

The Application of the Method of Dimensional Analysis When Solving Problems

Hrayr Ohanyan*

ASPU after Khachatur Abovyan

*Corresponding author: hrayrohanyan@gmail.com

Abstract When solving problems, a number of methods are applied in physics, that are targeted towards simplifying the problem solution and making it accessible for students to master. The method of dimensional analysis was observed in the article, as an effective means of solving lots of problems.

Keywords: *dimensional analysis, teaching physics, International System of Units*

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1. Introduction

When solving problems, a number of methods are applied in physics, that are targeted towards simplifying the problem solution and making it accessible for students to master. The method of dimensional analysis was observed in the article, as an effective means of solving lots of problems. The distinctiveness of the method is that it gives an opportunity to apply physical law resolutions and solve problems, without complex mathematical operations. This method is used in the new textbooks of physics by the authors [1,2], which is a novelty in the sphere of physics school educational programs. The method has been used during the application of a number of resolutions, though it is not widely used when solving problems, despite the fact, that the result is frequently positive. Hence, not only the way of method application is represented, but also the effectiveness is proved with the help of examples. In my opinion, this article will support both learners and teachers in the process of teaching physics, and can become a part of methodological base for teachers.

The method of dimensional analysis: During the school courses, the learners deal with the International System of Units (Si). All the derivative values can be expressed by the basic value units. It is necessary to remind the learners, that when doing mathematical operations with resolutions, in this case the changes affect the dimensions of physical values [[6], pages 10-13], as well as it is worth to mention, that the non-dimensional values have 1 dimension, and not 0.

For indicating the dimension, let us use the following:

$$F = LMT^{-2}$$

If we are aware of all the physical dimensions describing a specific phenomenon, then linking their dimensions [[7], pages 66-67] and causing the right and left parts of seekable equation to have the same dimensions, [[5], pages 17-18], we can find the required

functional relationship $f = \alpha(L^m M^n T^k)$ Here the m, n, k can be integers, fractional numbers or zero, and α is a coefficient of variation, which, in particular cases, can be equal to one.

Examples of problem solutions: A number of examples of problem solutions are introduced below, the solutions of which become easier and simpler when the method of dimensional analysis is applied. The “traditional” way of problem solution is introduced, with the purpose of making the effectiveness of the method of dimensional analysis obvious.

Problem 1: What speed should the earth rotate, for the body weight to be equal to zero on the equator? Let the density of the earth be $5,6 \cdot 10^3 \frac{kg}{m^3}$, the radius $R = 6400 km$.

It is known, that the linear speed value is based on the radius vector of the rotating object. Besides, we know that the body weight depend on the acceleration. Thus, the body will not have a weight, if its centripetal acceleration $a = g$. So, $v = r^m g^n$. Now let us insert each value in dimension:

$$LT^{-1} = L^m (LT^{-2})^n$$

Equaling the same value indexes at the right and the left, we can write:

$$\begin{cases} m+n=1 \\ -2n=-1 \end{cases} \Leftrightarrow \begin{cases} m=\frac{1}{2} \\ n=\frac{1}{2} \end{cases}$$

Inserting the result in the resolution, we will have:

$$v = r^{\frac{1}{2}} g^{\frac{1}{2}}$$

$$v = \sqrt{Rg} = 8000 \text{ m/s}$$

Now, let us consider the “traditional” way of problem solution and compare it to the proposed method. On one hand, due to the universal law of attraction, we have

$$F = G \frac{mM}{r^2}$$

moreover, we can consider for the body on the equator to be $R = r$, thus, $F = G \frac{mM}{R^2}$. The earth

mass will be $M = \rho V = \frac{4}{3} \pi R^3 \rho$ indicated by density.

Inserting this value, we will have $F = G \frac{4}{3} \pi R \rho m$.

