Metacognitive Strategy Knowledge Use through Mathematical Problem Solving amongst Pre-service Teachers

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Abstract Metacognition-related studies do often give focus on the regulation or experience components but little on the knowledge component. In particular and especially within the Philippine context, not much focus is given with regards to a clear and coherent academic framework that fortifies the metacognitive strategy knowledge in mathematical problem solving amongst students. Using an evolved grounded theory, the purpose of this study is to look closely into the metacognitive strategy knowledge of preservice teacher education students. Twenty-three students participated and initial data were collected using the prepared problem solving test. Subsequently, interviews were conducted to supplement the initial data. Pandit's grounded theory methodology and the constant comparison method were used to analyze the data collected. Findings revealed an emerging three-phased categorization of metacognitive strategy knowledge thru problem solving: preparatory, production, and evaluation. The multi-distinct yet related categorization were neither linear nor just cyclic in nature but is experienced and underwent by problem solvers with varying degree of creativity and flexibility depending on the problem at hand, beliefs, attitudes, and learning style. The findings shed some light on the distinct role of metacognitive strategy knowledge and some ensuing factors during authentic problem solving.

Keywords: *metacognition, metacognitive strategy knowledge, preservice, teacher education, mathematical, problem solving*

1. Introduction

Problem-solving is an important part of intellectual behavior of the individual. It has a special importance in the study of mathematics [24]. Mathematical problem solving is taught to the students to develop a generic ability in solving real life problems and to apply mathematics in real life situations. In solving, frameworks were created to model the step-step process. Most formulations of a problem solving framework attribute some relationship to Polya's problem solving stages. These stages were described as understanding the problem, making a plan, carrying out the plan, and looking back. Failure in problem solving is generally resulted from failing, to choose the most effective method, to organize the mathematical operations, to understand the point of problem, to analyze, and to monitor and control operations carried out [21]. On the other hand, metacognition has been found essential to come to successful learning. Studies on metacognition have proven that there is a strong correlation between problem solving and metacognition. It has been identified by many researchers that a key factor in the problem-solving process is metacognition.

The notion "metacognition" was further developed by the end of the 1970's and through the 1990's by many researchers who were interested in the psychology of metacognitive thinking: Brown [2,3], Brown et al. [1], Garofalo & Lester [9], Wellman [23], Schoenfeld [17,18], Campione, Brown, & Connell [4], Lester [12] and many Others. During these years metacognition became a successful tool for researchers investigating thinking processes in the instructional domain. There are different types of metacognition, namely: metacognitive skills, metacognitive experience and metacognitive knowledge. Metacognitive knowledge has three general types that are of particular importance. Strategy Knowledge refers to students' knowledge of general strategies for learning and thinking ,Task Knowledge refers to knowledge of cognitive tasks as well as when and why to use these different strategies and Person Knowledge refers to knowledge about the self (the person variable) in relation to both cognitive and motivational components of performance. Metacognitive strategy knowledge include knowledge about both cognitive and metacognitive strategies, as well as conditional knowledge about when and where it is appropriate to use such strategies. It also involves skills needed to solve a problem such as prediction/orientation, planning, monitoring and evaluation.

Related studies in the literature focused on naming and categorizing of metacognitive strategies to propose appropriate frameworks. Although such approaches can help develop understanding in metacognition, and uncover potential associations between cognition and metacognition, they fail to provide a research base to facilitate students to improve metacognitive strategy knowledge during problem-solving activities. Such a research base is mainly vital for preservice teacher education students who need to be trained to develop problem-centered approaches in their teaching. This is to facilitate the development of metacognitive strategy knowledge in their future students.

This study focused on understanding metacognitive strategy knowledge employed by a sample of preservice education students as they engaged in mathematical problem solving.

2. Method and Sources of Data

This descriptive and qualitative study employed method of grounded theory guided by Pandit's approach [15]. Also, Techniques and analytical tools by Strauss and Corbin [20] were used in coding process and the constant comparison method by Glaser and Strauss [10].

The research design phase in accordance with Pandit's grounded theory methodology consisted of six phases with nine steps namely: the review of technical literature consisted of two steps namely definition of research question and a priori constructs; selecting cases; the data collection phase consisting of development of rigorous data collection protocol and entering the field; data ordering phase consisting of data ordering; the data analysis phase consisting of analyzing data relating to the first case and theoretical sampling; and the last phase consisting of reaching closure and comparing emergent theory with extant literature.

Step 1: Definition of Research Question and A Priori Constructs

A. Definition of Research Question

Problem solving is the process of interpreting a situation mathematically that usually engages several repetitive cycles of expressing, testing, and revising mathematical interpretation ([11], p. 782)

Problem-solving is a form of inquiry learning where knowledge that is existing is applied to unfamiliar of innovative situations in order to gain new knowledge (Killen, 1996; Sternberg, 1995, cited by [7]). It also refers to a vehicle for learners to evaluate, construct and refine their own beliefs and theories about mathematics as it relates to the beliefs and theories of others [14]. Engaging in problem-solving involves finding an answer for a particular problem and encouraging learners to develop their own ability to think mathematically [19]. The processes involve use of content knowledge, procedures, strategies, language, and reflections [9,17,18].

Metacognition is a skill often studied and associated with problem solving. Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them. It also refers to active monitoring and consequent regulation and orchestration of cognitive processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective [8].

Metacognition includes metacognitive knowledge, metacognitive skills and metacognitive experience.

There are three types of metacognitive knowledge related to mathematical problem solving: person knowledge; task knowledge; and strategy knowledge. The focus of this study is almost exclusively on the mathematical strategy knowledge component of metacognitive knowledge. Metacognitive strategy knowledge includes knowledge about both cognitive and metacognitive strategies, as well as conditional knowledge about when and where it is appropriate to use such strategies.

The main question in this present study aimed at understanding the use of students' metacognitive strategy knowledge in mathematical problem solving.

B. Definition of a Priori Constructs

This includes the discussion on scope of mathematical strategy knowledge.

Mathematical strategy knowledge (Strategic Knowledge) naturally consists of knowledge of algorithms, heuristics and person's awareness of strategies to aid comprehending problem statements, organizing information or data, planning solution attempts, executing plans and checking results [9].

Strategy Knowledge is knowledge of general strategies for thinking, learning and problem solving. There are a number of general strategies which represent the various heuristics individuals can use to solve problems for problem solving and thinking in mathematics. In the problem-solving area they can include the knowledge of means-ends analysis, working background and others while in thinking, they may use general strategies for inductive and deductive thinking, such as evaluating the validity of different logical statements, avoiding circularity in arguments, making appropriate inferences from different sources of data, and drawing on appropriate samples to make inferences. In addition, students can have knowledge of various metacognitive strategies that can be useful in planning, monitoring, and regulating their learning and thinking. These include ways individuals plan their cognition (e.g., set subgoals), monitor their cognition (e.g., checking their answer to a math problem; asking themselves questions as they read a piece of text), and regulate their cognition (e.g., reviewing and re-compute calculating mistake in a math problem, re-read something they don't understand). Metacognitive Strategy knowledge consists of various strategies students might employ to memorize materials, to extract meaning from text, and to comprehend what they hear in classrooms or what they read in books and other course materials. Although there are a large number of different learning strategies, they can be grouped into three general categories: rehearsal, elaboration, and organizational [22]. Rehearsal strategies refer to the strategy of repeat terms or words to be recalled over and over to oneself. Generally, it is not the most effective strategy for learning more complex cognitive processes. In contrast, elaboration strategies include a variety of mnemonics for memory tasks, as well as strategies such as paraphrasing, summarizing and choosing major thoughts from texts. These result in deeper processing of the material to be learned and better comprehension and learning. Finally,

organizational strategies include various forms of note taking, outlining and concept mapping where the student makes connections between and among content elements. Like elaboration strategies, these organizational strategies usually result in better comprehension and learning than rehearsal strategies.

Step 2: Selecting Cases

Based from the research design of Pandit's model, theoretical sampling, which is one of the types of purposive sampling techniques, was utilized in the selection of the cases. This enabled the researcher to focus on theoretical useful cases that either test or broaden the emergent theory. Theoretical sampling is based on emerging concepts, with the goal of being able to investigate the dimensional range or varied conditions along which the properties of concepts differ [20]. Hence, theoretical sampling is unplanned. The specific sampling decisions progress during the research process itself.

In the initial sampling, the researcher was interested in generating categories as possible [20]. The identification of the first respondent was done through identifying who among the students enrolled in problem solving subject last summer 2011 had the highest grade. In case of very close grades, the recommendation of their teacher in Problem solving was considered. The above statement is the basis of the type of problem solver that the researcher interviewed first.

The rest of the participants were theoretically sampled. Moreover, the participants of this study were the students of Saint Mary's University who were officially enrolled in problem solving subject during summer 2011.

2.1. Data Collection Phase

Data collection phase included development of rigorous data collection protocol and entered the field by exploring compound data sources to put on different types of data.

Step 3: Development of Rigorous Data Collection Protocol

Case study data base was created in this phase and the multiple data collection methods were implemented to address the reliability and construct validity of the case study.

In this study, the case study data base was created and the multiple data collection methods like semi-structured interviews, questionnaires and videotapes were initially planned and applied. Moreover, the use of students' actual mathematical problem solving outputs was deemed useful especially in the triangulation of data.

Step 4: Entering the field

In this step, the overlapping of data collection and analysis was done in making adjustments to the ongoing data collection. In this study, the researcher entered the field as a researcher, all she did was to conduct semi-structured interview and let the participants fill up the questionnaire.

2.2. Data Ordering Phase

As the third phase of the grounded theory-building process, data ordering phase facilitated easier data analysis and permitted examination of processes.

Step 5: Data Ordering

Data ordering includes sorting and arranging events chronologically to allow examination of processes and to facilitate data analysis. Data in this study were organized according to selective and specified set of properties and dimensions since it underwent a nonmathematical process of interpretation that were carried out to discover concepts and relationships in raw data.

2.3. Data Analysis Phase

Pandit [15] asserted that data analysis is central to grounded theory approach. This makes use of the process of coding in which data are broken down, conceptualized and put back together in new ways [20].

