

Teaching Practices of Academic Language Support. A Video-based Analysis of Mathematics Lessons in Germany and in Switzerland

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Abstract Language teaching and learning is important in all school subjects. This is the underlying idea of this paper, which analyses teaching practices in mathematics lessons filmed during ‘The Pythagoras Study’. Qualitative methods are engaged to reconstruct how the teachers draw the pupils’ attention to linguistic features and language use in functional contexts of subject teaching. Specifically, it is investigated how teaching practices open students’ access to a subject-specific genre: a mathematical theorem. The theoretical framework includes the language concept of systemic functional linguistics. Theoretical concepts that are employed to identify promising teaching practices are a model for Introducing and Scaffolding Genres in the Classroom, the concept of Academic Discourse Practices and the concept of Dialogic Teaching. The findings inform us about teaching practices of academic and technical language support. They show how academic discourse practices of language observation as well as a cumulative classroom discourse can support the students’ access to a subject specific genre.

Keywords: *academic language, Pythagoras lessons, Germany, Switzerland*

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1. Introduction

“Im rechtwinkligen Dreieck ist die Summe der Flächeninhalte der Kathetenquadrate gleich dem Flächeninhalt des Hypotenusenquadrats” [[4] p. 184], translation from German: “In a right-angled triangle the sum of the areas of the squares of the *Katheten* equals the area of the square of the hypotenuse”¹.

According to Barbara Drollinger-Vetter, this sentence is a technical representation and an “important aid to thinking and communication” for those who already understand Pythagoras’s theorem [[4], p. 184, t.f.G.²] – but for pupils who are completely unfamiliar with this mathematical theorem, the statement is not at all helpful.

“We should end up with a proper German sentence, shouldn’t we?” (teacher).

This is a quote from a mathematics teacher striving with his class to express Pythagoras’s theorem in words that his pupils will understand.

The purpose of this article is to analyze teaching practices of academic and technical language support. The objects of the analysis are mathematics lessons from the

Pythagoras study [16], a classroom video study conducted during the 2002/2003 school year. The sample consisted of 20 Swiss and 20 German classes from secondary schools. According to the design, all classes were filmed during their first three lessons of introduction to the Theorem of Pythagoras. In 2014, the research team of the Pythagoras study, Eckhard Klieme, Christine Pauli and Kurt Reusser, invited me and other academics to analyze selected lessons from the Pythagoras study from different research perspectives. Results from my analyses of the selected lessons are presented in this paper.

My specific interest is in the promising practices of teachers who endeavor to extend the linguistic repertoire of their pupils and who embrace the idea of language learning and teaching in all school subjects. This idea has received renewed attention in German education policy and research because of educational inequalities in the context of linguistic and socio-cultural diversity. Social differences in language use and literacy practices are considered a challenge for language and subject teaching, and language education is connected with the idea of equal opportunities and social justice [5]. My perspective for analysis in this article relies on findings from the research project “Teaching practices of academic language support. A video study in multilingual classrooms”, also known as the BilLe video study³. The BilLe video study was conducted in classrooms with high percentages of

¹ In German, the sides of a right-angled triangle that are not the hypotenuse are called *Kathete(n)*, and this word is used in the formulation of Pythagoras’ theorem. It is important that pupils familiarize themselves with this word. Since there is no exact equivalent in English, I shall use the German term in the following.

² t.f.G = ad-hoc translation from the German original.

³ The project “Bildungssprachförderliches Lehrerhandeln (BilLe)” was funded by the German Research Foundation (2012-2014, grant FU 586/4-1) and was carried out at the University of Münster.

pupils who are in the process of acquiring German as a second language and of pupils who are not familiar with academic language registers at home, whether or not German is the language they usually use at home. In these classrooms, expert teachers were filmed and their teaching practices were analyzed.⁴

The focus of analysis in this article is the teaching and learning of subject-specific genres. The theoretical framework will be illustrated with two examples from the BilLe study, a biology lesson (the genre is a behavioral description) and a Latin lesson (the genre is an ancient didactic poem). In the analysis of the Pythagoras lessons, I will then scrutinize the teaching and learning of a mathematical genre: the mathematical theorem, specifically the theorem of Pythagoras.

