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# Examples as Mediating Artifacts in Conjecturing

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**Abstract** In this paper we report a study concerning examples as mediating artifacts in conjecturing. We used activity theory as an analytical framework to analyze students' behavior and the process of conjecturing. The results show that through the guidance of proceduralized refutation model, most of the students could find the common properties by generating examples, and develop the ability of generalization and further made conjectures.

***Keywords:*** *activity theory, example, proceduralized refutation model*

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## 1. Introduction

Generating examples is typical of a mathematician's work in different activities. The literature has underlined the importance of this activity also in mathematical education, as a learning and teaching strategy [4,20,21], with relation to the construction of concepts [10,22], to the production of conjectures [1,2], and of proofs [8,9]. Thus learning mathematics can be seen as a process of generalizing from specific examples. More than forty years ago, Skemp wrote about the role of examples in the teaching and learning of mathematics [15]. Skemp's basic model for the learning of mathematical concepts was abstraction from examples, which meant that the teachers' choice of which examples to present to pupils was crucial. His advice on this topic includes consideration of *noise*, is the conspicuous attributes of the example which are not essential to the concept, and of *non-examples*, which might be used to draw attention to the distinction between essential and non-essential attributes of the concept. Once the concept is formed, later examples can be assimilated into that concept [16] and a more sophisticated *concept image* formed [17].

In this paper we focus on the process of generating examples in the process of conjecturing, seen as a special case of argumentation. Our points of view are that of Mason & Johnstone-Wilder that claim that "Participating in a conjecturing atmosphere in which everyone is encouraged to construct extreme and paradigmatic examples, and to try to find counter-examples (through exploring previously unnoticed dimensions-of-possible variation) involves learners in thinking and constructing actively. This involves learners in, for example, generalising and specializing" [14]. Lin stresses that a good lesson must provide opportunities for learners to think and construct actively, in addition, conjecturing is not merely the core of mathematising, but the driving force for mathematical proficiency [13]. And Lin claims that "Proceduralized Refutation Model (PRM)" is one of entries of conjecturing. According to such standpoint, the aim of this paper presented here was to describe how students generating and classify all kinds of examples to formalize mathematical conjectures.

## 2. Theoretical Framework

PRM can be applied to design a conjecturing activity by substituting each students' misconception into the first item in the worksheet which follows student's activities step by step in the model. The activity of conjecturing encompasses many specific phases. These phases (PRM) are listed here below [13]:

1. Teacher introduces a false statement, and students show images of the statement.
2. Encourage students generate more examples to make sure they comprehend the statement.
3. Encourage students generate more types of examples and make distinction of supporting and rejected examples.
4. Finding the common properties of supporting and rejected examples.
5. Reconfirming the correctness of the given statement.
6. Making conjectures and more conjectures.

Examples may be useful in each of these phases. First of all, they may help to understand the text and get into the problem. Afterwards, they may be worked out in order to discover a property. Once perceived the property, the examples may be useful to check its validity and to formulate and communicate the conjecture. Reflection on examples may also help to find some arguments for which the property holds. Many studies showed that students are keen to use examples in argumenting and conjecturing. According to Goldenberg and Mason, examples can usefully be seen as cultural mediating tools between learner and mathematical concepts, theorems, and techniques [6]. Hence we stress the importance of meaningful construction of examples as they are mediating artifacts serving as bridges for the conjecturing. The notion of example as mediating artifacts within a social community is also congruent to the activity theory.

Activity theory is a cross-disciplinary framework for studying different forms of human practices as developmental processes, both at the individual and social levels interlinked at the same time, including the use of artifacts [11]. Although Vygotsky did not dwell specifically on the concept of activity in any great detail, we understand that his work had great ramifications on the theory of activity in its current form, e.g., his notions of mediation by tools and signs. It was Leont'ev who developed an integrated framework for the theory of activity [12]. An activity always contains various artifacts (e.g., instruments, signs, procedures, machines, methods) and these artifacts have a mediating role. In other words, examples can be classified as artifacts under the notion of signs. Activity theory also emphasizes social interaction within an activity context and the processes of conjecturing that take place through interaction and mediation. Using activity theory as a framework for analyzing interactions between students in our study, we have the following components: subjects, community, tools, rules, division of labor, and object (see Figure 1).

