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The Effects of Training on Computational Fluency and Working Memory on Students' Achievement and Retention in Algebra

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Abstract Many students, despite a good understanding of mathematical concepts, are hindered by simple calculations. Recent cognitive researches claimed that working memory which is the limited capacity system of the brain that allows simultaneous storage and processing of temporary information may be improved through trainings and, consequently, improved mathematics performance. The purpose of this study is to investigate the effect of the dual task training on computational fluency with a load of working memory task compared to the traditional basic mathematics drill on the students' achievement and retention in high school algebra. This research made use of two randomly chosen intact groups of fourth year high school students who were given both pretest and posttest using three validated instruments. A 3-week follow-up test was administered to assess algebra retention. This study found that the students who received the dual task training have significantly higher gains in their computational fluency and working memory ability. However, both groups were comparable in terms of their algebra achievement and retention after the 6-week training. The computational fluency was also found to be a predictor of the students' achievement and retention in algebra. Thus, the working memory training has no direct effect on the students' algebra achievement but mediated the effect of computational fluency. A longer and uninterrupted experimental period is recommended for further studies with a focus on binary number tasks as this was found to predict computational fluency.

Keywords: *working memory, computational fluency, training*

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1. Introduction

Among educators, it is widely accepted that effective teachers hold a unique knowledge of students' conceptions and preconceptions in learning. This knowledge on the learner is a tool for teachers to plan for better instruction and intervention that would eventually result to students' higher achievement. There is no doubt that a mathematics teacher plays a crucial role in the progress of every learner in learning a new topic. However, no matter how well the teacher presents a new lesson, the problem is not on comprehending the new lesson at hand. Many students, despite a good understanding of mathematical concepts, are hindered by simple calculations. The National Research Council [23] suggested that poor computational fluency may interfere with mathematical comprehension in much the same way that poor decoding skills interfere with reading comprehension. Unfortunately, majority of high school students including those in the senior year, lack the computational fluency. This has been the perennial problem of high school teachers struggling to teach Algebra to students who have weak computational skills and are expected to produce students who should be mathematically competent. High stake standardized national achievement tests are conducted annually by the Department of Education to fourth year high school students and more often than not, the result in mathematics is way below the national target. Research examining specific subtypes of learning disabilities has found that working memory deficits underlie the difficulties of students with reading and mathematical disabilities [3,12,27,28]. Working memory is the limited capacity system that allows simultaneous storage and processing of temporary information [2]. A growing body of literature shows that one's working memory (WM) capacity can be expanded through targeted training. These trainings in working memory have produced promising results in other cognitive abilities including mathematics skills [13,15,30]. Though time does not allow in high school mathematics to have routines on basic mathematics operations since each day is loaded with a competency that should be achieved, the need for drill is necessitated. The working memory tasks are added to the drill in the belief that this working memory drill may have a positive effect on the students' learning ability. It could be that this limitation on mastering the basic operational skills is due to the student's limitation on memory capacity.

2. Framework

Understanding cognitive obstacles in the learning process is drawn by Herscovics [11] from Piaget's theory of equilibration. This process of equilibration involves not only *assimilation* – the integration of the things to be known into some existing cognitive structure- but also *accommodation* - changes in the learner's cognitive structure necessitated by the acquisition of new knowledge. But, the learner's existing cognitive structures are difficult to change overnight, their very existence becoming cognitive obstacles in the construction of new structure. From this perspective, cognitive obstacles are the natural consequences of the accommodation process of the elements that is beliefs, way of communicating and other factors of the new culture. According to Piaget (1949, 1958), children learn primarily by manipulating objects until the age of 12. Early transition to abstract instruction may cause children's mathematical disabilities in the next developmental period of formal propositional operations. Concept Learning involves the recognition of objects by their abstract characteristics.