Step 6: Analyzing Data Relating to the First Case

In this study, open coding, axial coding and selective coding were utilized. Open coding means labeling and categorizing phenomenon as suggested by the collected data. It is in this stage that concepts, the building blocks in the construction of grounded theory, emerged. It utilizes constant questioning, fracturing the data, and making comparisons. In this study, data were compared and similar incidents were grouped together and were given the same conceptual label. Grouped concepts were categorized into a higher more abstract level. Axial coding, according to Pandit [15], refers to the process of developing main categories and their sub-categories. It was used in this study to put the data together and to develop connections between category and its sub-categories. Moreover, selective coding was done to integrate categories and to build the initial theoretical framework. A story line was generated from descriptive narrative about the central phenomenon of the study. When analyzed, the core category emerged from the story line. Strauss and Corbin [20] stressed that the core category must be the sun, standing in orderly systematic relationship to its planets. The emergent categories are related to the said core category according to the paradigm model that allows a researcher to think in a systematic manner as questions are asked back and forth, propositions are generated and comparisons are frequently implemented. An essential component of the coding process is writing theoretical memos that are very useful in the formulation and revision of the theory during the process. Hence, the memos written by the researcher in her study were very useful in containing the directions relating to the evolving theory building process.

First, data that were gathered from the first case were analyzed using open coding. This started by drawing out concepts from the raw data. Concepts that were derived from these raw data were analyzed further and related concepts were grouped together to form categories using the axial coding. Microanalysis of the data was done in word-for-word, line-by-line or sentence-by-sentence through the constant comparison method to generate categories with their properties and dimensions and to suggest relationships among categories. Microanalysis is actually a combination of open and axial coding. Conceptualization or labeling of the data was also used while doing the microanalysis. Ideas emerged from these were categorized. The researcher wrote memos in sentence or two but these could possibly run on to a paragraph in few occasions. After determining the concepts from the raw data, these were analyzed and related concepts were grouped together. And for each sub categories of concepts which the researcher called

categories, the researcher drew out common properties. Common properties were labeled as subcategories. The resulting sub categories were referred to as properties of teachers' knowledge.

After the open coding, axial coding was done to draw out relationships among categories, sub categories and concepts. This coding was made to draw comparison from events and situations where the concepts were drawn. When categories emerged, then selective coding was done to determine the core category out of the categories formed from axial coding. The core category was known by analyzing the categories and sub-categories, and identifying one element in the case that hold the categories and sub-categories together. The core category functioned as a theme, or a principle that orchestrated the interaction of categories and sub-categories towards a common path or purpose.

After open coding, axial coding and selective coding of the raw data gathered from the first case, a paradigm or framework showing relationships of concepts emerged. This became the initial, tentative framework for metacognitive strategy knowledge in mathematical problem solving which was presented in a form of proposition. The result that came out from the open coding of the first case served as a guide in data collection and analysis in the other cases.

Step 7: Theoretical Sampling

Strauss and Corbin [20] define theoretical sampling as sampling on the basis of emerging concepts with the aim of exploring the dimensional range or varied conditions along which the properties of concepts vary. That is, this step confirmed, extended and strengthened the theoretical framework through the replication across cases. In this study, selecting additional cases was done to confirm, broaden and sharpen the theoretical framework that emerged from the data analysis in the first case. It involved testing and developing the initial theoretical framework by selecting additional cases according to the principle of theoretical sampling.

Data gathering was guided by the intention, on the part of the researcher, to determine whether the same categories and additional ones emerged as cases are added. The same coding process of data analysis was performed in the other cases. Cross-case analysis was done to determine similarities and differences among the participant as regards to categories, sub-categories or properties, and their relationships. The model that emerged from the findings in all cases after cross-case analysis contained all similar and unique categories in all the participants.

Step 8: Reaching Closure

This is the principle of theoretical saturation when the marginal value of the new data is minimal which then served as the basis on deciding when to discontinue adding cases and when to stop iterating between theory and data.

Theoretical saturation was attained when there are no more additional categories that emerged after the coding process on a case. That is, "there is no more juice that can be extracted from the oranges". The final model contained all the similarities and differences of the categories of all cases after constantly comparing concepts categories and subcategories as a result of constant comparison method by Strauss and Corbin [20].

Triangulation of data sources were observed in this study. Thus, aside from the semi-structured interviews complemented by videotaped proceedings and data were gathered through participant's actual mathematical problem solving output, filled up questionnaires.

Semi-Structured Interview complemented by a Videotaped Proceedings. This was employed during the semi-structured interviews in order to determine the sincerity of their answers during the actual interview and to make sure they were not just bluffing. Semi-structured interview for all cases was videotaped. This refer to interview between the researcher and respondents that is flexible, allowing new questions to be brought up during the interview as a result of the guide questions prepared. Examples of questions asked includes what they know about problem solving, how they solved problems, what strategies they used in solving and how do they know that they solved the problems correctly.

Actual Participants' Outputs in Mathematical Problem Solving Set. These refer to the solution of the students on mathematical problem written in paper. These documents were utilized to validate participants' responses on some exercises in mathematical problem solving strategy knowledge in the semi-structured interview for the purpose of data triangulation. The mathematical problem solving set was a researcher-constructed and adopted test composed of 2 items in arithmetic and algebra, 1 item in trigonometry, geometry, sets, probability, number theory and puzzle problem/logic. The test was tried out with 12 Master of Science in Teaching major in Mathematics students enrolled at Saint Mary's University. The first 10 items solved correctly by the majority were considered.

Filled up Questionnaire. This refers to the Metacognitive Strategy Knowledge Questionnaire. This questionnaire was based on the Mathematics Motivated Strategies Learning Questionnaire (MMSLQ) by Liu and Lin [13] and the prospective/retrospective metacognition questionnaire used in evaluating and improving mathematics teaching-learning process through metacognition by Desoete [6] that was filled up by the respondents to better understand their metacognitive strategy knowledge.

3. Results and Discussion

To solve the problems on this study, the grounded theory approach step by step is presented. Starting with the data analysis of the initial framework of "Using Metacognitive Strategy Knowledge in Mathematical Problem Solving Process" of Case 1 and this was developed by adding more cases or the theoretical sampling. Thus, the same step was undergone and presented across all cases to come up with framework.

Pre-service Teacher Education Students' Framework of Metacognitive Strategy Knowledge that Emerges during Mathematical Problem Solving: Case of Isagani

To solve this problem on identifying a pre-service teachers' framework of metacognitive strategy knowledge that emerges during mathematical problem solving, the grounded theory approach continues on data analysis.

Developing the Initial Framework from the Analysis of Isagani

In this section, the initial framework derived from the analysis of Isagani is presented. Prior to the presentation of the analysis, a description of the problem solving heuristics of Isagani is discussed and presented.

Describing the Metacognitive Strategy Knowledge of Isagani

Figure 1 illustrates the metacognitive strategy knowledge of Isagani as integrated in his problem solving process.
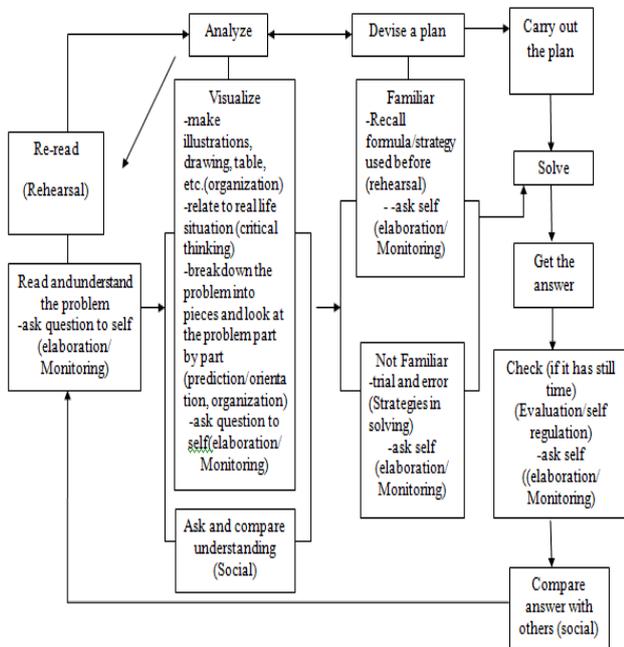
Figure 1. Metacognitive Strategy Knowledge of Isagani in his Problem Solving Process

Isagani's process is greatly affected by the way the problem is being presented whether dictations or written, individual or group work. He claimed that in solving mathematical problems he cannot understand problems being presented orally. He prefers to see it written. He first reads and understands the problem by rereading it. In analyzing the problem, he visualizes the setting of the problem by drawing, singling out needed details in the problem by listing it down and not necessary using all the given because for him some problems include distracters to test the comprehension and critical thinking skills of the solver. He also establishes connections amongst the part of the problem to decide what strategy to use and what fits the problem, make illustrations if necessary, break down the problem into pieces, look at them part by part and relate them to real life situations. An example of this is calculating mentally if the change is correct after buying something. He may solve some problems based from real life situation through applying the strategy he uses in solving problems he encounters in outside environment. The way he attacks the problem depends if the problem is familiar or not to him. If the problem is familiar, he recalls formula or strategy he used to solve similar problem given before, follow the GRESA (Given, Required, Equation, Solution Answer) method and consider several strategies in mathematics but uses the strategy that fits the problem if it is necessary. If the problem is not familiar, he reads the problem again and again. If this fails, he asks other students and compares how they understand the problem with how he understands the problem. If he has no other choice anymore, he uses trial and error strategy. He knows

that he understood the problem when he is confident of the strategy that he planned to do, he knows what to do and he knows the step by step process of coming up with the answer. Sometimes he has mental calculations before actually solving the problem or sometimes he just uses common sense in times when the answer is obvious and he doesn't have to go through the whole process anymore. Once he arrives at an answer, he goes back to the problem and sees if the answer satisfies what is asked in the problem by checking it. He commonly uses substitution especially when a formula is involved to get the answer. If he has an estimated outcome or estimation, he compares it with his final answer. He knows that he has solved the problem correctly if he tried to check it, the answer in his question, "does my answer makes sense?" is yes and tried to compare with others already. While doing the whole process of solving, he asks himself some questions like "what is really needed in the problem", "what are the things that I must include in my solution", "what strategy will I use?" and "does my answer make sense?". During the interview, he claims that he is a slow problem solver that is why he keeps on re-reading. When he thinks he is already somewhat sure of his analysis, he proceeds in planning and applies whatever strategy he formulated and finally solves the problem. If he has still time he checks his answer by looking back at the entire problem, rereading the problem, analyzing, devising a plan, carrying out the plan, and so on. Since he takes too much time in reading and analyzing the problem by looking back, when he already devises a plan, he sticks to that plan and seldom tries other strategy because he claims that in solving a problem especially during classes, the given problems have a time limit.