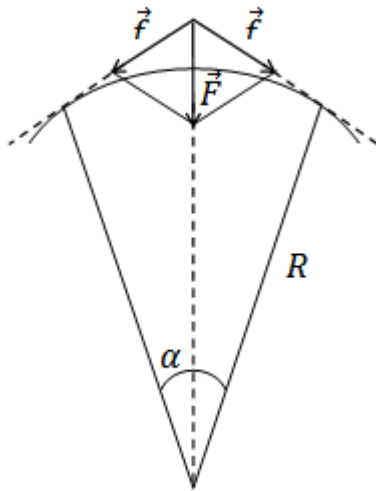
Besides, the force on the body on the equator during the rotation: $F = m \frac{v^2}{R}$. It is obvious here, that

$$G \frac{4}{3} \pi R \rho m = m \frac{v^2}{R}$$

, and from here $v = \sqrt{\frac{4}{3} \pi R^2 \rho G}$ or

$v = 7920 \frac{m}{s}$. It is vivid, that applying the method of dimensional analysis when solving the problem, we need only two steps, on the contrary, the “traditional” method requires a number of resolutions, and only after some changes we get the desired result.

Problem 2: The metal chain with l length and M mass is attached at centrifugal machine and rotates at frequency ν . Considering the chain breakdown as a circle during the rotation, find out the stretching force along its length.



With the purpose of making the effectiveness of the method of dimensional analysis obvious when solving a problem, at first let us consider the “traditional” way of solution.

As a result of rotation, the chain will have a ring shape with R radius, for the Δl part Δm mass of the length of which we can write: $\Delta M = \frac{M}{l} \Delta l = \frac{M}{l} R \alpha$. During rotation, the stretching forces, which affect the particular part by the other parts, will cause \vec{F} effective force, that will be equal to $F = 2f \sin \frac{\alpha}{2}$

according to Newton law $F = \Delta M \omega^2 R$.

$$2f \sin \frac{\alpha}{2} = \Delta M \omega^2 R \text{ or } 2f \sin \frac{\alpha}{2} = \frac{M}{l} R \alpha \omega^2 R$$

. Since $\omega = 2\pi\nu$ and in case of small α angles $\sin \frac{\alpha}{2} \approx \frac{\alpha}{2}$ thus

we will have $2f \frac{\alpha}{2} = \frac{M}{l} R^2 \alpha \omega^2$, from where

$$f = \frac{M}{l} \nu^2 (2\pi R)^2 = \frac{M \nu^2 l^2}{l}$$

, from where: $f = M \nu^2 l$. Now let us solve the problem with the help of the method of dimensional analysis. The problem requires to find out what the functional dependency: $F = f(M, \nu, l)$ will be like. According to the problem, the stretching force of the chain is viewed as a function from its mass, rotation frequency and length $F = f(M, \nu, l)$, thus there should be the following implication:

$F = M^m \nu^n l^k$, or expressed by the dimensions, we will have:

$$LMT^{-2} = (M)^m (T^{-1})^n L^k$$

$$\begin{cases} k = 1 \\ m = 1 \\ -n = -2 \end{cases} \Leftrightarrow \begin{cases} k = 1 \\ m = 1 \\ n = 2 \end{cases}$$

Inserting these values, we will have: $F = M \nu^2 l$.

Now we got the same result, but the way was much simpler.

I would like to mention, that the problems mentioned below have been solved only by the method of dimensional analysis, having the purpose of leaving the “traditional” way of solution for the individual work.

Problem 3: The moving spaceship, the velocity of which is $v = 9400 \frac{m}{s}$, the front section area is $S = 50m^2$, falls into micrometer fog. Each cubic meter of the space contains one universe powder, the mass of which is $m = 0,02g$. For keeping the velocity of the spaceship constant in those conditions, how much should the engine propulsion force be increased?