2. Theoretical Framework

2.1. Linguistic and Technical Registers in Schooling

A basis for analyzing teaching practices of academic language support is the language concept of systemic functional linguistics developed by M.A.K. Halliday. A simplified rendition of this concept is 'form follows function'. Halliday emphasizes that "texts [written or oral, S.F.] vary systematically according to contextual values". To illustrate "ways of using language in different contexts" Halliday gives examples such as "e-mail messages, inaugural speeches, service encounters in the local deli, news bulletins, media interviews, tutorial sessions" [13], p. 27]. According to Halliday, the language system serves innumerable functions and there are many varieties of language. The term register denotes "a functional variety of language" [13]. If we follow Halliday, language development and learning is not the acquisition of linguistic structures in itself, but the acquisition of linguistic function, which means learning the various uses of language.

Academic language is an important linguistic register for teaching and learning. It is characterized by linguistic structures, which serve to express complex and abstract content in isolation from its initial context. For this reason, it features characteristics of written language in oral texts too. In educational science, the discussion about the importance of academic language for schooling is based on Jim Cummins's distinction between "Basic Interpersonal Communication Skills" (BICS) and "Cognitive Academic Language Proficiency" (CALP) [3]. Cummins calls attention to the fact that pupils, especially those with home languages other than the language of schooling, need time

and support to acquire the academic language of schooling. In the German discourse of educational science, the term *Bildungssprache* is used to capture the concept of academic language as a register [12]. The following definition of *Bildungssprache* was a starting point for the BilLe video study:

Bildungssprache "is characterized by linguistic tools and structures which enable the speaker to express complex and abstract content regardless of the concrete interactive situation. It contains features of the written form and serves the linguistic construction of universal meanings. The register of academic language is used *inter alia* in linguistic activities typical of schooling." [7], p.42, t.f.G.]

There is no doubt that mastering the register of academic language is an important prerequisite of pupils' success in school [3]. At the same time, though, it would be misleading to assume that all learning, teaching and thinking in school lessons occurs in the register of academic language. As in everyday life situations, in school, too, a variety of linguistic registers have their proper place depending on the social contexts of language use. A broad definition of multilingualism, which takes into account the internal varieties of each language and calls attention to the challenges of language teaching and use in school lessons was proposed by Mario Wandruszka [20]:

"The teacher must recognize that he is educating children to be multilingual. He must recognize and acknowledge the value of the languages, vernacular, regional and social dialects the children already possess; he must then introduce the children to a new language, the academic language [*Bildungssprache*], and make them aware of their growing multilingualism, the richness of our linguistic possibilities" [20], p.18, t.f.G.].

Wandruszka describes an internal multilingualism, which is possessed by every individual. He highlights the teacher's responsibility for initiating the pupils' acquisition of the register of academic language. However, he also emphasizes the value of all the other linguistic varieties pupils use inside and outside school. With regard to school success, teachers are not only in charge of academic language support, they should also enable pupils to use their linguistic range as appropriate to different situations. From this perspective, the challenge in teaching is to provide transitions between the registers, that is, to bridge informal language use and academic language use in classroom interaction. There is a well-known example of appropriate shifts between linguistic registers by Pauline Gibbons from a lesson on magnetism in a primary school. The example illustrates how a teacher creates different situations for language use in the classroom in order to support transitions between linguistic registers.

1. Pupil experimenting with magnets in a work group with other pupils: "Look, it's making them move. Those didn't stick."

2. The same pupil telling her teacher about the experiment: "We found out that pins stuck on the magnet."

3. From the same pupil's written report: "Our experiment showed that magnets attract some metals."

4. From a children's thesaurus: "Magnet attraction occurs only between ferrous metals."

[10], p. 3]

⁴ Ten teachers from five schools were video-taped during their regular lessons. These teachers were considered to be 'experts' in promoting pupils' language skills due to the fact that their pupils had achieved good results in the language categories of assessment tests. Their selection was based on a person-centred approach in schools in a major German city. All the schools have a specific language education programme and a high percentage (above average) of multilingual pupils and pupils from low income families. The total recording time was 32 hours and included regular lessons in grades 1-10 (age range from 6 to 16 years) in two primary schools, one secondary school and two advanced secondary schools (*Gymnasien*). A wide range of subjects was recorded. The study's analytical approach combines quantitative methods (low-inference video coding) and qualitative methods (interpretive video interaction analyses) [8].

