fluency in using division ACs. Specifically, in the control group, the improvement was significant and the effect of intervention moderate (Z = -2.644, p = 0.008, r = -0.42), while in the experimental groups improvement was significant and the effect of intervention large (1st experimental group Z = -3.731, p = 0.00, r = 0.60 / 2nd experimental group Z = -4.300, p = 0.00, r = -0.60 / 3rd experimental group Z = -3.757, p = 0.00, r = -0.54).

The results regarding performance of ACs after the intervention showed that there were no statistically significant differences among groups in fluency (χ^2 = 2.701, df=3, p=0.44) (Table 2), maintenance (χ^2=3.929, df=3, p=0.27) and generalization (χ^2 =2.536, df=3, p=0.47) of multiplication ACs (Table 3).

Table 3. Results of comparisons in maintenance and genergalization

		Maintenance		Generalization	
Multiplication	N	M	SD	M	SD
Control group	20	7.25	2.24	7.65	2.7
Exp. Group 1	19	7.89	1.72	8.21	3.19
Exp. Group 2	26	7.23	2.58	7.69	2.73
Exp. Group 3	24	8.35	2.03	8.43	2.12
Division					
Control group	20	7.61	3.68	5.94	3.71
Exp. Group 1	19	7.05	1.92	7.11	2.25
Exp. Group 2	26	7.58	2.8	6.92	2.82
Exp. Group 3	24	6.14	2.71	6.13	3.29

Also for division ACs there were no statistically significant differences in fluency (χ^2=0.385, df= 3, p=0.943) (Table 2), maintenance (χ^2=5.178, df=3, p=0.16) and generalization (χ^2= 0.859, df = 3, p = 0.83) among groups (Table 3).

Overall, the results showed that students with LD in all groups of the research, regardless of the intervention they had received, presented a significant improvement on learning the ACs of both operations, while there are no significant differences among them.

Table 4 presents the effect (r factor) of interventions on fluency of multiplication ACs, comparing the final performance of groups in pairs (control vs. 1st, 2nd and 3rd experimental group, 1st vs. 2nd and 3rd experimental group, 2nd vs. 3rd experimental group).

Table 4. Effect size among all groups

Effect size (r)						
	Control group			Exp. G. 1		Exp.G. 2
Fluency	Exp 1	Exp 2	Exp 3	Exp 2	Exp 3	Exp 3
Multiplication	0.05	0.15	0.21	0.11	0.19	0.08
Division	0.14	0.09	0.06	0.01	0.01	0.02

For the multiplication ACs comparisons between control group and experimental groups showed that the effect of alternative grouping was negligible (0.05). The effect of combining alternative grouping and fact strategies was small (0.15), and adding components of number sense in all of the above had also small effect (0.21). Teaching fact strategies had small effect (0.11), and the combination of fact strategies with components of number sense had also small effect (0.19). Finally the effect of dimensions of number sense was also negligible (0.08). (Table 4).

As for the division ACs, the pair wise comparison between the control group and the experimental groups showed that the addition of alternative grouping to the basic intervention had small effect (0.14), while the effects of all other combinations were negligible (Table 4).

4. Discussion

The first aim of this study was to examine the effect of an intervention based on fundamental principles of effective instruction on LD students' ability to learn ACs of multiplication and division. The second aim was to establish the extent to which the three interventions resulting from the gradual enrichment of the basic intervention, initially with an alternative grouping of ACs based on a common feature, then by teaching fact strategies, and finally by teaching certain components of number sense, differ in effectiveness among them and from the basic intervention.

The results showed that the intervention which was based on the six principles of effective instruction had a positive impact on the acquisition of ACs of multiplication and division by students with LD in the control group, since the number of ACs that the participants of this group could fluently recall almost doubled. This finding is consistent with researches and meta-analyses that have examined the impact of interventions based on principles of effective instruction and concluded that these interventions are especially effective with regard to basic mathematical knowledge, such as ACs [5,6,13,21,32]. The effectiveness of interventions based on the principles of effective instruction, just as the control group intervention of the present study, can be attributed to several factors, such as (a) the use of systematic measurements for the initial and the final performance of student, (b) the establishment of clear learning goals, (c) the control of prerequisite knowledge, (d) the promotion of conceptual understanding and (e) the facilitation of memorization and direct recall by organizing a network of inter-correlated knowledge of ACs [19,33,34,35].