In any activity that learners have to be engaged in, there must be a pedagogical object that perhaps produces an outcome, e.g., formulating the conjectures to a false statement. In the process of engaging in the activity, there are also rules for the accomplishment of the object. These ''rules'' can include notions such as common properties of different kind of examples, or backing [18]. The community of people that can be involved could include other students and teacher. Division of labor includes the specific roles and actions to be carried out by particular individuals in the activity, e.g., rebuttals, qualifiers [18].

Activity systems are not stable and harmonious systems; instead they can be described by inner contradictions caused by tensions among the components of the system [3]. These tensions arise when the conditions of the components cause the subjects to face the contradictory situations that hamper the attainment of the object. In other words, the working conditions that a subject faces in an activity system may not be in favor of the subject attaining the object because of the conditions that one component brings to other components. To reiterate, our central claim in this paper is that examples serve as mediating artifacts for conjecturing from the social to individual levels.

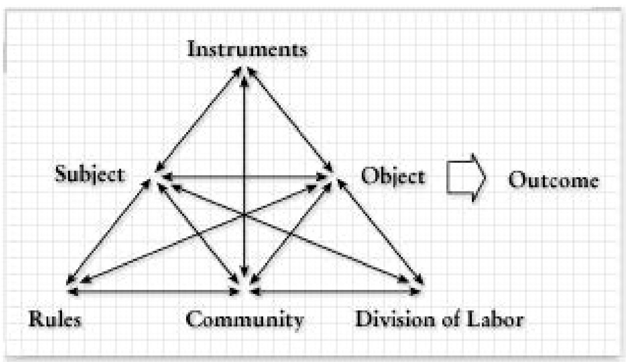


**Figure 1.** Structure of an activity

# 3. Method

The main methodological goal in this research was to capture data that would enable the examination of examples played as a mediating artefact that was injected into the research participants' activity system. To reach this goal, students worked out the statement individually and then together with group members, and following the process of PRM. Classroom observations were audio-recorded and transcribed. The observation notes and students' learning profiles are collected. These data were targeted to reveal the influence that examples had on the interpersonal interactions among students.

We observed the processes carried out by eighth - grade aboriginal students with low attainment. There were 21 students were divided into seven groups and their mathematical teacher involved in the study. The students were given the following statement:

The sum of n consecutive numbers is even.

Then teacher guided students follow the process of PRM. The processes proceeded individual argumentation and group argumentation alternatively. Students proceeded individual argumentation at phases 1, 2, 6. Data analysis was in line with conversation analysis (CA) and pragmatics [7]. Specifically, it is argued that conversational interaction provides a means for students to construct increasingly sophisticated approximations to scientific concepts collaboratively, through gradual refinements of ambiguous, figurative, and partial meanings.

# 4. Empirical Data and Analysis

According to the data analysis, we found that Examples as raw material for understanding mathematical statement, Students can distinguish between supportive and rejected examples by constructing their own example spaces, and Students can develop the ability of generalization by looking for the common properties and then make conjectures.

## 4.1. Examples as Raw Material for Understanding Mathematical Statement

Under the condition of no hint, no students generated examples to make a judgment but wrote down "Yes" (38.1%), "No" (9.5%) and "I don't know" (52.4%). However, after the teacher suggested generating examples to explain, all students could give one or more examples. Students also judged whether the given statement was correct or not according to the examples generated by them. Five students (23.8%) considered it was correct and 16 students (76.2%) thought it was not correct. After taking the examples as the mediating artifacts, students could initially understand the statements, and thus examples became the important artifacts to connect the students with the statement. Moreover, the internal contradiction of the students at this stage was generated by the tension between the mediating artifacts and the rules. Although students could give supportive and rejected examples (mediating artifacts), all of them took examples as inferential basis and theoretical support was the characters of the odd and even numbers (rules).