Isagani's problem solving process is a cycle of read-analyze-plan-solve-and-check phases. He usually reflects and looks back through engaging himself in self-questions. Thus, monitoring and evaluating himself in every step he does. When he finds the problem difficult and unfamiliar to him, he tries to ask questions to others. He also claimed that he prefers group activity in solving problem rather than individualized approach because he can convene with others or he can brainstorm with how others think about the problem. With the whole process case one has underwent to solve a mathematical problem, he uses metacognitive strategy knowledge such as self-questions, re-reading the problem, listing the important details and others. In addition, the act of asking question to others and wanting to convene with others through group work is another kind of strategy called "affective/social" strategy. This strategy is not covered by identified metacognitive strategies of this study.

Isagani's problem solving process is in a way similar to Polya's stages of problem solving. These stages were described as understanding the problem, making a plan, carrying out the plan, and looking back. Since case one's problem solving heuristic is somewhat cyclic in nature due to constant reflection, it also shows the simplified Polya's stages making use of problem posing. Brown and Walter have provided the major work on problem posing. Case one's problem solving process is somewhat similar to Schoenfeld and Lester's problem solving process since they develop Polya's stage by combining Polya's stages with information-processing theories. This resulted to five

stages of problem solving, namely: reading, analysis, exploration, planning and implementation and verification.

In addition, there was some metacognitive strategy knowledge which was not included in the illustration presented in Figure 1. Table 1 illustrates other metacognitive strategy knowledge of Isagani as it was obtained through his responses in metacognitive strategy knowledge questionnaire.

In addition, there was some metacognitive strategy knowledge which was not included in the illustration presented in Figure 1.

The Use of Grounded Theory in Data Analysis

After the review of technical literature consisting of two steps namely definition of research question and a priori constructs; selecting cases; the data collection phase consisting of development of rigorous data collection protocol and entering the field; and data ordering phase consisting of data ordering, the data analysis phase will be the next step consisting of analyzing data relating to the first case and theoretical sampling; and the last phase consisting of reaching closure and comparing emergent theory with extant literature.

Constant Comparative Method in Four Stages

These stages aim to discover emergent themes and categories.

First Stage: Comparing Incidents to Form Categories (Open Coding)

The first stage of constant comparison began by comparing data with a group of incident in a single interview/sector to form categories or what is usually called open coding. This is where concept, categories and subcategories were developed.

Table 1 illustrates open coding of an interview transcript as it appears in the open coding of an interview of Isagani.

Table 1. Incidents, Behaviors/Type of Metacognitive Strategy Knowledge/ Dimensions/Others and Category of Isagani

Incidents	Behaviors/ Type of Metacognitive Strategy Knowledge/ Dimensions/Others	Category
Mathematical problem solving as I have learned is a way of somewhat threshing out or using details of the given problem to come up with an answer and I think it is a systematic approach towards solving problems.	-Analysis of information - following step by step method in solving (heuristic)	Preparatory metacognitive strategy knowledge Strategies in solving
I used some method just like the old GRESA method and heuristics method.	-awareness of the method -systematic approach (monitoring) -heuristics	-strategies in solving -Use of Metacognition Strategy Knowledge

Second Stage: Integrating Categories and their Properties (Axial Coding)

It is in this stage that memoing for paradigm features and drawing out relationships between categories and subcategories took place.

Questions like what, when, where, why and how and with what result of consequences were asked in this stage. This is to understand the relationships among the categories. This stage made use of Conditional Relational Guide format, a reflective coding or formulate matrix to ask and answer each relational question about the category named in the far-left column.

- What is [the category]? (using a participant's words/memos helps avoid bias.)
- When does [the category] occur? (Using "during…." helps form the answer.
- Where does [the category] occur? (Using "in…" helps form the answer.)
- Why does [the category] occur? (Using "because…" helps form the answer.)
- How does [the category] occur? Using "by…" helps form the answer.)
- With what Consequences does [the category] occur or is [the category] understood?

Almost all questions in Conditional Relational Guide were used to this study except the question "where does the category occur?" because it was found to be inappropriate to be used. Below is the Conditional Relational Guide used in this study.

Table 2 illustrates a Conditional Relational Guide.

Table 2. Extract from Conditional Relational Guide

Phenomenon	What is the category	When does the category occur	Why does the category occur	How does the category occur (Use of Metacognitive Strategy Knowledge)	Consequences
Preparatory metacognitive strategy knowledge	Constructing meaning and developing interpretation	during first phase of the problem solving	primary encounter and sense making	-listing -making drawing, illustrations, tables, chart -reading the problem again and again	To understand the problem
Production metacognitive strategy knowledge	Exploring/ Discovering	During the second phase of the problem solving	Planning what to do	-Using trial and error -visualizing the situation -establishing connection amongst part of the problem -analyzing the problem part by part	Preparatory to design a plan. For better analysis

The Conditional Relationship Guide tries to relate the structure to the process.

The data obtained in this stage by answering the questions in conditional relationship guide were further developed or enhanced through selective coding and sorting categories.

Third Stage: Delimiting the Theory through Selective Coding and Sorting Categories

In this stage, there was a constant comparison of data, memoing and drawing out relationship among distinct categories and between the core category and the other categories to come up with a theoretical framework.

In this stage, a storyline was developed and formulated. This process includes writing a general descriptive overview, or story line, and verifying it with the data at hand.

Development of Story Line for Isagani (first case)

For the first case of this study, the main story line was:

How does Isagani solve mathematical problem? How does the types of metacognitive strategy knowledge affect mathematical problem solving and what metacognitive strategy knowledge does he involve in his solving? Isagani's problem solving process is a cycle of read-analyze-plan-solve-check phases. He has different ways of analyzing the problem such as reading it again and again, relating it to real life situation, recalling problems similar to problems solved before, visualizing the problem and others. He prefers to see the problem written rather than dictated by the teacher. Throughout the process, he uses self-questioning to monitor himself and evaluate himself on what he is doing. The ways he analyzes the problem and what he does before proceeding to his step-by-step problem solving process is an act of using his metacognitive strategy knowledge. Isagani's problem solving heuristics seems to be influenced by his beliefs, attitude and learning style. He believes that he is a slow problem solver and due to time limit, he fails to reflect or look back to the whole cycle of solving problems.

The story line for the first case serves as a guide to step back again to weave a version of the story at a higher level of abstraction, integrating structure and process in a single statement. Thus, the "Using Metacognitive Strategy Knowledge in Mathematical Problem Solving" emerged along the way.

In this study, Case one actually uses his metacognitive strategy knowledge to solve mathematical problems.

The following paragraphs illustrates the delimiting stage of the theory of the first case through the selective coding process.

Selective Coding for Core Categories for Isagani

"Using Metacognitive Strategy Knowledge in Mathematical Problem Solving" a theory that has emerged from three multi-distinct yet related main processes of categories:

1. Preparatory metacognitive strategy knowledge
2. Production metacognitive strategy knowledge
3. Evaluation metacognitive strategy knowledge

Some of the categories consisted of dimensions and sub-dimensions.

In solving mathematical problems, case one is greatly affected on the way the problem is being presented. He claimed that he cannot understand problems being presented orally. He prefers to see it written. He first reads and understands the problem by *rereading it*. In analyzing the problem, he *visualizes* the setting of the problem by *drawing, singling out needed details in the problem* by *listing it down* and *not necessarily using all given* because for him some problems includes distracters to test the comprehension and critical thinking skills of the solver. He also *establishes connections among the part of the problem* to decide what strategy to use and what fits the problem, *makes illustrations if necessary, breaks down the problem into pieces, looks at them part by part* and *relates them to real life situation*. An example is calculating mentally if the change is correct after buying something. He may solve some problems based from real life situation by applying the strategy he used in solving problems he encountered in the outside environment. The way he attacks the problem depends if the problem is familiar or not to him. If the problem is familiar, he *recalls formula* or strategy he use to solve similar problem given before, follows the *GRESA (Given, Required, Equation, Solution Answer) method* and consider several strategies in mathematics but uses the strategy that fits the problem. If the problem is not familiar, *he reads the problem again and again*. If he is still confused of the problem, he *asks his classmates and compares* how they understand the problem with how he understands it. If he has no other choice anymore, he uses trial *and error strategy*. He knows that he understood the problem when *he is confident of the strategy that he planned to do, he knows what to do and he knows the step by step process of coming up with the answer*. Sometimes he has *mental calculations* before actually solving the problem or sometimes he just uses *common sense* in times when the answer is obvious and he does not have to go through the whole process anymore. Once he arrived at an answer, *he goes back to the problem* and sees if the answer satisfies what is asked in the problem by *checking it*. He commonly uses *substitution* especially when a formula is involved to get the answer. If he has an *estimated outcome* or estimation, he *compares it with his final answer*. He knows that he has solved the problem correctly if he *tried to check it*, the answer in his question, *"does my answer make sense?"* is yes and *tried to compare* with others already. While doing the whole process of solving, he *asks himself some questions* like "what is really needed in the problem", "what are the things that I must include in my solution", "what strategy will I use?" and "Does my answer make sense?". During the interview, he claims that he is a slow problem solver that is why he keeps *on reading and reading the problem again and again*. When he thinks he is already somewhat sure of his analysis, he proceeds in planning and applies whatever strategy he formulated and finally solves the problem. If he has still time he *checks* his answer by *looking back at the entire problem, rereading* the problem, *analyzing, devising a plan, carrying out the plan, and so on*. Since he takes too much time in reading and analyzing the problem by looking back, when he already devises a plan, he sticks to that plan and seldom tries other strategies because he claims that in solving a problem especially during classes, the given problems have time limit.
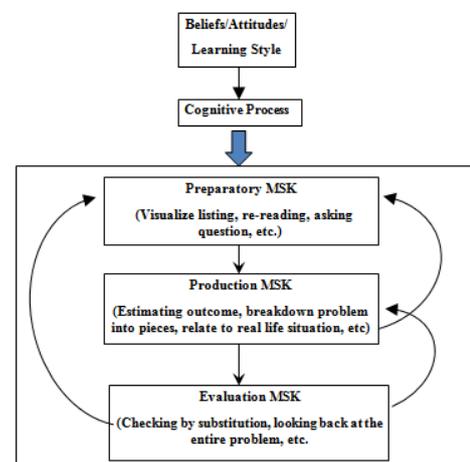

Figure 2. Initial Framework of "Using Metacognitive Knowledge in Mathematical Problem Solving" from Isagani

Synthesis

The initial framework of using metacognitive strategy knowledge in mathematical problem solving consists of three multi-distinct yet related main processes of categories. The framework does not follow linear path from preparatory MSK to evaluation MSK because the cognitive process of a solver is affected by some factors such as belief, attitudes and learning style. The solver may go back or jumped from one dimension to another.