In comparing the four interventions as to their effectiveness in supporting students with LD in ACs acquisition, it was observed that all groups improved their performance. Although the 2nd and the 3rd experimental groups showed higher levels of fluency in ACs of both operations compared to the 1st experimental group and the control group, the differences among groups were not statistically significant. The absence of statistically significant differences was also evident in reference to maintenance and generalization of ACs of both operations. The results are consistent with studies which compare interventions for ACs and find no significant differences among them in fluency, maintenance and generalization of ACs, when all the interventions are of enhanced quality, and there is no comparison with the qualitatively ambiguous common instruction of ACs in ordinary general classes, e.g. [7,13,36,37,38]. In the present research, the control group received a high- quality teaching support based on the principles of effective instruction, which alone yielded positive results that cannot be easily overcome by experimental interventions, especially if these interventions differ from the basic one in a small number of special techniques. It should be noted that the design of the present study allows the significant limitation of the possibility to attribute more advantages to the specific techniques used than those they actually have or might have in case of a basic intervention with significant shortcomings and general low quality [21,32].

The results showed that when effective instruction is combined with alternative grouping of ACs, teaching of strategies, and acquisition of number sense dimensions, separately or in conjunction, the resulting differences in the fluency of ACs are small compared to effective instruction that does not include these techniques. The results agree with meta-analyses which suggest that when the control group receives a specific instructional support that differs from the everyday instruction, then the effect of the experimental intervention is low or moderate, while when the control group does not receive a specific support (but continues to be instructed in the usual way), then the effect of the experimental intervention is large [19,33,34]. In particular, the alternative grouping, as shown in the comparison between the control group and the 1st experimental group, has hardly any effect on the fluency of ACs. The results are in line with reference [38] who showed that the alternative grouping is not sufficient for the students to improve fluency of addition and subtraction ACs. However, researches such as reference [5] and reference [7] which had included the alternative grouping in the interventions for fluency of multiplication ACs, found that the results were positive. This difference may be due to the differences in the sample and the research design.

The teaching of strategies has positive, albeit small, effect on the fluency of multiplication ACs, as shown in the comparison between the 1st and the 2nd experimental groups. The results agree with other studies [7,23,25,37] and suggest that students with LD need direct teaching of strategies, either because they do not usually develop them spontaneously, as students without LD usually do [1,8,28] or because they do not use them regularly with ease and efficiency even when they have developed some of them [23,38,39].

Just like the alternative grouping, the number sense has minimal effect on the fluency of multiplication ACs, as shown in the comparison between the 2nd and the 3rd experimental groups. The results are consistent with researches which show that the number sense is not significantly correlated with the fluency of Acs [40,41]. The results partially support the view of reference [9]), who claim that number sense affects the fluency of ACs according to the characteristics of operations (e.g. subtraction). Perhaps what applies to addition ACs, might also apply to multiplication ACs, because both operations share common features. Specifically, the ACs of both operations are usually recalled as phonological information from memory, in the form of *six, eight ... forty-eight* [42] and there is no interference of the number sense [43,44]. Furthermore, the common strategy conventionally used for finding an unknown multiplication AC is repeated addition (e.g. 6x4 is 4 + 4 + 4 + 4 + 4 + 4 = 24). Also, the commutative property applies to both operations (a + b = c and b + a = c, a x b = d and b x a = d) [45].

For the ACs of division, the isolated effect of alternate grouping is small (0.14), as shown in the comparison of the 1st experimental group with the control group, while the effect of teaching strategies and number sense is neutral, as shown in the comparison of 1st with 2nd experimental group (0.01) and the comparison of the 2nd to the 3rd experimental group (0.02), respectively. The small effect of the alternative grouping on the fluency of division ACs may be a function of the imperfect groups used, as students recalled the most prominent and easily memorized groups (e.g. multiplications *x1* or *twin-numbers*), but had difficulty in learning the 35 division ACs for the memorization of which they had to use relevant multiplication ACs.