According to the social context, the language becomes more explicit and decontextualized with each sentence (from one to four). Sentences three and four show features of the academic language register: The language is impersonalized, more abstract, and denser; generalizations are expressed and technical terms are used. These features make it difficult for many pupils to master the transitions between the registers. The necessity to acquire linguistic features that allow for an increasingly context-reduced language use is a given in all school subjects. Furthermore, besides the cross-curricular challenges of academic language use, there are also specific linguistic requirements for teaching and learning the different school subjects, due to technical language registers that are part of specific knowledge domains. In the following, this will be scrutinized for the case of mathematics.

In the teaching and learning of mathematics, linguistic and mathematical comprehension processes interact. To understand mathematical concepts and problems, verbal forms of presentation as well as nonverbal representations are important. Susanne Prediger and Lena Wessel [19] include both in the term ‘registers of presentation’ (t.f.G.). To illustrate the challenges of teaching and learning mathematics, they propose a model of registers of presentation that comprises all possible forms of representation: verbal forms of presentation (informal language, academic language, technical language) and nonverbal representations (iconic and symbolic-formulaic representations). They emphasize that *all* these registers of presentation should be considered in mathematics lessons and that to support the pupils’ comprehension it is expedient to *interlink* different registers of presentation within the process of teaching and learning. Findings from cognitive-psychological research and research on mathematical didactics indicate that to deliberately translate from one register of presentation to another contributes to the pupils’ comprehension of mathematical concepts [19, p. 168]. Therefore, pupils should be given the opportunity to verbalize iconic and symbolic-formulaic representations, using different linguistic registers including informal registers. The general features of academic language, such as decontextualization and generalization, apply to the technical mathematical register as well. Additionally, the mathematical register is characterized by a high degree of precision, especially with regard to definitions, and there are technical terms that differ considerably in meaning from the everyday meanings of the words [19, p. 165]. The sequences from the Pythagoras lessons analyzed in this article will illustrate that pupils need to grasp the particularities of the academic language register *and* the particularities of the technical mathematical language register as a basis for mathematical understanding. My analyses focus on the question as to how teachers deal with the variety of representations of a mathematical theorem, specifically the theorem of Pythagoras, in their classrooms.

One purpose of the lessons analyzed in this article is to lead the pupils to an understanding of the technical representations of the theorem of Pythagoras. According to Drollinger-Vetter [4], pupils have to know and understand three types of technical representation to fully comprehend the theorem of Pythagoras: iconic representation, symbolic-formulaic representation and symbolic-verbal representation (technical language register). At the same

time, Drollinger-Vetter highlights a fact that is fundamental to understanding the importance of non-technical linguistic registers for teaching mathematics: “Technical representations are not self-explanatory” [4], p. 185, t.f.G.]. Pupils can only work with technical representations after having understood their meaning. So they should be enabled to explain the meaning in their own words. This is where the focus of the BiLe video study comes into play: Many pupils need help to enunciate explanations of technical representations with the necessary precision, and teaching practices of language support are required. With regard to the theorem of Pythagoras, Drollinger-Vetter points out that the theorem consists of various components that have to be understood in order to fully comprehend the theorem. The pupils have to understand, for example, that they are talking about right-angled triangles only and that the theorem contains a precondition and an assertion. Drollinger-Vetter calls such components “elements of understanding” and claims that they have to be adequately interlinked in teaching and learning [4], p. 186 *et seq.*, t.f.G.]. For this reason, in the classroom it is expedient to verbalize not only the whole theorem but also the specific elements of it that need to be understood.

Hereinafter, the mathematical theorem will be looked at as a *subject-specific genre*, and the teaching of subject-specific genres as a challenge for all teachers. Drawing on the theoretical framework of functional grammar, the “notion of genre” is broad: It describes

“all the language events, spoken or written, that we participate in as members of our particular society and culture. [...] [A] genre is shared by members of the same culture and is ‘recognized’ by them as a genre” [11], p. 109].

This definition helps us to understand the importance of making the different genres of school subjects accessible to pupils who are not yet members of subject-specific cultures. A teacher of mathematics knows the subject-specific rules, including linguistic norms, of a mathematical theorem implicitly. But “explicit teaching” of these rules [11], p. 109] is necessary to enable the pupils to take part in the culture of mathematics.