Working memory has been shown to underlie performance on a variety of tests of mental abilities, including measures of general intelligence [4,8,18]. The ability to integrate diverse information in working memory is central to human reasoning and problem-solving task [7,20,24]. If working memory is a limited capacity "mental workspace" then increasing its capacity should have wide-ranging benefits-making people better able to perform complex cognitive task, to learn from experience, and generally become more intelligent [19]. Working memory training is still at its infancy and is only popular among cognitive psychologists with its focus on helping people with cognitive impairments. A potential explanation for the mechanism underlying improvements via working memory training is that such interventions may improve the child's ability to divide attention appropriately between processing and storage during tasks that require both skills concurrently. Such training is believed to improve the child's ability to choose and retain relevant information whilst suppressing other information not currently required [14]. These skills are directly relevant for learning mathematics particularly algebra where rules and concepts are applied together while deciding which rule or concept should be correctly applied.

The claim about the possibility of learning mathematics meaningfully without some mastery of basic procedures may thus be compared to the claim about possibility of successfully building a brick house without bricks [26]. Brain research and "laws of forgetting" assure that even things learned to a machine like level tend to defy automatic recall if not practiced on a regular basis. It is in this concern that practice on the basic four operations on integers, fractions and decimal numbers was conducted through daily short drill. This approach is supported by Thorndike's theory of Connectionism which states that learning is the result of associations formed between stimuli and responses. One of the theory's three primary laws is the Law of Exercise. This law states that connections become strengthened with practice and weakened when practice is discontinued.

3. Objectives

Specifically, the study aimed to answer the following questions:

1. How do the training methods affect the students' computational fluency, working memory, algebra achievement and algebra retention?
2. How do students' computational fluency and working memory relate to their algebra achievement?
3. How do students' computational fluency and working memory relate to their algebra retention?

The result of this investigation is important primarily as a contribution to the growing body of knowledge in understanding the student's thinking and learning in mathematics. The study parallels the attention pointed by standard documents including the National Council of Teachers of Mathematics-NCTM and the National Board for Professional Teaching Standards-NBPTS on the importance of teachers holding knowledge of students as learners and being able to recognize the preconceptions and background knowledge that students typically bring to each subject [6]. This study will also give a significant contribution to the latest studies on working memory training and how it affects the performance of regular high school students particularly in mathematics.

4. Methodology

The study made use of the Pretest-Posttest Quasi Experimental-Control Group design. Two intact fourth year classes were randomly chosen for the study with 33 students per class. The training period was conducted during the second quarter of the school year 2014-2015 at Misamis Oriental General General Comprehensive High School. Three validated instruments were used to gather the data. First, is the working memory test which is a modified noncomputer-based test adopted from the computer-based test developed by cognitive psychologists. The test has six parts and divided into the *auditory task* and the *visual-spatial task*. Under the *auditory task* are the *digit span* and the *n-back test*. For the *visual-spatial task* are the *digit span, binary number, n-back and visual-matrix tests.* Second, is the basic mathematics test which is an open-ended test on the four basic operations on integers, fractions and decimals. Third, is the algebra achievement test which is a multiple choice test covering the topics in quadratic function and polynomial function.

At the onset of the training, both groups were pretested on their working memory, computational fluency, and algebra achievement. The experimental group received the training on basic mathematics drill with working memory task while the control group received the basic mathematics drill only. The training consumed 10 to 15 minutes of class hour everyday for 6 weeks. The training for the control group was a daily 15-item drill on the four fundamental operations of integers, fractions and decimals. Flash cards were used in the drill with 5 seconds per card. Students wrote their answers in a paper after each card was flashed. Answers were immediately checked and scores were recorded. Students knew right away their scores in the drill. The training of the experimental group, on the other hand, was dual task. Similar to the control group, the experimental group was given the computational fluency drill but with 10 items only. Right after the computational fluency drill, 10 items of working memory training task were given. In the working memory training, visual stimuli were presented to students to memorize. Students were asked to hold their pens up while each visual stimulus was shown for 10 seconds and were required to remember colors of circles and semi-circles, letters and figures. A response was required after 10 seconds from each time the visual stimulus was no longer shown. Responses were written on the students' paper. The tasks required in the working memory training vary each day. One particular working memory task required students in the experimental group to memorize the colors of the circle and the semi-circle, memorize the geometric figure and the letter inside this figure within 10 seconds only. The teacher then prompted the students to write on their paper the last letter of the colors of the 3 circles in correct order, plus the first letter of the color of the semi-circle and the exact geometric figure with the correct letter inside the figure. If the displayed 3 circles were of the colors blue, green and yellow, the students would write "E N W". If the semi- circle is black, the students would write "B" and if the figure is a triangle with the letter A inside, the students were expected to exactly draw a triangle with the letter A inside the triangle. For other days, the responses in the working memory training were to write the second letter of the colors of the three circles then the first letter of the color of the semi-circle and the next letter in the alphabet immediately following the letter printed inside the geometric figure. Other variations of the visual stimuli were employed tasking the students' working memory to process everyday. Trial tests were provided to students before the working memory training started each day to ensure that they understand the responses required. The responses were written on the same paper which the students used for their computational fluency drill. One week after the treatment period ended, the posttests on the basic math skills, working memory and algebra achievement were administered at one test each day for 3 consecutive days. Three weeks after the algebra achievement posttest, the algebra achievement test was administered again to both groups to get the algebra retention scores. All the tests and the trainings were administered by the researcher herself.