The problem requires to find out what the functional dependency $F = f(\rho, s, v)$ will be like, where ρ is the density. Hence:

$$F = \rho^m s^n v^k$$

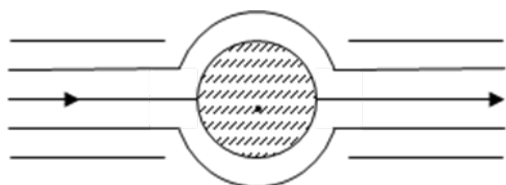
$$LMT^{-2} = (L^{-3}M)^m L^{2n} (LT^{-1})^k$$

$$\begin{cases} -3m + 2n + k = 1 \\ m = 1 \\ -k = -2 \end{cases} \Leftrightarrow \begin{cases} n = 1 \\ m = 1 \\ k = 2 \end{cases}$$

By inserting these values, we will have:

$$F = \rho s v^2 = 0,02 \cdot 10^{-3} \cdot 50 \cdot (9400)^2 = 88360N$$

Problem 4: The ball is in a stationary condition in flowing ideal liquid, the velocity of which is v . Find out the dynamic pressure on the ball. Does it depend on the radius of the ball?



It is obvious, that the pressure will depend on the density ρ of the liquid, flow velocity v and the size of the ball. $p = f(\rho, v, R)$ or $p = \alpha \rho^m v^n R^k$: It follows from here that:

$$L^{-1}MT^{-2} = \alpha (L^{-3}M)^m (LT^{-1})^n L^k$$

$$\begin{cases} -3m + n + k = -1 \\ m = 1 \\ -n = -2 \end{cases} \Leftrightarrow \begin{cases} k = 0 \\ m = 1 \\ n = 2 \end{cases}$$

Accordingly, it follows from the resolution $p = \alpha \rho v^2 R^0$ that the dynamic pressure does not depend on the ball size, though at the beginning we had that assumption: $p = c \rho v^2$.

If we consider the static pressure as well, we will have $p = p_0 + \alpha \rho v^2$ ($\alpha = 2$).

2. Summary

As it was proven in the article, the method of dimensional analysis is an effective one when solving a number of problems. The introduction of problem solutions through comparative analysis of the proposed, as well as the "traditional" method makes the effectiveness of the method of dimensional analysis obvious. Besides, mastering the method of dimensional analysis, not only the teachers, but also the students can have an opportunity to:

- Solve a number of problems by a simple method;
- Get some physical equations without complex mathematical calculations;
- Clarify the functional relationship of the physical quantities within a dimensionless coefficient of proportionality;
- Check the homogeneity of the equation, reflecting the functional relation between the quantities.

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Appendix 1: The dimensions of the basic physical values are mentioned

The name of Quantity	Describing Formula	The Dimension
Area	$S = a \cdot b$	L^2
Volume	$V = a \cdot b \cdot h$	L^3
Velocity	$v = \frac{s}{t}$	LT^{-1}
Acceleration	$a = \frac{\Delta v}{t}$	LT^{-2}
Density	$\rho = \frac{m}{V}$	$L^{-3}M$
Force	$F = ma$	LMT^{-2}
Pressure	$P = \frac{F}{S}$	$L^{-1}MT^{-2}$
Work	$A = F \cdot s$	L^2MT^{-2}
Power	$N = \frac{A}{t}$	L^2MT^{-3}
Force constant (spring constant)	$k = \frac{F}{x}$	MT^{-2}
Temperature	T	θ
Potential difference	$U = \frac{A}{q} = \frac{A}{It}$	$L^2MT^{-3}I^{-1}$
Resistance	$R = \frac{U}{I}$	$L^2MT^{-3}I^{-2}$
Induction	$L = \frac{\varepsilon \Delta t}{\Delta I}$	$L^2MT^{-2}I^{-2}$
Capacitance	$C = \frac{q}{U}$	$L^{-2}M^{-2}T^4I^2$
Planck's constant	$h = \frac{E}{\nu}$	L^2MT^{-1}
Electric constant	$\varepsilon_0 = \frac{Cd}{S}$	$L^{-3}M^{-1}T^4I^2$
Heat	$Q = A$	L^2MT^{-2}
Gravitational constant	$G = \frac{FR^2}{m^2}$	$L^3M^{-1}T^{-2}$
Heat capacity	$C = \frac{Q}{m\Delta T}$	$L^2T^{-2}\theta^{-1}$