2.2. Teaching Practices and Classroom Discourse

Findings from video-based classroom observations in the BiLe study show *how* teaching practices can support pupils’ access to subject-specific genres such as an ancient didactic poem [9] or a behavioral description [7]. Theoretical concepts that help us to understand and analyze such supportive teaching practices are a model for “Introducing and Scaffolding Genres in the Classroom” [11], p. 114 *et seq.*], the concept of “Academic Discourse Practices” [14] and the concept of “Dialogic Teaching” [1]. The model is directed at the macro-level of classroom organization. The two concepts are concerned with interaction at the micro-level of the classroom. The analysis of teaching practices of academic language support includes both the macro- and the micro-level of the classroom. All three theoretical concepts cited provide a basis for the analysis of the Pythagoras lessons in this article.

Introducing and Scaffolding Genres in the Classroom

Macro-scaffolding, according to Gibbons, is directed at classroom organization, namely lesson planning and the flow of lesson phases. The construction of different lesson phases is supposed to relate technical and linguistic learning to one another in a sensible way and to facilitate transitions between registers. The underlying idea is that “planning a sequence of tasks that require a gradual shift along the mode continuum⁵ is one way to move from familiar everyday language toward the abstractions of academic texts” ([11], p. 155). The specific model “Introducing and Scaffolding Genres” is “particularly relevant for content-based language teaching” ([11], p. 114) and therefore for all subject teaching. It comprises four phases: *Building the Field*, *Modeling the Genre*, *Joint Construction* and *Independent Writing*. In the following, I will explain the phases and illustrate a possible implementation of the model using the example of a biology lesson from the BiLe video study, where the subject-specific genre is that of a behavioral description [7]. The illustration shows a slight variation of the model.

According to Gibbons, during the first phase, *Building the Field*, the teacher supports the pupils’ information-building about the writing topic. The phase centers on the content and not on the language. However, one of its aims is to introduce specific vocabulary and technical terms. In the biology lesson, the teacher announces “*an exciting theme*”, the new subject of behavioral biology. In a short discussion the meaning of *behavior* is negotiated. A ten-second film featuring a monkey is shown seven times. The pupils are asked to take notes on the behavior of the monkey. The teacher announces that these notes are meant to be the basis for writing down a behavioral description later on.

The second phase, *Modelling the Genre*, focuses on language, especially on the form and the function of the subject-specific genre. Appropriate model texts are discussed. The phase lends itself to metalinguistic activities: talking about linguistic features of the genre. In the biology lesson, the pupils examine the text of an unknown pupil describing the behavior of the monkey in the film. In pair work, the pupils highlight successful and not so successful wording in the unknown pupil’s text, and they discuss rules for a successful behavioral description.

The third phase, *Joint Construction*, focuses on both content and language. The teacher and the pupils jointly work on appropriate wording, according to Gibbons’ model. Both aspects, the process (i.e. the formulation) and the product (i.e. the text) are important. Co-constructive interaction (which I will come back to) is a special characteristic of this phase. In the biology lesson, the *rules* of a successful behavioral description are discussed (but not yet the behavioral description itself). The teacher collects suggestions on the blackboard (keywords) and these are evaluated in a whole-class discussion. This process does not result in a common text, but in a common blackboard design.

Gibbons claims that the pupils are well prepared by the first three phases to formulate and write on their own in the fourth phase, *Independent Writing*. The pupils are then asked to consider the demands of the subject-specific genre. In the biology lesson, the pupils do not yet write a

behavioral description, but instead they explicitly deal with the demands of the genre. They write down, in whole sentences, the rules of a successful behavioral description.

As the example of the biology lesson from the BiLe video study illustrates, Gibbons’ model can be used to create an awareness of different ways of introducing subject-specific genres in the classroom. The purpose is not to verify whether the model is fully implemented, but to investigate the variations of the model that are employed for academic language support. The same approach will be followed in the analysis of the Pythagoras lessons.

I now come to teaching practices on the micro-level of classroom interaction and introduce the concepts of Academic Discourse Practices and Dialogic Teaching.

Academic Discourse Practices

Vivien Heller and Miriam Morek [14] introduce the concept of academic discourse practices in demarcation from “academic language”. They suggest the term *academic discourse practices* to emphasize the importance of “socioculturally evolved, routinized procedural solutions for recurrent communicative problems” (“practices”, [14], p. 181) and “discursive activities above the sentence level” (“discourse”, [14], p. 181).

“By ‘academic discourse practices’ we refer to spoken and/or written communicative procedures that are situated in educational activities and serve institutional purposes of knowledge construction and knowledge transfer” ([14], p. 182).