The mean and standard deviation were used to describe the level of the students' working memory ability, computational fluency, and algebra achievement. Descriptive ratings for the mean scores were considered for a qualitative interpretation. Scores before and after the training intervention were compared. The One-way Analysis of Covariance or ANCOVA was applied to test for significant difference between scores at the .05 level of significance. The ANCOVA was also employed to test for retention effect in algebra achievement due to the treatment.

To determine the influence of the working memory ability and computational fluency of students on their achievement and retention in algebra, the Regression Analysis was utilized. Regression Analysis was also used to extend the analysis on which part of the working memory test predicts the students' mathematics performance.

5. Results and Discussion

The results of the analysis are presented in Table 1.

The data in Table 1 presents the students' mean scores in the Basic Mathematics Test before and after the treatment. Both groups are in the beginning level before the training started which means that they are comparable in terms of their basic mathematics skills. This low pretest result in basic mathematics conforms to the findings of Calhoon, Emerson, Flores & Houchins [5] on the computational fluency performance profile of high school students with mathematics disabilities. Their study revealed that high school students with mathematics disabilities showed computational weakness. Their addition is equivalent to 4^{th} grade level, subtraction at 3^{rd} grade level, and multiplication and division at 3^{rd} grade level.

Even if the participants in this present study were regular learners, their computational fluency level is apparently the same as those with mathematics disabilities.

After the treatment period, the posttest showed a slight increase in their mean scores. The control group had an increase of 1.73 while the experimental group got an increase of 2.61. A higher mean score compared to the group that did not receive the working memory drill. This shows that the students who received the dual training performed higher in the computational fluency test. Their descriptive rating also improved from beginning to developing while the control group remained in the beginning stage. The standard deviation of both groups indicates that the scores were dispersed. To test if the difference of scores is significant, the one-way ANCOVA was used. This statistical tool removes the influence of the pretest which was used as covariate. The ANCOVA is the best process since the participants were intact groups and the ANCOVA removes from the criterion scores the part which is predictable from covariates. Table 2 shows the result of this analysis.

Table 1. Mean, Standard Deviation and Descriptive Rating of Students' Basic Mathematics Test Scores

	Control Group		Experimental Group	
	Pretest	Posttest	Pretest	Posttest
Mean	4.42	6	5	8
Standard Deviation	2.95	3	3	3
Descriptive Rating	Beginning	Beginning	Beginning	Developing

Table 2. Summary Table of the One Way Analysis of Covariance (ANCOVA) Equal N's of Students' Basic Mathematics Test Scores

Sources of Variation	Adjusted Sum of Squares	Df	Adjusted Mean Squares	F-Ratio	Probability
Treatment Between	24.68	1	25	6.14*	0.016
Error Within	253.39	63	4		
Total	520.86	65			

$p < 0.05$.