As for the neutral effect of teaching strategies for fluency of division ACs results are consistent with studies such as reference [28] and reference [29], which showed that strategies improve accuracy, but not the response speed and, consequently, fluency. Therefore, in measurements of fluency, where students answer a multitude of ACs in minimum time (1 or 2 minutes), the results are likely to be low and to not reflect the real level of student's knowledge. The results seem to justify the debate about the appropriateness of using fluency (correct answer in <3 " seconds) as a measure of ACs' possession in all four operations. A possible explanation of these results is that for addition and multiplication ACs the direct recall may be the most effective strategy; in contrast, for subtraction and division ACs the use of corresponding addition and multiplication ACs may be more effective, although it requires longer time or more practice to reach the desired levels of performance (e.g. the result of 56 : 7 = ? may be found through the corresponding multiplication 7 x 8 = 56) [7]. However, in the present study students were taught the use of corresponding multiplication ACs for finding division ACs, but the result was not as satisfactory as expected. One possible explanation is that this technique presupposes accurate and immediate recall of multiplication facts, and if students have not reached the necessary performance level, then, even if they understand the complementary relationship of operations, they cannot recall the appropriate multiplication ACs and use them for finding the corresponding division ones [7,8]. Due to reasons of research fidelity, a number of participants of the present study had to move on to the next phase of the intervention before reaching the criterion of 80% accuracy in multiplication ACs' recall. This feature of the research design may have undermined their ability to perform at the desired level regarding the division ACs. In other words, it is possible that if longer time had been devoted to practicing multiplication ACs or teaching division ACs, the results might have been different [7]. Moreover, the results support the view that learning of addition and multiplication ACs at the direct recall level should precede the instruction of corresponding subtraction and division ACs [8,46,47].

The findings that the number sense is not associated with fluency of division ACs contrasts with researches like reference [9], who argue that the number sense promotes the learning of difficult ACs of subtraction and division, as they are more difficult operations requiring complex conceptual and quantitative manipulations. A possible explanation of the result may be that the specific components of number sense chosen to be taught the participants of the present study may not play such an important role in ACs' acquisition. The issue certainly warrants further exploration.

With regard to the maintenance and generalization of ACs of both operations there were no significant differences among groups. As for the maintenance, the result can be attributed to the fact that the four groups had similar average performance at the end of the intervention, which they retained for the next three weeks. In reference to generalization, higher performance levels were rather expected for the 2nd and the 3rd experimental groups, in comparison to the 1st experimental and the control groups, because interventions included teaching fact strategies that have been found as promoting generalization of ACs [28,45]. However, the overall results on generalization of ACs in the present study is not in line with previous research. A possible explanation might be that the generalization task of the present study was conceptually relevant to the tasks of the main intervention, with small transfer requirements of the possessed knowledge to higher tasks (near transfer). So, perhaps the task was not suitable to show the advantages of including strategies in the 2nd and the 3rd experimental interventions. Another reason for the same result might be connected to age differences between our sample and that of other studies e.g. reference [28].

A key limitation of the study is the relatively small number of participants with LD. Even though in interventional studies it is not easy to have large samples, the conduct of research with larger populations per intervention group is essential in order to confirm the trends that have emerged here [7,28,45].

Another limitation, that might affect the results, relates to the decision that the transition from multiplication to division be based on the need to complete the experiment in a specified period of time, without necessarily students having mastered fluency of ACs at accuracy level > 80%. The choice may affect the results mostly for division ACs, as multiplication ACs are prerequisite knowledge for them in the context of certain techniques. For example, the application of complementary relationship as a strategy for division ACs requires mastering multiplication ACs. If the students do not pose this knowledge, they have to manage a wealth of new information of data, when they are taught strategies and concepts which link operations between each other, and it is probable that they fail to respond successfully.

An important general conclusion is that the design and implementation of interventions based on the principles of effective instruction may support decisively student's efforts to learn ACs of multiplication and division. Various special techniques can contribute to the improvement of interventions, depending on the features of the operations. Further research is required in order to establish the best methodological syntheses for teaching effectively the ACs of each operation to students with learning disabilities.

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