Heller and Morek point out that the pupils do not simply have to acquire linguistic tools of academic language such as syntactic constructions. They refer to research that rather shows that “difficulties can be traced back to their [the pupils’, S.F.] inability to ‘contextualize’” ([14], p. 182). The authors emphasize the importance of *contextualization competence*, not only for conceptualizing academic language practices, but also for teaching and learning. This corresponds with the conclusions drawn from Halliday’s functional linguistics perspective (see section 1.1). Pupils have to learn to “adequately interpret social and communicative contexts in terms of their local and/or global sequential implications for the utterances to be produced” ([14], p. 182). This is a useful perspective for the analysis of teaching practices of academic language support.

The concept of academic discourse practices is well-suited for video-based observation of classroom interaction because it captures the dynamics and social contexts of academic language use. Linguistic norms of schooling are established in everyday classroom interaction, and the video-based analyses in the BiLe study focus on the reconstruction of linguistic norms implied or explained by the teachers during their regular subject teaching activities. In the analysis of two Latin lessons taught by the same teacher, the concept of academic discourse practices was applied to the classroom activity of language observation [9]. We analyzed *how* the teacher draws the pupils’ attention to linguistic features and language use in functional contexts of subject teaching. Through analysis of a whole-class discussion of a translation of an ancient didactic poem from Latin to German we reconstructed that the teacher engaged the pupils in the following three *academic discourse practices*:

1. Text revision on the level of linguistic expression (example: teacher says “*Now we have to see if we can improve the way we express some points*”).

⁵ Gibbons relies on M.A.K. Halliday’s definition of mode differentiating between written and oral texts and considering the corresponding linguistic features on a mode continuum.

2. Translation between linguistic registers (example: teacher says “*What does that mean? Perhaps we can just put it in your own words again*”).

3. Explanation of and reasoning behind proposals of formulations (example: teacher says: “*‘simulate’ fits in quite well here, doesn’t it? In the context of ‘how genuine are these feelings really’*”. A pupil interrupts the teacher: “*But I think ‘feigned’ is better with regard to feelings. In my opinion it seems to fit better*”).

It was concluded that these are three socially established *practices of language observation* that help the pupils to gain access to the subject-specific genre of an ancient didactic poem. The assumption was made that such practices of language observation support the pupils’ access to subject-specific genres not only in the teaching of languages, such as Latin, but also in any other subject lessons, including mathematics.

Dialogic Teaching

Pupils can only acquire academic discourse practices if they have the opportunity and are encouraged to take part in classroom interaction. From classroom research we know that in classrooms all over the world this is far from being the norm because teacher/pupil exchanges are teacher-centric [1], p. 14 [18], p. 152]. This includes allocation of the right to speak by the teacher, major share of speaking by the teacher and typical question and answer behavior in the form of IRE (Initiation, Response, Evaluation) discourse. Whole-class discussions, in particular, are characterized by this pattern of interaction and the corresponding teaching activities. In this article, teacher-directed class discussions from the Pythagoras study will be analyzed. Christine Pauli [18] calls attention to the fact that there is great potential for development in the style of teacher-directed class discussions. This would include expanding the IRE discourse and developing co-constructive discussion patterns. Even in teacher-directed class discussions it is possible for teachers to take pupil contributions and “expand and accentuate them and make them the point of departure for further consideration” [18], p. 149, t.f.G.]. As we have found in the interpretive video interaction analyses of the BilLe study [2,6,9], expert teachers move on from “the traditional discouragement of pupil-(to)-pupil talk in whole-class, chalk-and-talk teaching” [17], p. 89].

The concepts of *Educated Discourse* [17] and *Dialogic Teaching* [1] reflect the way interaction with pupils is structured and encouraged by the teachers filmed in the BilLe study very well. Neil Mercer defines discourse as “language as it is used to carry out the social and intellectual life of a community” [17], p. 79]. He distinguishes between *educational discourse* and *educated discourse*. Teachers’ engagement in *educational discourse* corresponds to their institutional role in the classroom: In educational discourse, teachers instruct, evaluate and guide the pupils’ learning. It is widely agreed that pupils in school acquire educational discourse and learn to “use language in some typical and conventional ways” [17], p. 79]. There is no schooling without educational discourse. However, the BilLe teachers were observed to additionally engage in *educated discourse* with their pupils quite frequently and by that, according to Mercer, to pursue “the important goal of education”:

“It is to get students to develop new ways of using language and to think and communicate, ‘ways with

words’ which will enable them to become active members of wider communities of *educated discourse*.” [17], p. 80, emphasis in original].