The result of the analysis shown in Table 2 have an F-Ratio of 6.14 which yielded a probability value of 0.016 that is less than 0.05. This means that there is a significant difference in the students' computational fluency level after the treatment. This implies that those who were given the daily drill on the basic operations of integers, fractions and decimals plus the working memory tasks have improved significantly in these skills. Although the control group also received the computational fluency drill, in fact they received more items in the drill than the experimental group, yet their performance in the posttest did not result in significant increase. The National Mathematics Advisory Panel [22] suggests that practice allows students to achieve automaticity of basic skills. Woodward [31] stated that drill and practice programs have demonstrated positive effect on improving retrieval speed for facts already being recalled from memory. Gersten [10] recommends at least 10 minutes devoted daily toward developing computational fluency. However, this study shows that the computational fluency drill with working memory tasks gives better result in improving the students' computational fluency as compared to drill only as evidenced in the significantly higher score of the treatment group. The study of Vander Molen, Van Luit, et al. [30] on computerized working memory training in adolescents with mild intellectual disabilities resulted in higher scores at follow-up in arithmetic. According to Geary [9], poor working memory resources lead to inadequate representations of arithmetic facts in long-term memory. The addition of the working memory training in the treatment group may have caused a positive effect on their retrieval speed for fact already being recalled from memory. The working memory tasks in the training consist of visual-spatial tasks which may have contributed significant independent variance to students' mathematics ability [17].

Table 3 shows the mean scores, standard deviations and qualitative rating of the students' working memory test scores. The working memory test has six different working memory tasks.

Table 3 shows the mean of the total scores of students in the working memory test. It can be seen that the control group had a higher working memory mean score by 1.15 as compared to the experimental group before the training started. Both scores fall into the average ability range. After the training, the posttest results show that the experimental group's mean score is higher by 1.39 compared to the control group. Though higher, the scores remained in the average ability range for both groups. The standard deviation indicates that both groups are heterogeneous in their working memory skills. To test if this difference is statistically significant, the ANCOVA was again employed. Table 4 shows in summary the result of the analysis.

Table 4 shows the result of the analysis of the participants' working memory test scores. The analysis yielded an F-ratio of 4.49 with a probability value of 0.038 which is less than 0.05 level of significance. This means that there is a significant difference in the students' working memory ability as influenced by the method of training. The dual task training must have accounted for the increase in the working memory ability of students compared to those who did not undergo the working memory tasks. This result confirms the studies of near transfer effects on training working memory. This result supports the study of Alloway, Bibile & Lau [1] who found that working memory training showed gains in both verbal and visou-spatial working memory tasks for the group who received the training four times a week compared to those who were trained once a week only and those who were not trained at all. Holmes, Dunning & Gathercole [13] also found in their study on adaptive working memory training of children with poor working memory, a substantial improvement on the children's working memory after completing intensive training and these gains were maintained six months out. Similar results were also found by Henry, Messer & Nash [14] on significantly larger gains in WM for the trained group following short, face-to-face adaptive WM training intervention for typically developing children.

Table 3. Mean, Standard Deviation and Descriptive Rating of Students' Scores in the Working Memory Test

	Control Group		Experimental Group	
	Pretest	**Posttest**	**Pretest**	**Posttest**
Mean	15	17	14	18
Standard Deviation	4.59	4	5	4
Descriptive Rating	Average Ability	Average Ability	Average Ability	Average Ability

Table 4. Summary Table of the One Way Analysis of Covariance (ANCOVA) Equal N's of Students' Working Memory Test Scores

Sources of Variation	Adjusted Sum of Squares	Df	Adjusted Mean Squares	F-Ratio	Probability
Treatment Between	60.51	1	61	4.49*	0.038
Error Within	848.89	63	13		
Total	1177.03	65			

$p < 0.05$.

Table5. Mean, Standard Deviation and Descriptive Rating of Students' Scores in the Algebra Achievement Test

	Control Group		Experimental Group	
	Pretest	**Posttest**	**Pretest**	**Posttest**
Mean	2.91	13	4	13
Standard Deviation	1.38	4	3	4
Descriptive Rating	Beginning	Beginning	Beginning	Beginning

Table 6. Summary Table of the One Way Analysis of Covariance (ANCOVA) Equal N's of Students' Algebra Achievement Test Scores

Sources of Variation	Adjusted Sum of Squares	Df	Adjusted Mean Squares	F-Ratio	Probability
Treatment Between	0.2	1	0	0	0.911
Error Within	982.21	63	16		
Total	986.26	65			

p> 0.05.

The studies mentioned earlier also considered the far transfer effects of the working memory training which is likewise, the intention of this present investigation. To determine the effect of the training on the students' algebra achievement, Table 5 and Table 6 show comprehensively the important figures needed in testing the null hypothesis.