At this point of the grounded theory process, higher level of theoretical sensitivity had been reached in terms of the ability to identify and select only those variables that are related to the core variable or basic process that accounted for the variations in resolving the main concern of the first participant in the problem solving process.

Fourth Stage: Writing the Theory

How the core categories have been developed in elaborating the story line and generating the theory "using metacognitive strategy knowledge in mathematical problem solving" was explained in writing in this concluding stage. The rest of the participants were included in the coding process and extant literature or theories in each category were utilized to strengthen the theory's explanatory power. In grounded theory research, this is called "supplementation" [11]. As cited by Laguda [11], stated that supplementation is a way of constructing new categories for possible inclusion in developing theory. Conceptually, it lies between coding (which names categories and specifies the properties associated with them), and theoretical sampling (which tells us what kinds of site or situation we want to look at next). Supplementation starts with extant category, and systematically elaborates contrasting categories in order to provide "raw material" for theoretical sampling, crosscutting and densifying theories, and testing hypotheses. The focus of supplementation is thus on categories not on data; on "might be" rather than "is". The result of supplementation and elaboration is the condition halfway between the beginnings of an inquiry in a situation that is being studied, and its conclusion in a new situation.

The succeeding paragraphs which answer the statement of the problem number two of this study utilized "supplementation and elaboration" to develop the theory "Using Metacognitive Strategy Knowledge in Mathematical Problem Solving".

The Meaning of "Using Metacognitive Strategy Knowledge in the Mathematical Problem Solving among Pre-service Teacher Education Students"

This study was based on a main emergent concern of the participants in the study, which asked how pre-service teacher education students solve mathematical problems. Qualitative method of Pandit [15], a grounded theorist, and the constant comparison method of Glaser and Strauss [10] were used.

Liu and Lin (April 2010) conducted an intensive study entitled, "The Survey Study of Mathematics Motivated Studies for Learning Questionnaire for Grade 10-12 Taiwanese Students". This study draws attention in understanding senior high and vocational school students' mathematics learning motivation and strategies. Two questionnaires about motivation and learning strategies were developed based on Motivational Strategies for Learning Questionnaire (MSLQ). One is the Mathematics Learning Strategies Scale which was developed by both creating new items and adapting items from MSLQ. It consisted of four subscales namely cognitive, meta-cognitive, non-informal and informal resources management. Each subscales has corresponding factors included such as for cognitive, the factors are rehearsal, elaboration and organization. For meta-cognitive, it includes critical thinking and self-regulation. Together with the study of Desoete [6], she cited that Brown [2,3] distinguished between four types of skills: prediction, planning, monitoring and evaluation.

On the other hand, Garofalo and Lester [9] described problem solving behavior as consisting of four phases of distinctly different metacognitive activities: orientation, organization, execution and verification. In describing their framework, Garofalo and Lester indicated that shifts from one phase to the next commonly occurred when metacognitive decisions resulted in some form of cognitive actions.

Additionally, the problem solving model of Wilson, Fernandez and Hadaway [24] that shows the non-linearity of problem solving that shows clockwise and counterclockwise nature of the cycle suggesting the process can go top-down or bottom up with reference to Polya's model.

"Using Metacognitive Strategy Knowledge in Mathematical Problem Solving" which emerged from the data, represents the core category or basic cognitive and metacognitive processes by which pre-service teacher education students solve mathematical problems. The research findings of Liu and Lin (April 2010), Garofalo and Lester [9], the citation of Desoete [6], and the problem solving model of Fernandez, Hadaway and Wilson [24] have shown some important bearing on the emergence of this theory.

The theory is now presented in four parts. The first part is the presentation of metacognitive strategy knowledge in problem solving among pre-service teacher education students officially enrolled in problem solving subject. The second part is the presentation of the coding process of the rest of the participants. The third part is the outline of what is meant by "Using Metacognitive Strategy Knowledge in Mathematical Problem Solving" which was also presented as coding process. The fourth part presents a grounded typology for pre-service teacher education students in how they solve mathematical problems using their metacognitive strategy knowledge.

Table 3. Extract From Open Coding of Interview Transcripts of All Cases

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Others	Categories
Mathematical problem solving, there are numbers that are related to ..like money or interest	Relate to real life Critical Thinking	Use of Metacognitive Strategy Knowledge
Ahhhmmm.. I will read the problem then find the given and the equation needed to get the right answer..but if I really don't know it...	-read and select data needed(planning and critical thinking) -choose related formula/equation (Rehearsal, self-regulation) -assess degree of difficulty	-use of metacognitive strategy knowledge -preparatory metacognitive strategy knowledge

Table 3 illustrates the open coding of the interview transcripts as it appears in the open coding of the interview of all cases.

Memos were made and coded by from open coding. This is to determine if the coding made was accurate.

Table 4 illustrates the coding of the memos done by the researcher.

Table 4. Extract from the First Open Coded Memos

Memos	Behaviors/ Type of Metacognitive Strategy Knowledge/ Category/Subcategories Others
1. The way a solver attacks or solves a certain problem is affected by certain factors like beliefs, learning styles, attitude and even time limit.	Factors affecting a solver
2. Mathematical Problem solving process was assumed to be a systematic process of solving or a step by step process.	Use of Metacognitive Strategy Knowledge / Monitoring
3. Some students relate mathematical problem solving to real life situation	Use of Metacognitive Strategy Knowledge/critical thinking
4. Some students relate math problems to number, formulas and strategies. Thus a certain problem needs a formula in order to be solved.	Use of Metacognitive Strategy Knowledge/ awareness of strategies
5. Some students named their method as read, analyze, solve and check or read, solve, check and etc.	Systematic approach/ Use of Metacognitive Strategy Knowledge / Monitoring

After the coding process of all the cases in the investigation, the data revealed that that pre-service teacher education students solve mathematical problems "using metacognitive strategy knowledge". It involves stages namely: preparatory metacognitive strategy knowledge, production metacognitive strategy knowledge and evaluation metacognitive strategy knowledge. It involves dimensions in each stage such in preparatory metacognitive strategy knowledge, the dimensions involve are constructing meaning and developing, analysis of information and others. The dimensions were created based on the role of metacognitive strategy knowledge in solving mathematical problems. It seems that there are some factor such as attitude, belief, learning style, time limit and others that affects the pre-service teacher education students in solving a problem. Thus, the success or failure in solving may be influenced by these factors.

The stages of metacognitive strategy knowledge have something to do with the problem solving process. The process as emerged from the data is a mixed-nature of Polya [16], Schoenfeld [18] and Garofalo and Lester [9] problem solving process and framework of problem solving of Fernandez, Hadaway and Wilson [24]. With the problem solving framework of Fernandez, Hadaway and Wilson [24], this is similar to its cyclic and dynamic nature of problem solving. Although it is similar with Polya, Schoenfeld and Lester problem solving process and framework of Fernandez, Hadaway and Wilson, there exists a striking difference between and among them. In this study, the categories involved are not actually the mathematical problem solving process but rather they are the stages in problem solving using metacognitive strategy knowledge. The stages are preparatory metacognitive strategy knowledge, production metacognitive strategy knowledge, and evaluation metacognitive strategy knowledge. Each phase has its dimensions classified by what action can be done by doing the metacognitive strategy knowledge which will be helpful in solving the problem.

The study of Desoete [6] in evaluating and improving mathematics teaching-learning process cited that Brown [2,3] distinguished between four type of skills: prediction, planning, monitoring and evaluation. Together with the Mathematical Strategy Scale of Liu and Lin (April 2010) consisting of four subscales namely cognitive, meta-cognitive, non-informal and informal resources management.

The study of Polya [16], Schoenfeld [18] and Garofalo and Lester [9], Fernandez, Hadaway and Wilson [24], Liu and Lin (April 2010) and citation of Desoete [6] on four types skills have important bearing on the present study. Their similarities and differences with the present investigation are described and presented as categories and sub-categories and illustrated in the succeeding paragraphs.

Figure 3 illustrates the emerged integrative construct of the core categories of using metacognitive strategy knowledge in the mathematical problem solving among pre-service teacher education students.
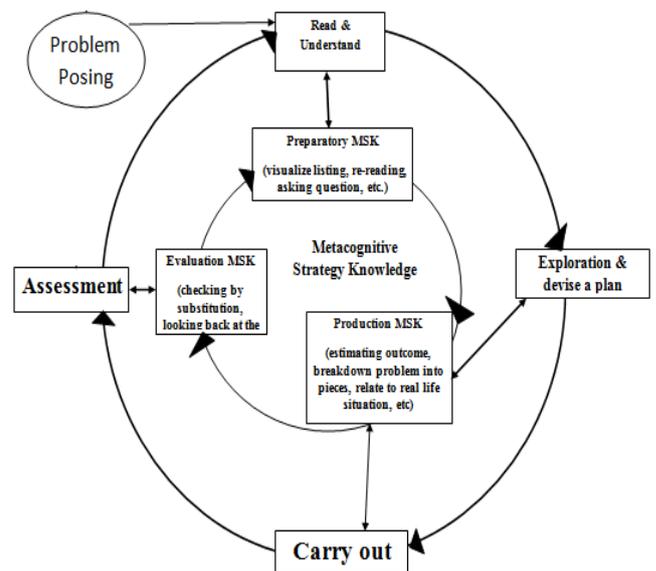

Figure 3. Emerged Integrative Construct of Core Categories

Synthesis

The emerged integrative core categories show that during the stages/phases of problem solving such as read and understand, exploration and devise a plan, carry out and assessment. The metacognitive strategy knowledge, which was located at the center of the problem solving, affects the stages of problem solving. Thus, the movement of skipping some stages or returning back from previous stage was because of the metacognitive strategy knowledge.

Through problem posing, a problem was formulated that may trigger a solver to undergo the whole process of problem solving and may end up with another problem.

The following paragraph discussed the categories generated from the coding processes.

Category 1: Preparatory metacognitive strategy knowledge

Preparatory metacognitive strategy knowledge among the pre-service teacher education students comes as the first stage in solving using metacognitive strategy knowledge. It includes dimensions where metacognitive strategy knowledge are likely to influence cognitive actions namely: constructing meaning and developing interpretation, analysis of information and looking back on the problem.