To pursue this goal means, first of all, to listen carefully to the pupils and then to open up opportunities for them to develop and practice new forms of language use and thinking. We can regard *dialogue* as an important feature of educated discourse. Robin Alexander [1] refers to Mikhail Bakhtin’s concept of dialogue and applies it to classroom talk:

“For Bakhtin dialogue is essential to discourse – to a world where meanings are neither fixed nor absolute, and where the exchange, acquisition and refinement of meaning is what education is centrally about. Indeed [...] dialogue is about helping children to locate themselves within the unending conversations of culture and history” [1], p. 25].

According to Alexander, in classroom talk “dialogue becomes not just a feature of learning but one of its most essential tools” [1], p. 25]. Alexander describes five important features of dialogic teaching [1], p. 28]: Dialogic teaching is collective, reciprocal, supportive, cumulative and purposeful. A focus of the analysis of the Pythagoras lessons will be on the *cumulative* quality of teaching and learning: “teachers and children build on their own and each other’s ideas and chain them into coherent lines of thinking and enquiry” [1], p.28]. The extent to which teachers and pupils in the Pythagoras lessons engage in cumulative dialogue to develop a common understanding of the theorem of Pythagoras and its singular elements of understanding will be investigated. At issue is whether teachers enable pupils to access the community of mathematical discourse by supporting their participation in dialogue.

3. Analysis of Mathematics Lessons from the Pythagoras Study

Based on the theoretical concepts presented in the first section, the following analysis of teaching practices of academic language support considers the lessons from the Pythagoras study on the macro- and micro-levels. The following questions guide the analysis.

Macro-Scaffolding: How does the flow of the lesson phases support the linguistic and technical comprehension of the mathematical theorem?

Academic Discourse Practices: How is the teaching and learning of the mathematical theorem embedded in academic discourse practices of language observation?

Dialogic Teaching: How do the teachers involve the pupils in a co-constructive and cumulative dialogue?

Methodical Procedure

The database comprises the films and transcriptions of six mathematics lessons in total: three lessons from a classroom in a ninth grade of a German *Gymnasium* (henceforth referred to as Piepenbrink class) and three lessons in an eighth grade of a higher-track school in Switzerland (henceforth referred to as IKEA class).⁶ The

⁶ The names “Piepenbrink class” and “IKEA class” refer to the examples from outside the world of mathematics used in the lessons to illustrate Pythagoras’s theorem. I chose these terms because I do not want to refer to the national settings (‘German’, ‘Swiss’) that are irrelevant for my analysis.

films and transcriptions were selected and provided by the research group of the Pythagoras study and the transcriptions used for my analysis were translated from German to English. I have included in the analysis only the lesson phases focusing on the theorem of Pythagoras itself. Not included were lesson phases involving calculations and exercises or phases dealing with the mathematical proof of the theorem. Analysis of the selected phases from the two classes will show that comparatively, more promising teaching practices of academic language support can be observed in the Piepenbrink than in the IKEA class.

For analysis on the macro-level, lesson phases were defined on the basis of Gibbons's model 'Introducing and Scaffolding Genres in the Classroom' (see section one). The purpose is not to verify whether the phases in the Pythagoras lessons are congruent with those of the model. But rather the model serves to sensitize us to the question as to *how* the phasing of the lessons supports the pupils' access to the mathematical theorem. The differentiation of the phases is based on video-based observation as well as on close reading of the transcription texts.

On the micro-level of the lessons, interpretive video interaction analyses [15] were conducted to foster a deeper understanding of the nature of academic discourse practices of language observation and classroom interaction, taking special account of the concept of Dialogic Teaching (see section one). From each class, sequences from teacher-directed whole-class discussions about the wording of the theorem of Pythagoras were chosen for sequence analysis with a focus on teaching practices. Sequence analysis was performed according to the following principles: the sequences are interpreted word by word. Thus, what was said before is taken into consideration for interpretation, but not what is said subsequently. The actual wording of the teachers' utterances is compared with other possible wordings for the purpose of discussing what is actually said as opposed to what could have been said. In order to achieve a triangulation of researchers' perspectives, different readings were discussed in the working group of the BiLe video study. The transcriptions of the analyzed sequences are appended to this article. Within the space limitations of this article, selective data (short extracts from the transcriptions) are used to exemplify the findings from the sequence analyses.