As presented in Table 5, both groups showed low pretest mean scores. The students at the start of the experimental period had no prior knowledge on Quadratic Functions and Polynomial Functions. At the end of the training period, there was an obvious increase in the posttest mean scores of both groups. The control group had an increase of 10.21 in the posttest while the experimental group had an increase of 9.27, a slightly lower increase compared to the control group. Though the mean scores have increased in the posttest, these scores still fall under the descriptive rating as beginning. The standard deviation in the posttest scores indicates that the scores are widely spread. It means that some students got much higher or lower scores from the posttest mean score. In particular, 6% of the participants from both groups were proficient, 12% from both groups were approaching proficiency, 15% from the experimental group and 18% from the control group were developing.

Looking at the posttest results, the scores are seemingly equal. A non- significant difference of scores may immediately be inferred. To determine if there is a significant difference in the students' algebra achievement as influenced by the training method, further analysis was done and Table 6 shows the summary of the One-Way ANCOVA.

The one way ANCOVA yielded an F-ratio of 0.01 only with a probability value of 0.911 that is much higher than 0.05 significance level. The training methods have no differential effect on the students' performance in the algebra test. However, it can be noted that both groups received training on computational fluency which could be a reason why the groups performed at the same level in the algebra posttest. This study shows that the working memory training has no effect on the students' learning in algebra. The result conforms with other studies on WM training like that of Henry, Messer & Nash [14]. Their test on far transfer effect of the working memory training in typically developing children showed that those who received the training performed significantly higher only in reading comprehension but did not differ from the untrained group in their gains in mathematics particularly in the 12-month follow-up. Another study with similar result is that of Karbach, Strobach & Schubert [16] who found that the working memory training conducted to middle- aged children has resulted in transfer to reading ability test but not on a mathematics test. Unlike in the study conducted by Holmes, Dunning & Gathercole [13], the working memory training that they employed to children with working memory impairments resulted in improvements in mathematics scores several months after training. The difference here perhaps is the working memory ability and the age of the research participants. In the present investigation, the participants were high school students with average working memory ability. Though a few of these participants have low working memory ability, the majority are in the average. Trainings on working memory have not resulted in increased performance in algebra for students with average working memory ability compared to untrained students with equal working memory ability. Reports on working memory training show a certain degree of maintenance effect weeks after the treatment; this study likewise sought its effect on retention in algebra.

To determine the retention of students' learning in algebra, the algebra achievement test was administered again to both groups three weeks after the posttest. Table 7 shows the mean score in the algebra posttest and in the retention test. The data in the algebra posttest were presented again alongside the retention test for a better comparison of results.

From Table 7, it can be observed that the experimental group apparently had a higher mean in the retention test by 1.22 compared to the control group. Moreover, the experimental group had increased their mean score in the retention test over the posttest while it was the other way around for the control group. However, all the mean scores remained in the descriptive level as beginning which is less than 50% of the perfect score. The experimental group's scores were more dispersed than that of the control group as shown in their retention test's standard deviation. In fact, none from the control group was proficient in the retention test while 3% from the experimental group were proficient. Only 3% from the control group were approaching proficiency which is much lower than in the experimental group which is 9.4%. In the developing rating, 21.9 % were from the control group while 25% were from the experimental group. To test whether this difference is statistically significant, the One Way ANCOVA was done. Table 8 shows the result.

This one way ANCOVA used the algebra posttest as covariate. The ANCOVA was employed to test the hypothesis that there is no significant difference in the students' algebra retention as influenced by the method of training used. The ANCOVA revealed that there is no significant difference in the algebra retention test scores. The probability value of 0.073 is found to be greater than the acceptable level of significance which is 0.05. This implies that the training method has no significant effect on the students' retention in algebra. Although the experimental group who received the working memory training have higher retention score, it failed to reach significant level. Those who received the computational fluency drill remembered relatively the same as those who received the computational fluency drill and working memory training. The presence of the working memory training in the experimental group did not significantly improve the students' retention in algebra over the students' retention in the control group. Similar to the studies mentioned earlier [14,16], the working memory training did not result in significant gains in mathematics but only to reading comprehension. Though there was maintenance effect after the working memory training in arithmetic [30], this study found no maintenance effect in algebra. Achievement gains associated with better working memory would be expected to take time to develop and not anticipated immediately after training [13].