During this stage, the students start by reading the problem. They select relevant details such as given in the problem by underlining it or listing it to make sure of what was asked. The strategy they will use depends on the nature of the problem. Sometimes they visualize the problem by simply drawing the setting of the problem or how it is described or by representing the given information with tables. Then, they reread the problem not only once but as often as they want to ensure they made the correct illustration. They use their previous knowledge in mathematics domain for understanding the given information and relate it to relevant mathematical concepts that might be used in the problem. They reflect through recalling similar problems or familiarity of the problem and assess the degree of difficulty.

This stage affects the read and understand phase of problem solving which is similar with first phase of Polya's [16] problem solving phase of understanding the problem. This stage of Polya is recognizing what is asked for. This is also similar with Schoenfeld [18]; and Garofalo and Lester [9] first and second phase in problem solving called reading and analysis. Polya [16] explained this stage in his book "How to Solve It" by indicating the guide questions and things to consider in this stage, first you have to understand the problem, find what is the unknown, the data, the condition, determine if the possibility of satisfying the condition and whether or not the condition is sufficient to determine the unknown, and others. However, he did not consider dimensions. The act of answering those questions by means of using one's creativity in solving is actually using metacognitive strategy knowledge. The fact that this does not just talked about the typical problem solving stages makes it different from any other stages. In addition, the existence of the dimensions involved in this stage makes this different from others. Though the dimensions of this stage seem the same with the explanation on reading and understanding or analyzing, however, since the subcategories are created through expanding the explanation through identifying what the solver really does to understand to the problem namely: constructing meaning and developing interpretation, analysis of information and looking back on the problem, the term "preparatory metacognitive strategy knowledge" was coined.

Table 5 illustrates how pre-service teacher education students continue to the next level of preparatory metacognitive strategy knowledge in the problem.

Table 5. Extract from the Coding of Showing How Pre-service Teacher Education Students Proceed to the Constructing Meaning and Developing Interpretation Dimension of Preparatory Metacognitive Strategy Knowledge.

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
I'm thinking about how I could answer the question	-asking question(Elaboration) -constructing meaning and developing interpretation	Use of Metacognitive Strategy Knowledge -Preparatory metacognitive strategy knowledge	Asking question is one way to
Sometimes, but if I really don't know it…I read it again and again	-reading repeatedly(rehearsal, prediction/orientation) -Constructing meaning and developing interpretation	-Use of Metacognitive Strategy Knowledge -Preparatory metacognitive strategy knowledge	Reading the problem repeatedly is an act of constructing meaning or developing interpretation

In constructing meaning and developing interpretation, preparatory metacognitive strategy knowledge takes place through listing, making drawing, illustrations, tables, chart and others.

Table 6 illustrates how pre-service teacher education students proceed to the next level of preparatory metacognitive strategy knowledge.

Table 6. Extract from the Coding of Showing How Pre-service Teacher Education Students Proceed to the Analysis of Information Dimension of Preparatory Metacognitive Strategy Knowledge.

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
First aaahhhhhhh I do read the problem as a whole and then I list down the important data and formula which I need in solving the problem	-read, select relevant data by listing(planning and critical thinking) -think of appropriate formula(rehearsal, self-regulation) -Constructing meaning and developing interpretation	-preparatory metacognitive strategy knowledge, production metacognitive strategy knowledge -use of metacognitive strategy knowledge	Listing relevant data or given is one way of analyzing the information

In analysis of information, preparatory metacognitive strategy knowledge takes place through selecting relevant information and relating it to a certain mathematical field.

Table 7 illustrates how pre-service teacher education students proceed to the next level of preparatory metacognitive strategy knowledge.

In the process of preparatory metacognitive strategy knowledge, before proceeding to the next process, the solver looks back on the problem through recalling similar problems and assessing degree of difficulty

Based on some of the participants' statements, there are some problems that need not proceed to the next stage since the answer is obvious. They ended by writing the

answer after this stage with no written solution at all. Some participants proceed immediately to the next stage which is the production metacognitive strategy knowledge and do not mind the degree of their understanding. Their statements resulted to another dimension of preparatory metacognitive strategy knowledge which is complete or incomplete. With these, some stop in some processes.

Table 7. Extract from the Coding of Showing How Pre-service Teacher Education Students Proceed to the Looking Back on the Problem Dimension of Preparatory Metacognitive Strategy Knowledge

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
Yes ma'am..we just think of what we have done before	Recall and Apply method to similar problems solved before(rehearsal/ Speculation)	-use of metacognitive strategy knowledge -production metacognitive strategy knowledge	Recalling similar problems as a way to look back at the problem
	-reading repeatedly(rehearsal, prediction/orientation) -assessing degree of difficulty(looking back on the problem) Giving up(attitude)	-preparatory metacognitive strategy knowledge -use of metacognitive strategy knowledge -factor affecting solver -looking Back	Looking back by assessing difficulty

Based on the statements of the participants, a summary of the emerged category of preparatory metacognitive strategy knowledge in solving mathematical problem is illustrated in Figure 4.
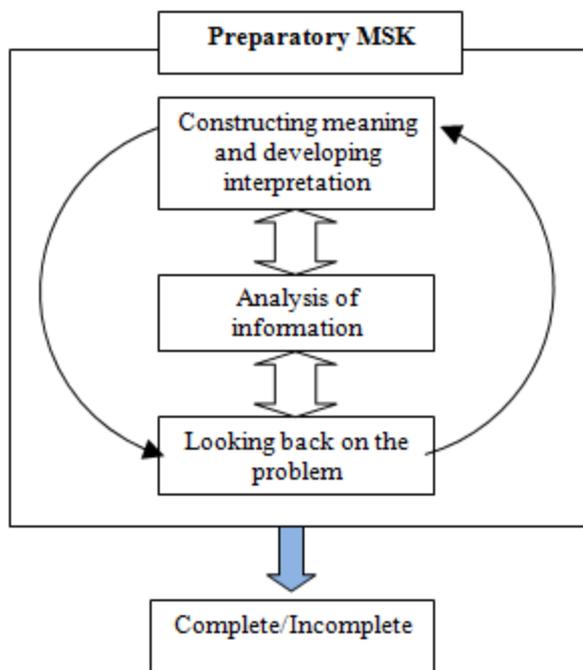
Figure 4. Emerged Framework for Preparatory Metacognitive Strategy Knowledge

Synthesis

The category of preparatory MSK consists of dimensions such as constructing meaning and developing interpretation, analysis of information and looking back on the problem. The path that a solver takes is not necessary linear. S/he may jump from one dimension to another or go back to the previous dimension. After undergoing the preparatory MSK, this may result to complete or incomplete understanding of the problem.

Category 2: Production metacognitive strategy knowledge

During this stage, the pre-service teacher education students try to find ways to solve the problem. They visualize the situation of the problem and explore or discover possible method/strategy/formula to solve the given problem. They may relate the problem to real life situations or to similar problems encountered before. Until such time that they may devise a plan or strategy. After proceeding to the next step, they may reflect at their decision for sometime if their action is practical or if it is feasible.

The production metacognitive strategy affects the phase of Polya's [16] problem solving stage of devising a plan. Through the production metacognitive strategy, Polya's stage is responding to what is asked for. Polya mentions that there are many reasonable ways to solve problems such as guess and check, make an orderly list, draw a picture and others. Likewise the phase affected by the metacognitive strategy knowledge is also similar to the third phase of Schoenfoeld [18] and Lester [12] which is exploration and the fourth phase which is planning. However, this stage is also different in a way that it does not mean like what other mathematical solving process is all about but rather the metacognitive strategy knowledge that affects the mentioned stages of problem solving. Production metacognitive strategy knowledge stage includes the phase of Polya, Schoenfoeld [18] and Lester [12] in its dimensions, formulating a plan and exploring/discovering respectively. As mentioned this stage has dimensions which make it different from others. This process involves dimensions namely: explore/discover, speculate, reflect on the discovery and speculation, formulate a plan and reflect practically of the plan.

Table 8 to Table 12 show sample statements that explain each dimension of category production metacognitive strategy knowledge.

Table 8 best illustrates statements that explain the explore/discover dimension of production metacognitive strategy knowledge.

Table 8 Extract from Coding the Emerged Explore/Discover Dimension of Production Metacognitive Strategy Knowledge

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
I use the different strategies that I know, for example trial and error	-recall formula (rehearsal/self-regulation) -awareness of different strategies like trial and error -discover/explore	- Use of Metacognitive Strategy Knowledge -production metacognitive strategy knowledge	Use of trial and error to discover/explore right strategy.

Some ways that show explore/discover category of production metacognitive strategy knowledge are the using trial and error, visualizing situation, establishing connection among part of the part or analyzing the problem part by part.

Table 9 best illustrates statements that explain the speculation dimension of production metacognitive strategy knowledge.

Table 9. Extract from Coding the Emerged Speculation Dimension of Production Metacognitive Strategy Knowledge

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
Yes I usually do that, for instance…we had a subject before in problem solving which you could relate…like for example the RT building..sort of like that	-Relate to real life situation (critical thinking) -Speculation	-use of metacognitive strategy knowledge -Production metacognitive strategy knowledge	Relating problem to real life situation is one way of speculation.

Speculation is done through relating the problem to real life situation and relating it to problems similar to the problem solved before.

Table 10 best illustrates statements that explain the reflecting on the discovery and speculation dimension of production metacognitive strategy knowledge.

Table 10. Extract from Coding the Emerged Reflecting on the Discovery and Speculation Dimension of Production Metacognitive Strategy Knowledge

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
I well for me ahhmmm I do not choose too but it comes to you when you already met similar problems. For example you do not plan to use that same strategy but when you read the problem it comes to you that ahhh aha this is what we solved before so I will use the same method in solving	Relate and Carry out the same strategy used to similar problems -rehearsal -speculation -Reflecting on speculation	Production metacognitive strategy knowledge Carrying out Use of Metacognition Strategy Knowledge	Reflecting was done after speculation

After speculation or discovery, the participant may reflect on it, making sure that their discovery/exploration and speculation is feasible to come up with a plan.

Table 11 best illustrates statements that explain the formulating a plan category of production metacognitive strategy knowledge.

Table 11. Extract from Coding the Emerged Formulating a Plan Dimension of Production Metacognitive Strategy Knowledge

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
Yes ma'am, then I list the given ma'am or underline it and then I also outline what is asked (Soledad).	-list, select, outline (critical thinking, planning) (constructing meaning and developing interpretation) -formulate a plan	Preparatory metacognitive strategy knowledge Production metacognitive strategy knowledge	The solver list, select and outline implies that s/he has something in his/her mind to attack the problem. Something like a plan
So what I do, I choose the best strategy (Helen).	Formulate a plan	Production metacognitive strategy knowledge	

It is in this stage that formulating a plan or devising a plan or method was done. The participants claimed that the strategy they used depends on the nature of the problem.