## 4.2. Students Can Distinguish between Supportive and Rejected Examples by Constructing Their Own Example Spaces

The teacher asked students to give more examples and encouraged them to distinguish supportive examples from rejected examples and their common properties. Most of the students (81%) did not have any difficulties in distinguishing supportive and rejected examples. When further asking students to search for the common properties of these examples, they could not find the common properties of supportive examples, and only two students could find the common properties of rejected examples: the sum of two consecutive numbers is not even. Therefore, from the distinction between supportive and rejected examples to find out the common properties, it cannot be transited directly. It needs to have some bridgeworks of thought. From the observation in the classroom, it found that students wrote examples in disorder. They mixed supportive and rejected examples without classifying. Hence, the teacher asked students to classify the examples, the supportive ones wrote down on the left side and the rejected ones wrote down on the right side, and encouraged them to find out the common properties of these two kinds of examples.

After group argumentation, amongst seven groups, six groups found the common properties of rejected examples; e.g., when the first number is even, the sum of three consecutive numbers is not even. Five groups found the common properties of supportive examples: when the first number is odd, the sum of three consecutive numbers is even. When checking the process of group argumentation, three groups inducing and finding out the common properties were based on the results. Their inferential basis was the result pattern generalization focuses on the adjustment of the results of generating examples. Another three groups used deduction according to the process of generating examples to find out the common properties. Their inferential basis was the process pattern generalisation which focuses on the adjustment of the process of generating examples. Dali was as an example:

Dumen: The sum of three numbers sometimes is even, sometimes not even.

Nwolai: The sum of three numbers sometimes is even, and sometimes is odd. Why is it different from the sum of two numbers?

Dali: Two plus three plus four is an even number plus an odd number plus an even number, and the sum is an odd number.

Nwolai: Three plus four plus five is an odd number plus an even number plus an odd number, and the sum is an even number. If there are two even numbers, the sum is an odd number. If there is one even number, the sum is an even number.

Dali: Is this an answer? It is strange. I have one odd number and you have two odd numbers.

Nowlai: If there is one even number then the sum is even. And why is it an odd number if there are two even numbers?

Dali: Oh, that’s right. Isn’t even number plus even number an even number? Why is it an odd number?

Dumen: No, it needs to add one more odd number to be an even number.

Dali: So the problem isn’t in the even number but in the odd number. It should say if there are two odd numbers the sum is an even number. If there is one odd number, the sum is odd.

Nwolai: You’re saying that an even number plus an even number is an even number and an odd number plus an odd number is an even number.

Dumen: Yes! So an even number plus an odd number is an odd number.

Dali: So we just need to see there are how many odd numbers in the consecutive numbers.

According Davydov’s claim [5], that generalization comes from comparisons between examples, i.e. how their differences enable you to see critical common features, rather than seeing properties, relevant or irrelevant, which they happen to have in common, appears to be upheld.

At this stage, due to the improvement of the rules and division of labour, these three groups started the changes and developments of activity system. In terms of the rules, they found out the common properties of the sum of odds and of the sum of even and odd. In terms of the division of labour, the role of rebuttals is to reject the fact of the sum of the even numbers, and the role of qualifiers is that under the special condition of the number of odd is even, the sum of three consecutive numbers is even.

## 4.3. Students Can Develop the Ability of Generalization by Looking for the Common Properties and then Make Conjectures

Finally, the teacher encouraged students to make conjectures based on the common properties. Fourteen students (66.7%) could make definite conjectures, and five students (23.8%) gave indefinite conjectures (not sure or possible). The indefinite conjectures that students made focus on were the type of “P→Q’”, e.g., the sum of n consecutive numbers is not necessarily an even number. However, the definite conjectures that students made can be divided into two types: “P’→Q and “P’→Q’”, as shown below (Table 1). There are 14 students made at least two or more definite conjectures, and thus, the sum of the percentage is over 66.7%.