To further understand how the students' working memory and computational fluency relate to their achievement and retention in algebra, the multiple regression analysis was done. Table 9 presents the results for the algebra achievement with the computational fluency and working memory as predictors.

As shown in Table 9, only the students' computational fluency can predict the students' performance in algebra. It bears a correlation coefficient of 0.330. The result further revealed that for every unit increase in the computational fluency, the predicted increase in the algebra achievement is 0.483 holding the working memory constant. Furthermore, 12.1% of the variability of the students' achievement in algebra is explained by the students' computational fluency and working memory. These results suggest that a student who is computationally fluent performs better in algebra and the working memory ability of a student does not affect his/her performance in algebra. Whether a student has high ability or low ability in working memory, it bears no effect on the student's algebra performance. On the other hand, a student with high computational fluency would indicate a high achievement in algebra while a student with low computational fluency can predict a low performance in algebra. This result agrees with the study of Tolar, Lederberg & Fletcher [29] that computational fluency had the strongest effect on algebra achievement though they also found moderate effects of working memory. Both groups received the computational fluency drill which could have accounted for the non significant difference in their performance in the algebra test. Working memory is found to be related to mathematical performance in adults and in typically developing children [25], but this present analysis found otherwise, at least not in algebra.

To determine the influence of the students' computational fluency and working memory on their retention in algebra, Table 10 presents the results of the multiple regression analysis.

The regression analysis shown in Table 10 reveals that between the two skills, it is the students' computational fluency that predicts their retention in algebra. As seen in the table, the computational fluency bears a correlation coefficient of 0.431. The result further revealed that for every unit increase in the computational fluency, the expected increase in the algebra retention is 0.497 holding the working memory constant. Furthermore, 18.9% of the variability of the students' retention in algebra is explained by the computational fluency and working memory. On the other hand, the analysis showed that the working memory is a weak predictor of students' retention in algebra. There are studies showing working memory training to have maintenance effect in mathematics [13,30]; however, there are also studies that do not show retention effect in mathematics [14,16]. In this study, it was found that the retention of students' learning in algebra is independent from their working memory ability. Those exposed in the working memory training and those who were not, remembered basically the same at 3-week follow-up.

Putting together the regression analyses results, the computational fluency of a student can predict both his/her achievement and retention in algebra whilst the working memory cannot. But, the working memory bears an effect on the learner's computational fluency through the dual task training. Since the group that was exposed to the dual task training which has the working memory task earned significantly higher scores in the computational fluency test, the researcher endeavored to understand which of the parts in the working memory test best predicts the computational fluency of the student. Regression analyses were performed on computational fluency with each part of the working memory test as predictors. The six parts of the working memory test are: the digit span auditory test; the visual digit span; the binary number test; the n-back auditory test; the visual n-back; and the visual matrix test. Among the six WM tasks, the regression analyses results revealed that only the binary number test can significantly predict the students' computational fluency. It yielded a probability value of 0.027 which is smaller than 0.05 significance level. It has a correlation coefficient of 0.295. The analysis also showed that for every unit increase in the binary number test score, the expected increase in the computational fluency is 0.901. However, only 8.7% of the variability of the students' computational fluency is explained by the binary number task in the working memory test. Established standardized computer-based WM tests used by cognitive psychologists consist of different tasks that involve storage and processing tasks. The working memory test used in this study, however, was non computer-based and was conducted with the researcher's facility. The test also consists of storage and processing tasks which are not the same with the trained tasks. Nonetheless, among these tasks, the binary number task was found to be a predictor of students' computational fluency. Those who can correctly store binary numbers in their memory tend to be more computationally fluent. This task requires strategy on how to store several digits of 0's and 1's and retrieve them in correct sequence in a short period of time. Rules in basic mathematics requires strategy as well with all the many rules they need to remember like the rules of signed numbers, operating on fractions with equal or unequal denominators, remembering how to reduce fractions, operating with mixed fractions, operating with decimals and remembering the rules for the decimal point, and other basic concepts. All these must be loaded and retrieved correctly in the memory of the students. For those who cannot store and process these information, a memory load may occur and a high chance of committing errors.