Table 12 best illustrates statements that explain the reflecting practicality of the plan dimension of production metacognitive strategy knowledge.

Table 12. Extract from Coding the Emerged Reflecting Practicality of the Plan Category of Production Metacognitive Strategy Knowledge

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
Or I don't solve it anymore using the first idea I made use then I try other formula.	-reflecting on the practicality of the plan	-production metacognitive strategy knowledge evaluation metacognitive strategy knowledge/realization	The solver has the tendency to change plan after reflecting practicality of the plan. Thus trying other strategy to solve the problem
	-Relate to real life situation (critical thinking) --reflecting on the practicality of the plan	-use of metacognitive strategy knowledge -production metacognitive strategy knowledge	The solver reflect what plan is practical to use.

After formulating a plan, the solver reflects again if the plan is practical or not. This is where the solver should decide whether to carry out the plan or not.

In this stage, it does not mean that since there are dimensions involved in production metacognitive strategy knowledge, all categories must be complied. A solver may or may not execute some of dimensions.

It is also in this stage that a solver may break down the plan, assess the plan, apply the plan and reflect on correction of the action before proceeding to the next phase.

Production metacognitive strategy knowledge affects Polya's [16] problem solving stage of carry out the plan by developing the result of the response. In this stage, care and patience are needed together with necessary skills for solving. There are times that the strategy chosen during this stage was not right and was just discovered after reflection on correctness of action. Thus, carrying out of the plan may be done several times depending on the success in this stage. If the first selection of the strategy did not work, formulating another plan and carrying out of the strategy are done again. This is also similar to Schoenfeld's [18] and Lester's [12] fourth phase.

This stage is also different because of dimensions developed. Table 13 to Table 15 present sample coding that emerged the subcategories of carry out stage.

Table 13 best illustrates the emerged category of breaking down plan dimension of production metacognitive strategy knowledge.

Table 13. Extract from Coding the Emerged Breaking Down Plan Dimension of Production Metacognitive Strategy

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
This is to know if all the data /numbers given are just enough to find the unknown because there are problems that you still need to solve for other data needed in order to solve for what is really asked in the problem.	Make connections (organization, evaluation) -break down plan	-use of metacognitive strategy knowledge -carry out	There are some problems that require sub-plans in order to solve what is the unknown in the problem.
Yes, because there are some given that are not supposedly needed to solve the unknown in the problem. Rather, you have to find the missing number needed to solve for what is really asked in the problem	Make connections(organization,evaluation) -break down plan	-use of metacognitive strategy knowledge -carry out	

There are problems that require sub plans before one could solve for what is really asked in the problem. This category of breakdown plans is only necessary depending on the nature of the problem.

Table 14 best illustrates the emerged category of assessing and applying plan dimension of production metacognitive strategy knowledge.

Table 14. Extract from Coding the Emerged Assessing and Applying Plan Dimension of Metacognitive Strategy Knowledge

Incidents	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
I used strategy that I already know if they could solve the problem (Helen).	assess and apply plan	Carry out	Before actually applying the plan, the solver assesses the plan. In this particular incident, the solver has shown that s/he is sure of the strategy/plan.
Ahhmm I actually chose a strategy which most fits the given problem	Assessing and applying the plan Critical thinking	Development and Carry out Use of Metacognition Strategy Knowledge	Assessing the plan and applying the plan after finding out that it fits the problem.

Immediately after the assessment of the plan, applying the plan is the next step. This requires careful execution of the basic skills in operation and solving while checking the work and logic at every step.

Table 15 best illustrates the emerged category of reflection on correctness of action category of carry out.

Table 15. Extract from Coding the Emerged Reflection on Correctness of Action Category of Carry out

English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
Just like for example in a certain problem that I have solved, sometimes I'm doubtful of it so I'm thinking of another way to solve it. But when it does not match with my first answer, I stick with my original or first answer	-try other strategies (critical thinking) -apply plan -drawing, table(organization) awareness of math strategies like drawing, making illustrations, trial and error, etc -decision to accept or reject answer -reflection on the correctness of action	-use of metacognitive strategy knowledge -development -carry out -assessment/ realization	The solver doubt on his/her action because he/she is reflecting on the correctness of his/her action

Reflection on the correctness of the action category of carry out was created before proceeding to the assessment stage. This category is a preparatory before assessing the answer. It is in this stage that the step by step process on solving or in this particular carry out stage was check while doing the action in a way that a further assessment will be done on the next stage.

Based on the statements of the participants, and the constant comparison method to arrive at a higher level of abstraction, framework for production metacognitive strategy knowledge category in solving mathematical problem emerged. This is illustrated in Figure 5

Synthesis

The emerged framework for production metacognitive strategy knowledge category was divided into two parts. In the first part, the solver may explore or speculate to formulate a plan and may go back or move forward after reflecting on his discovery/speculation. S/he may also reflect on the practicality of plan. If s/he is convinced of the practicality of the plan, s/he may proceed to decide

what plan to do or returns back to explore/discover/speculate again. If s/he decides to proceed s/he may breaks down the plan or assesses the plan before applying it. Finally, s/he may reflect on the correctness of the action. If there is something wrong or s/he is not satisfied with the result, s/he may go back again.

Category 3: Evaluation metacognitive strategy knowledge

A solver undergoes evaluation metacognitive strategy knowledge to evaluate or check the appropriateness of plan, actions and solutions to the problem. Ideally, given enough time, the pre-service teacher education students re-read the problem again. They may assess the plan and ask if their answers make sense. This is also the stage where the solver makes decision whether or not to accept the answer. There are several ways to assess if the answer is correct. Some use substitution and others verify it using other strategies.

This affects Polya's problem solving stage of looking back. In this stage, it is actually checking the correctness of the answer or simply answering the question, "what does the result tell me?" It is Polya's fourth stage where a solver, examine the solution obtained, checks the result/argument and others. This is also similar with Schoenfeld's [18] and Lester's [12] fifth stage which is the evaluation. Just like other categories, this stage is also different from the stage of Polya, Schoenfeld and Lester because evaluation metacognitive strategy knowledge stage possesses dimensions, namely: re-reading the problem, assessing the plan, asking if the answer makes sense and deciding to accept or reject the solution. It also means problem solving stages use metacognitive strategy knowledge.
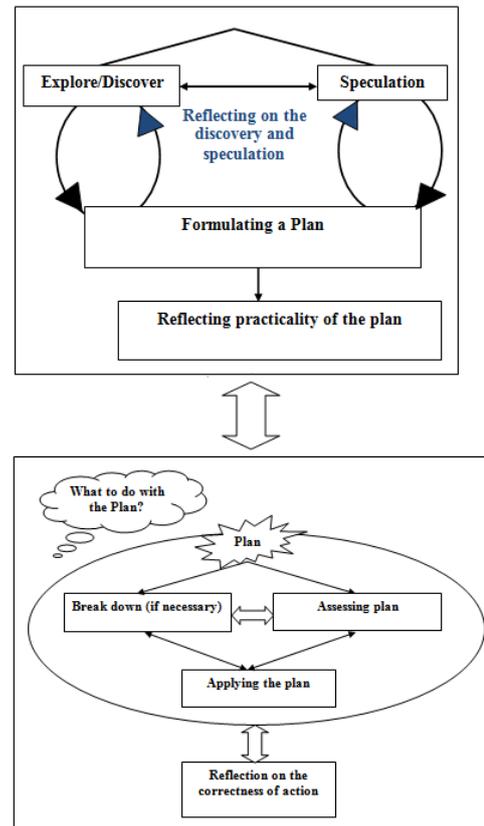
Figure 5. Emerged Framework for Production Metacognitive Strategy Knowledge Category

Table 16 illustrates the emerged dimensions of evaluation metacognitive strategy knowledge.

Table 16. Extract from Coding the Emerged Dimensions of Evaluation Metacognitive Strategy Knowledge

Incidents	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
I will know that I have solved the problem correctly and if it if I tried to check it already and I tried to compare with others already and if yeah if the answer to my question does my answer makes sense is yes (Isagani).	-evaluation -asking if answer makes sense -social -critical thinking -decision making	Use of Metacognition Strategy Knowledge -evaluation metacognitive strategy knowledge	One way of knowing if the answer is correct is through asking if the answer makes sense. It is in this stage that the solver needs to decide whether to accept or reject his/her answer.
It's very hard to decide but if you are sure that you have done it correctly..you will know it (Barbara).	-decision making to accept or not	Evaluation metacognitive strategy knowledge	The solver has its own criteria of identifying if answer is right or wrong. Thus, needed to make decision to accept or reject.

During the interview, the pre-service teacher education students claimed that they usually do checking by substitution, by comparing their estimation with the obtained result and by checking the computations. There were some who did not bother and immediately proceeded to the next item without checking. The reason might be the time limit given to them and others accepted that they were not confident of their answers because they did not fully understand the problem or they did not have any idea on how to solve the problem. With these, they did not give time anymore to check whether their answers are correct or not. Some assessed the problem through checking the computations only, other assessed the problem by going over the whole process.

In addition, evaluation metacognitive strategy knowledge also includes dimensions on reflecting on the entire solution process and reflecting on the mathematical

accuracy involved, one's confidence in handling the process and degree of satisfaction and reflecting on the entire solution process through looking back at the whole problem or going back to the whole process, identifying critical features and evaluating the solution. The solver looks back at the whole problem or goes back to the whole process by reviewing the whole process and checking every step. S/he identifies critical features by checking out which step or computation or method in the problem was complicated and hard to figure out. In realization, evaluating the solution does not just mean checking the computations but relating it to adaptability to other problems and finding other ways to solve the given problem.

Table 17 illustrates the emerged dimensions of evaluation metacognitive strategy knowledge.

Table 17. Extract from Coding the Emerged Dimensions of Evaluation Metacognitive Strategy Knowledge

Incidents	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
I can solve another problem similar to it (Clara).	Relate/reflect it to future problem	Realization	Students reflect on the problem solving process for future application
ahhhmmm… I cannot tell but I say somewhat they are similar because as we have been trained it is easier to use to established strategies in solving problems but sometimes when we solve problems we just use commonsense, if the answer is obvious something like that so I don't have to go through the whole process anymore I just I just arrived at an answer without necessarily ahhh identifying the give, etc. (Isagani)	-critical thinking -reflecting/ comparing the used of established strategies with using his/her commonsense	Use of Metacognition Strategy Knowledge evaluation metacognitive strategy knowledge.	The solver prefers not to undergo the whole process.