**Table 1. The categories of definite conjectures**

| Categories | Definite Conjectures |
|---|---|
| P’→Q (1) 3(14.3%) | The sum of n consecutive even numbers is an even number (modify P). |
| P’→Q (2) 2(9.5%) | The sum of 4n consecutive numbers is an even number (modify P). |
| P’→Q (3) 6(28.6%) | Except the number 2, the sum of consecutive even numbers is an even number (modify P and exclusion) |
| P’→Q (4) 10(47.6%) | If the first number of three consecutive numbers is odd, then the sum will be an even number (modify P and add a condition). |
| P’→Q (5) 3(14.3%) | If the number of odd is even, then the sum of n consecutive numbers will be an even number (modify P and add a condition). |
| P’→Q’ (1) 14(100%) | The sum of two consecutive numbers is an odd number (modify P and Q at the same time) |
| P’→Q’ (2) 12(85.7%) | If the first one of three consecutive numbers is an even number, the sum will be an odd number (modify P and Q at the same time and add a condition to P). |
| P’→Q’ (3) 3(14.3%) | If the number of odd is odd, the sum of n consecutive numbers will be an odd number (modify P and Q at the same time and add a condition to P). |

At the beginning, there was no student could generate examples to judge the statement, but most of them could make conjectures finally. It shows that the examples can be served as the mediating artifacts of the conjecturing activity. Students can think and negotiate within these examples. These examples served as the mediating role of the psychological instrument to guide mind and behavior [19]. Vygotsky considers that human can dominate their behavior or even self through the symbolic and cultural systems from outside. These systems let students able to internalize linguistic symbols and develop the psychological functions of higher level. If the mediating artifacts have different qualities, such as examples, they can lead to different types of psychological functions of higher level.

## 5. Discussion and Conclusion

From the examples of conjecturing activity elaborated in previous sections, we are convinced that participating in a conjecturing activity designed with PRM in which aboriginal students is encouraged to do the following activities:

- constructing and testing with different kind of examples
- organizing and classifying all kinds of examples
- realizing structural features of supporting examples
- finding counter-examples when realizing a falsehood
- experimenting
- formalizing a mathematical statement.
- imagining/ extrapolating/exploring a statement.

Concerning the use of examples in conjecturing, we observed how the exploration on examples might be fruitful, if it is carried out within a PRM (that influences the choice of examples and the way of looking at them) and not done “at random”. Students reflected on results, and on the internal structures of examples, in unfamiliar mathematical statement. In essence, examples are rich psychological and cognitive artifacts for co-construction, and when internalized, these examples serve as conceptual toolkits. With increased richness and flexibility in the use of this “mathematical toolkit,” students develop the peculiar genres for mathematical thinking.

Above all, learning with understanding is essential for enabling students to solve the new kinds of problems they will inevitably face in the future. According to their former learning experiences, most of the students are excessively focused on instrumental understanding without any extra energetic thinking. Consequently, the study is orchestrated for developing pupils’ conjecturing and convincing power as well as for facilitating students’ learning of mathematics towards relational understanding.

To summarize, according to the Vygotskian and activity theory perspectives, students should be enculturated with psychological “toolkits”. The results of this research confirm that generating examples is a strong psychological instrument which can help students to control their own behaviour from social plane and further affect personal psychological plane.

In an ongoing work we are also develop the argumentation learning activity of using examples as the mediating artifacts to investigate the developmental process of the competence of concept understanding amongst Taiwan aboriginal students after involving in the activity of mathematical conjecturing. The aims include developing the argumentation learning activity of using examples as the mediating artifact, the assessing tools of concept understanding which are reliability, validity and applicable.

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