Table 7. Mean, Standard Deviation and Descriptive Rating of Students' Scores in the Algebra Posttest and Retention Test

	Control Group		Experimental Group	
	Posttest	**Retention Test**	**Posttest**	**Retention Test**
Mean	13.12	12	13	14
Standard Deviation	4.15	3	4	3
Descriptive Rating	Beginning	Beginning	Beginning	Beginning

Table 8. Summary Table of the One Way Analysis of Covariance (ANCOVA) Equal N's of Students' Retention Test Scores

Sources of Variation	Adjusted Sum of Squares	Df	Adjusted Mean Squares	F-Ratio	Probability
Treatment Between	23.254	1	23	3	0.073
Error Within	425.711	61	7		
Total	622.859	63			

$p > 0.05$.

Table 9. Multiple Regression Results for Algebra Achievement with Computational Fluency and Working Memory as Predictors

Variable	Coefficient	Standard Error	Beta	t-value	p-value
Computational Fluency	0.483	0.193	0.330	2.499	0.016
Working Memory	0.055	0.122	0.059	0.449	0.655

R-square : 12.1%.

Table 10. Multiple Regression Results for Algebra Retention with Computational Fluency and Working Memory as Predictors

Variable	Coefficient	Standard Error	Beta	t-value	p-value
Computational Fluency	0.497	0.146	0.431	3.396	0.001
Working Memory	0.011	0.092	0.016	0.123	0.903

R-square: 18.9%.

6. Conclusions

This research found that the working memory tasks added to the computational fluency drill did not bear any significant difference on the students' achievement and retention in algebra as compared to those exposed to the computational fluency drill alone. Studies that review the effects of working memory training in improving cognitive skills have forwarded skepticism on the mechanism of far transfer effects including transfer in mathematics performance. According to Melby-Lervag & Hulme [19], WM training programs appear to produce short-term, specific training effects that do not generalize. They doubt that working memory training can be used as a method of enhancing cognitive functioning in typically developing children and healthy adults. Gains in academic achievement were not anticipated immediately post-training since achievement gains associated with better working memory would be expected to take time to develop [13].

On one hand, this study also found that the computational fluency drill with working memory tasks produce higher gains in the students' computational fluency as with those who have the computational fluency drill only. Moreover, the computational fluency was found to be a strong predictor of the students' algebra achievement and retention making the computational fluency a very important skill for students to be successful in high school algebra.

7. Recommendations

Based on the foregoing findings and conclusions, the researchers therefore recommend that basic mathematics operations should still be emphasized in high school mathematics because it is a predictor of their performance in algebra. Test on computational fluency should be given to high school students at the start of the school year as a diagnostic test to determine students at risk of mathematics failure. The basic mathematics drill with working memory tasks may be employed by high school mathematics teachers during their mathematics class to help improve the computational fluency of high school students. Modifications on working memory test with more focus on binary number tasks may be formulated and validated for the assessment of students' potential in improving their computational fluency. Working memory tasks that are related to the present topic taught in mathematics class may be employed in future studies on working memory training. Further studies may be conducted on working memory to improve binary number skills as a possible remediation to enhance computational fluency.

References

[1] Alloway, T.P., Bibile, V., & Lau, G. (2013). Computerized working memory training: Can it lead to gains in cognitive skills in students? *Computers in Human Behavior*, 29, 632-638.

[2] Baddeley, A.D. (1986). *Working memory*. Oxford University Press, UK: Clarendon.

[3] Bull, R., & Johnston, R. S. (1997). Children's arithmetic difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology*, 65, 1-24.

[4] Carpenter, P. A., Just, M. A. & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices Test. *Psychological Review* , 97: 404-31.

[5] Calhoon, M.B., Emerson, R. W., Flores, M., Houchins, D. (2007). Computational Fluency Performance Profile of High School Students with Mathematics Disabilities. Remedial and Special Education, 28(5), 292-303.

[6] Delany, S., Ball D.L., Hill, H. Schilling, S. & Zopf, D. (2008). "Mathematical knowledge for teaching": adapting US measures for use in Ireland. *Journal of Mathematics Teacher Education* V.2(3), 171-197.

[7] Duncan, J. (2001). An adaptive coding model of neural function in the prefrontal cortex. Nature Reviews: *Neuroscience*, 2:820-29.