Only few pre-service teacher education students have expressed that they used this process. This is somewhat similar with evaluation metacognitive strategy knowledge category, the only difference is in realization, the level or degree of assessing is higher than the usual evaluation metacognitive strategy knowledge.

Based on the statements of the participants, and the constant comparison method to arrive at a higher level of abstraction, the emerged category of realization in solving mathematical problem is illustrated in Figure 6.
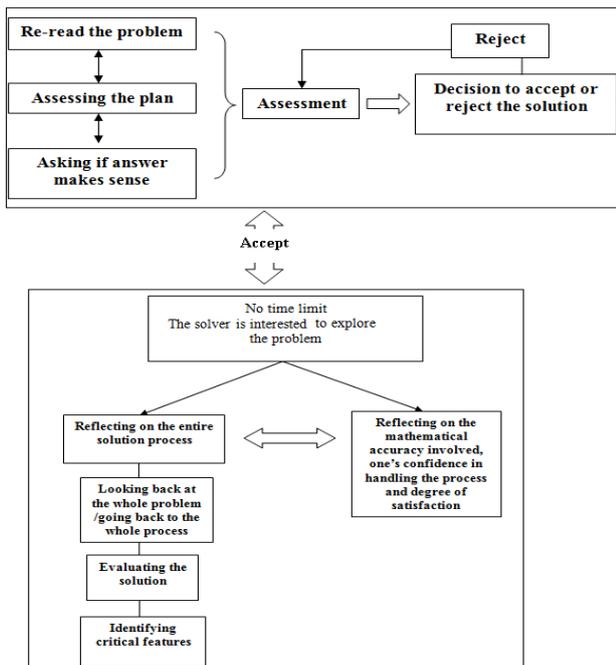

Figure 6. Emerged Category of Assessment Metacognitive Strategy Knowledge in Solving Mathematical Problem

Synthesis

The emerged category of assessment metacognitive strategy knowledge in solving mathematical problem starts with assessment. The solver may assess through re-reading the problem, assessing the plan, asking if the answer makes sense. S/he may decide whether to accept or reject the solution. If s/he rejects it s/he may go back to assessment and if s/he accepts it, s/he stops there. But if there is no time limit and the solver is interested to explore the problem, s/he may reflect on the entire solution process by looking back at the whole problem, evaluating the solution or identifying the critical features. S/he may also reflect on mathematical accuracy involved, one's confidence in handling the process and the degree of satisfaction.

Category 4: Factors affecting the solver

Before and during solving the mathematical problems, there are identified factors affecting the pre-service teacher education students. Table 18 illustrates sample coding of emerged category of factors affecting the solver.

Some of the identified factors affecting the solver are time limit, attitudes, beliefs, learning style, feelings and teacher factor. The attitude includes self-esteem and patience.

Category 5: Metacognitive Strategy Knowledge in Mathematical Problem Solving

The categories that make up metacognitive strategy knowledge in mathematical problem solving namely: preparatory metacognitive strategy knowledge, production metacognitive strategy knowledge and evaluation metacognitive strategy knowledge and the dimensions and sub dimensions involved, cause the movement from one process to another process. The mathematical problem solving process such as read and understand, explore, devise a plan, carry out and look back have a backward arrow because of the presence of metacognitive strategy knowledge used by the solver. This movement is similar to the problem solving model of Fernandez, Hadaway and Wilson [24]. The metacognitive strategy knowledge involved in each process is classified as cognitive, metacognitive and other strategies. The cognitive strategies consist of rehearsal, elaboration and organization. The metacognitive strategies consist of critical thinking and self-regulation. This is similar to the subscale and factors of Mathematics Learning Strategies Scale of Liu and Lin (April 2010) while other strategies include prediction/orientation, planning, monitoring and evaluation which is also similar to Brown's [2,3] four types of skills. Figure 8 shows the mathematical problem solving process of pre-service teacher education students revealing how metacognitive strategy knowledge affects their behavior. As students were engaged in solving the problems, their thinking processes did not proceed in a strictly linear order from the preparatory metacognitive strategy knowledge stage to the realization stage. Thus, an important result for this study was that this model is cyclical. Furthermore, the use of metacognitive strategy knowledge represented by broken or dashed lines in each of the stages may have been the activity that initiated them in vacillating among the stages as needed to work on the problem.

Table 18. Extract from Coding the Emerged Category of Factors Affecting the Solver

| English Translations	Behaviors/ Type of Metacognitive Strategy Knowledge/ Subcategory/ Others	Category	Memo
Yes ma'am, sometimes I begin answering it but I really don't know it	-giving up (attitude)	Factors affecting solver	Some solvers do not try to solve a certain problem and easily give up.
I solve it easily	-self-confidence (attitude)	Factors affecting solver	Some solvers are confident that they solve the problem easily
When time is limited then I I proceed to number two but when time is long then I try to check	Time limit-hindrance	Factors affecting solvers	Due to time limit, there some are process that are failed to undergo.
Sometimes if I do fell stressed	-attitude	Factors affecting solver	-
Yes, yes ma'am. Ma'am actually whenever I try to look at my answer at times I know that my answer are not correct because I'm not good in mathematics but in a way I can say that all my answer makes sense because I try to answer them and I exerted effort (Inday).	-talking or asking self (elaboration/evaluation/monitoring) -reflecting/looking back -belief	-use of metacognitive strategy knowledge -evaluation metacognitive strategy knowledge/realization -factors affecting solver	Because of certain belief that a solver is not good in mathematics, s/he is not confident on what s/he is doing and worst is that they feel that they are not doing it right.
Sometimes when I'm interested I do try to analyze first then keep on reading again and again then finally I try to answer it	-if interested (attitude) -keep reading and reading to analyze(rehearsal/prediction / orientation) -applying the plan	-factors affecting solver --use of metacognitive strategy knowledge -production metacognitive strategy knowledge	
Not not very well because ehh sometimes some problems are confusing..it seems that I need to analyze then eh to understand the question in the problem	- Confuse (feeling) -analyze and understand the problem and question (constructing meaning and developing sense)	- Factors affecting solver -preparatory metacognitive strategy knowledge	
If I have to solve a problem and it was not fully explained to me or I did not really understand how to solve it, I really find it hard to find the solution..but if I really understand the lesson…I get the answer	-teacher factor	Factors affecting solver	
Ma'am sometimes its so annoying but if you have no patience eh you will never solve the problem	Attitude	Factors affecting solver	
In solving mathematical problems ahhhmmm I cannot understand problems being presented orally I need to see something I need to see the figures, I need to see the numbers, that's how I think for example ahhh in our mathematics class if the teacher dictates the problem I cannot solve the problem that's one unique asset of me that I cannot solve something which is being told to me I need the black and white I need the print I need a picture if that is necessary. That is how I analyze problems not actually by hearing it (Isagani).	Learning style	Factors affecting solver	
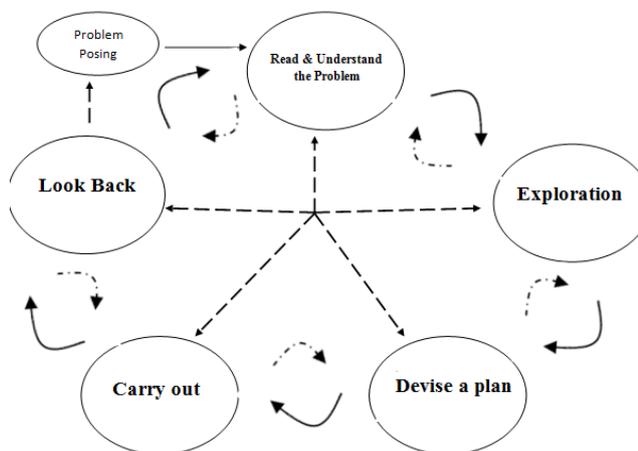
Figure 7. Mathematical Problem Solving Process Model Integrating the Use of Metacognitive Strategy Knowledge

Legend: ⟶ linear flow
— — ▶ use of metacognitive strategy knowledge flow

Synthesis

In solving mathematical problems, the step by step processes are read and understand, explore, devise a plan, carry out and look back. But because of the presence of metacognitive strategy knowledge, the solver may move from one process to another. S/he may also jump from one stage or phase to another.

The use of metacognitive strategy knowledge in the mathematical problem solving among pre-service teacher education students of Saint Mary's University is also supported by Garofalo and Lester's [9] cognitive-metacognitive framework for studying mathematical performance. Garofalo and Lester's [9] framework composed of four categories of activities in performing a mathematical task: orientation, organization, execution, and verification (see Chapter 2, pages 32-33). The table illustrates that distinctive metacognitive behaviors are associated with each category. The four categories are related to Polya's four phases, but are more broadly defined. The terms and categories used may not be the same but they may represent the same thing. Orientation,

organization, execution and verification categories in Garofalo and Lester's framework is tantamount to preparatory metacognitive strategy knowledge, production metacognitive strategy knowledge, production metacognitive strategy knowledge and evaluation metacognitive strategy knowledge in this study. Another example is the subcategories in Garofalo and Lester's framework such as comprehension strategies under orientation which may mean constructing meaning and developing interpretation. The categories are also different in a way that it does not just merely mean the stages or phrases in mathematical problem solving. The categories are coined from the metacognitive strategy knowledge embodied in it.

A Typology of Pre-service Teacher Education Students with Regards to How They Use their Metacognitive Strategy Knowledge in Solving Mathematical Problems

During the analysis of the data, a connection between incidents, concepts and categories was discovered. The typology, in particular, as frames of reference popped out while the researcher moved from data collection to coding and analyzing using the constant comparison method. The typology is based on two hypotheses which build the meaning of using metacognitive strategy knowledge in the mathematical problem solving process among pre-service teacher education students.

Hypothesis One

The success or failure of the problem solvers in doing mathematical tasks seem to be affected by factors like time limit, attitudes, beliefs and learning styles.

Hypothesis Two

The success or failure of the problem solvers in doing mathematical tasks also seems to be affected by the use of metacognitive strategy knowledge. Students have greater chance to solve the problems correctly if they use their metacognitive strategy knowledge on the whole process of solving because it is through this that they were able to assess and realize their action from stage to stage. On the other hand, failure to use the metacognitive strategy correctly may also lead to confusion or frustration.

Development of the Story Line for All Cases

In this study, the main story line was:

How do pre-service teacher education students solve mathematical problems and what are the types of metacognitive strategy knowledge that affect how they solve mathematical problems. On using metacognitve strategy knowledge in the problem solving process among pre-service teacher education students ,the students entered into a multi-dimensional process with its elements, namely: (1) preparatory metacognitive strategy knowledge through constructing meaning and developing interpretation, analysis of information, looking and looking back on the problem; (2) production metacognitive strategy knowledge through exploring/discovering, speculating, reflecting on the exploration/discovery and speculation, formulating a plan and reflecting on the practicality of the plan, by breaking down the plan if necessary, assessing plan, applying the plan and reflecting on the correctness of action; (3)evaluation metacognitive strategy knowledge through re-reading the problem, assessing the plan, asking if the answer makes sense and deciding whether to accept or reject the solution, reflecting on the entire solution process and the reflecting on the mathematical accuracy involved, one's confidence in handling the process and the degree of satisfaction. The problem solving process of the pre-service teacher education students becomes dynamic and cyclic. It also becomes flexible depending on the problem solvers creativity, and probably some other factors. The reason behind the dynamic, cyclic and flexible process in solving mathematical problem is due to metacognitive strategy knowledge present in an individual.

Selective Coding for Core Categories for All Cases

"Using Metacognitive Strategy Knowledge in the Mathematical Problem Solving" is a theory that has emerged from three multi-distinct yet related main processes of categories:

1. preparatory metacognitive strategy knowledge through constructing meaning and developing interpretation, analysis of information and looking back on the problem
2. production metacognitive strategy knowledge through exploring or discovering, speculating, reflecting on the exploration or discovery and speculation, formulating a plan and reflecting on the practicality of the plan, breaking down the plan if necessary, assessing plan, applying the plan and reflecting on the correctness of action,
3. evaluation metacognitive strategy knowledge through re-reading the problem, assessing the plan, asking if the answer makes sense, deciding whether to accept or reject the solution, reflecting on the entire solution process and reflecting on the mathematical accuracy involved, one's confidence in handling the process and the degree of satisfaction.

** Each of the categories consisted of dimensions and sub-dimensions. Mathematical problem solving phases or stages three multi-distinct yet related metacognitive strategy knowledge. In each stage, students frequently ask questions to themselves to help them throughout the process.

During preparatory metacognitive strategy knowledge stage or phase, the first thing solvers do is to ask questions like: what is the problem all about, what are the necessary given to be included, and others. They construct meaning and develop interpretation by reading the problem again and again, listing, making drawing, illustrations, tables, charts and other. They analyze information by selecting relevant information and relating it to a certain mathematical field. Then, they assess the degree of difficulty or maybe recall for similar problems as a way of looking back on the problem. Some solvers end up in this phase since they claimed that there are problems that need not undergo all the phases or stages. With just using their commonsense, the answer becomes obvious. Some solvers also prefer to write the solution rather than to compute mentally or vice versa. If the data in the problems are easy to manipulate they tend to use mental computation rather than written.

While analyzing and planning, they ask questions like "what application in real life can I relate?", "Did I already encounter this problem?" and "what method or formula will I use?" and again "am I doing it right?" This is during production metacognitive strategy knowledge stage/phase. The solvers explore or discover by using trial and error or using different mathematical strategies in solving,

visualizing the situation, establishing connection among the parts of the problem, analyzing the problem part by part and others. They may also speculate by relating the problem to real life situation or to the similar problems encountered before. Then, they reflect on the discovery and speculation before formulating a plan. Before they proceed to the next phase or stage, they reflect on the practicality of the plan, if it is feasible or not and if they are amenable to proceed to the next stage and carry out the plan or not.

After deciding if the problem is feasible and before carrying out the plan, still the students may break down the plan if necessary depending on the nature of the problem. Then, they assess plan to make sure that it fits the problem before applying the plan. After this, they reflect on the correctness of action.

In this stage, the solvers already have their answer to the problem but they want to make sure that they are correct that is why the evaluation metacognitive strategy knowledge stage or phase enters. In this stage the solvers may start evaluating or checking the appropriateness of plans, actions and solutions to the problems by re-reading to look back at the problem, assessing the plan by self-questioning and self-monitoring and asking if the answer makes sense. If they think that their evaluation metacognitive strategy knowledge is enough, they decide whether to accept or reject the solution. If they reject the solution they may look back to the other stages or what went wrong by assessing it again. If they decide to accept the solution, they may proceed to the next stage of the process or end up there. Some ways that can help them decide are checking the solution or the computations, identifying the difficult or complex parts and deciding to stick or trying other strategies. Some solvers reflect on the entire solution process by looking back at the whole problem /going back to the whole process or by identifying critical features and evaluating the solution. Some also reflect on the mathematical accuracy involved, one's confidence in handling the process and the degree of satisfaction.

In each of the stages, some students easily gave up and did not proceed to the next phase. Others continued solving and did something to solve the problem. Some students preferred to use the strategy introduced by their teachers, others made use of their own strategies. When they did not know where to go and what to do anymore, students had no other resort but to use trial-and-error or guessing. The process is dynamic and cyclical. It is also flexible where in a solver can go from one phase to another phase or jump from one phase to the next. This movement is because of metacognitive strategy knowledge. Metacognitive strategy knowledge triggers the problem solver to skip a stage or stages and to return back to some stages.

The mathematical problem solving as a whole seems to be influenced by some factors such as time limit, attitudes, beliefs and learning styles. Problem posing or problem formulation was also part of it. This takes place during evaluation metacognitive strategy knowledge that a solver was able to formulate another problem based on the previously solved problem. Thus, the pre-service teacher education students solve the problem by undergoing through the problem solving process with the use of his metacognitive strategy knowledge in all the processes involved before solving the problem.

After undergoing the whole process of grounded theory, using all the data gathered from interviews and written output, Table 19 demonstrate how these phases can be described in terms of indicators which took place in the solution processes used by the participants.

Table 19. Framework of Metacognitive Strategy Knowledge in Mathematical Problem Solving

Framework of Metacognitive Strategy Knowledge in Mathematical Problem Solving
1. Preparatory metacognitive strategy knowledge making
   A. Constructing meaning and developing interpretation
   B. Analyzing of information
   C. Looking back on the problem
2. Production metacognitive strategy knowledge
   A. Exploring/discovering
   B. Speculating
   C. Reflecting Exploring/discovering and Speculating
   D. Formulating a plan
   E. Reflecting feasibility of the plan
   F. Breaking down Plan (if necessary)
   G. Assessing plan
   H. Applying the plan
   I. Reflecting on correctness of action
3. Evaluation metacognitive strategy
   A. Re-reading the problem
   B. Assessing the plan
   C. Asking if answer makes sense
   D. Deciding to accept or reject the solution
   E. Reflecting on the entire solution process
   F. Reflecting on the mathematical rigor involved, one's confidence in handling the process and degree of satisfaction

In addition, to illustrate the bigger picture of the meaning of "Using Metacognitive Strategy Knowledge in Mathematical Problem Solving among Pre-service Teacher Education Teachers, Figure 8 emerged integrative construct of the theory.

4. Conclusion

The pre-service teacher education students' framework of metacognitive strategy knowledge that emerges during mathematical problem solving is a three-phase cyclic model mainly comprise of preparatory metacognitive strategy knowledge, production metacognitive strategy knowledge and evaluation metacognitive strategy knowledge. The primary role of metacognitive strategy knowledge is to facilitate the dynamic interdependence between the phases.

The importance of fostering metacognitive strategy knowledge during every phase/stages of mathematical problem solving is shown in in the figure 1. Without

metacognitive strategy knowledge, students are less likely to take one of the many paths available to them, and almost certainly are less likely to arrive at an elegant mathematical solution.

It was also shown that success and failures of the problem solvers in doing mathematical tasks are affected by factors like time limit, attitudes, beliefs and learning styles and by the use of metacognitive strategy knowledge. Students have greater chance to solved the problems correctly if they uses their metacognitive strategy knowledge on the whole process of solving because, it is through this that they were able to assess and realize their action from stage to stage. On the other hand, failure to use the metacognitive strategy correctly may also lead to confusion or frustration.
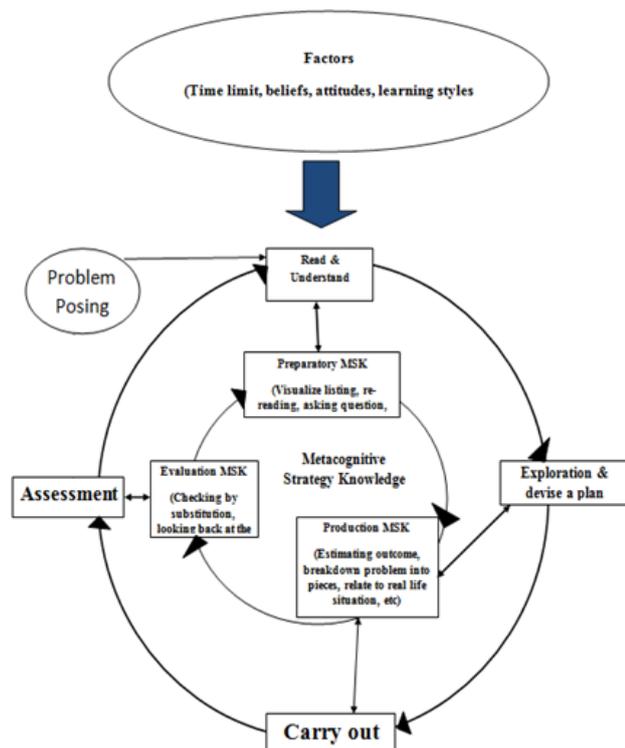
Figure 8. Emerged Framework on Using Metacognitive Strategy Knowledge in Mathematical Problem Solving among Pre-service Teacher Education Students with the Factors Affecting the Solver

5. Recommendations

In the light of the analyses and findings made from this study, the following are recommended.

1. This study may be further continued until developing framework for a successful problem solver.
2. Conduct further grounded theory research on "metacognitive strategy knowledge in mathematical problem solving process" that may include other participants. The kind of solver may also be considered as variable.
3. Consider grounded research on metacognition in general and may also consider certain fields and topics in mathematics such as statistics, geometry and others.
4. Further researches may include other tools like think-a-loud method and may limit the problem into one or two problem for more

concentrated/focused study on metacognition of a certain individual.

5. Teachers to design mathematical problem solving activities that promote metacognitive strategy knowledge or metacognition in general among students and may set classroom environment that promote development of metacognitive strategy knowledge or metacognition through using this study as a frame of reference.

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