[8] Engle, R. W., Tuholski, S. W., Laughlin, J. E. & Conway, A. R. (1999b). Working memory, short-term memory, and general fluid intelligence: A latentvariable approach. *Journal of Experimental Psychology: General*, 128(3), 309-31.

[9] Geary, D.C. (1993). Mathematical disabilities: cognitive, neuropsychological, and genetic components. *Psychological bulletin*, 114(2), 345.

[10] Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J.R., et al. (2009). Assisting students struggling with mathematics: Response to intervention for elementary and middle schools. Washington, DC: U.S. Department of Education Institute of Educational Sciences.

[11] Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner & C. Kieran (Eds.),. Research issues in the learning and teaching of algebra, 60-86. Reston, VA: National Council of Teachers of Mathematics.

[12] Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, 82, 375-386.

[13] Holmes, J., Gathercole, S.E., & Dunning, D.L. (2009). Adaptive training leads to sustained enhancement of poor working memory in children. *Developmental Science*, 12(4), F9-F15.

[14] Henry, L.A., Messer, D.J. & Nash, G. (2014). Testing for near and far transfer effects with a short, face-to-face adaptive working memory training intervention in typical children. *Infant and child development*, 23: 84-103.

[15] Jaeggi, S.M., Buschkuehl, M., Jonides, J. & Perrig, W.J. (2008). Improving Fluid Intelligence with Training on Working Memory. http://www.pnas.org/content/early/2008/04/25/0801268105.abstract.

[16] Karbach, J., Strobach, T. & Schubert, T. (2015). Adaptive working -memory training benefits reading, but not mathematics in middle childhood. *Child Neuropsychology,21(3)*, 285-301.

[17] Keeler, M.L. & Swanson, H.L. (2001). Does Strategy Knowledge Influence Working Memory in Children with Mathematical Disabilities?. *Journal of Learning Disabilities*, 34(5), 418, 17 pages.

[18] Kyllonen, P. C. & Christal, R. E. (1990). *Reasoning ability is (little more than) working memory capacity?! Intelligence*, 14:389-433.

[19] Melby-Lervag, M. & Hulme, C. (2012). Is working memory training effective? A meta-analytic review. Developmental Psychology, 49(2), 270-291.

[20] Miller, E. K. & Cohen, J. D. (2001). An integrative theory of prefrontal cortex function. *Annual Review of Neuroscience* 24:167-202.

[21] National Council of Teachers of mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: Author.

[22] National Mathematics Advisory Panel. (2008). Final Report: Foundations for Success. Western initiative for strengthening education in math. *WISE Math.* http://wisemath.org/us-national-advisory-panel/.

[23] National Research Council. (2001). Looking at Mathematics and Learning. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Adding it up: Helping children learn mathematics* (pp. 1-16). Washington, DC: National Academy Press.

[24] Prabhakaran, V., Narayanan, K., Zhao, Z. & Gabrieli, J. D. E. (2000). Integration of diverse information in working memory within the frontal lobe. *Nature Neuroscience*, 3:85-90.

[25] Raghubar, K.P., Barnes, M.A. & Hecht, S.A. (2009). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. Learning and Individual Differences, 20(2010), 110-122.

[26] Sfard, A. (2000). On reform movement and the limits of mathematical discourse. *Mathematical Thinking and Learning*, 2(3), 157-189.

[27] Siegel, L. S., & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development*, 60, 973-980.

[28] Swanson, H. L. (1993). Working memory in learning disability subgroups. *Journal of Experimental Child Psychology*, 56, 87-114.

[29] Tolar, T.D., Lederberg, A.R. & Fletcher, J.M. (2009). A Structural Model of Algebra Achievement: Computational Fluency and Spatial Visualisation as Mediators of the Effect of Working Memory on Algebra Achievement. *Educational Psychology*, 29(2), 239.

[30] Van der Molen, M.J., Van Luit, J.E.H., Van der Molen, M.W., Klugkist, I.,& Jongmans, M.J. (2010). Effectiveness of a computerized working memory training in adolescents with mild to borderline intellectual disabilities. *Journal of Intellectual Disability Research*, 54(5), 433-447.

[31] Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disabilities Quarterly.*