3.1. The IKEA class

3.1.1. Macro-Scaffolding

In the IKEA class, I distinguish three phases according to Gibbons's model 'Introducing and Scaffolding Genres' (see section one) in the following order: 1. Building the Field, 2. Independent Writing, 3. Joint Construction. In a deviation from Gibbons, the pupils have to produce a written form of the subject-specific genre on their own *before* the genre is discussed in the classroom (Joint Construction). Also, the phase Modeling the Genre as suggested by Gibbons does not occur as a distinct phase in the IKEA class.

The first phase, **Building the Field**, takes about 30 minutes. The teacher opens access to the field by taking a practical approach (the IKEA case) to the mathematical theorem. He presents the problem that an IKEA wardrobe

cannot be put up in a room because it touches the ceiling when being turned around. The pupils work on the problem in pairs using different methods. After that, in a whole-class discussion the length of the diagonal is found to be the crucial factor, and in the 23rd minute of the lesson the teacher sums up the *mathematical* problem: "*So in the end the problem is to determine the longest side of a triangle that has a right-angle if we know these two dimensions*". While speaking the teacher draws a right-angled triangle on the blackboard and points to the hypotenuse and the other two legs of the triangle (*Katheten*).

Some of the utterances that follow go beyond "Building the Field", but joint construction still does not take place: Even though the teacher has not yet named the theorem of Pythagoras, a pupil calls the theorem by its name and gives its symbolic-formulaic representation: "*A squared plus B squared equals C squared. That's Pythagoras's theorem*". The teacher reacts by pointing out that there is indeed a "*rule that says something about the squares on the sides*", and the teacher draws the squares on the sides of the triangle on the blackboard. The pupil repeats the formulaic representation. At this point, the teacher brings two technical representations of the theorem together, the iconic representation and the formulaic representation, by writing the formula on the blackboard too. Then he returns to the *practical* problem of the wardrobe without the theorem having been formulated in generally understandable words. Because of this lesson phase, the practical problem of the IKEA wardrobe is solved, and the teacher explains: "*This is a practical example of using Pythagoras's theorem*". The opportunity to translate between linguistic and technical registers was not taken, nor did the pupils have a chance to formulate the mathematical theorem in words. Even though two technical representations of the theorem were introduced and recorded on the blackboard (the iconic and the formulaic), there was no translation into informal or academic language.

In the second phase, **Independent Writing**, the pupils are asked to perform this translation individually by writing in their notebooks. The teacher prompts: "*formulate [...] the theorem of Pythagoras in words, in your own words*". He puts a heading for the pupils' version on the board: "My version". The teacher circulates in the classroom while the pupils are writing and he evidently finds that the formulation does not come easily to the pupils. Several pupils write down the formulaic representation instead of expressing the formula in words. Three times the teacher repeats his request to formulate in words to individual pupils. Finally, the teacher addresses all pupils: "*I can see that you are still finding it difficult to put the formula into words. So try to imagine that you don't have a drawing, a sketch you can refer to. Just formulate a sentence that expresses the same thing as the picture and the formula beneath it.*" At this point, the teacher explains the (far from simple) linguistic demands of the exercise explicitly: translation between non-verbal and verbal technical registers and the context-reduced quality of academic language. The teacher's explanation comes too late for the pupils, because it's time for a break and the Independent Writing phase is over.

The second lesson, after the break, begins with the third phase, **Joint Construction**: a whole-class discussion about the formulations the pupils have written down in

their notebooks. This discussion is analyzed on the micro-level of classroom interaction.

3.1.2. Extracts from the Sequence Analysis: Discourse Practices and Classroom Interaction

During the lesson phase of Joint Construction in the IKEA class, formulations are discussed by the whole class (phase of seven minutes and 40 seconds, see Appendix 1 for transcript). The following presentation of the analysis concentrates on the question as to whether and how a correct formulation is developed jointly. The element of dialogic teaching which is especially relevant is the *cumulative* course of the conversation (see section one). In the extracts from the transcript, the details important for the **cumulative** course of the formulation are put into **bold** letters.⁷

T: Right, I'll be very interested to hear your formulations. It's quite a challenge, to formulate something like that, ... a formulation to reflect the sentence we wrote on the board. ... So, ... **let's try**. ... FLORIAN, what did you write?

Thus, the teacher opens the phase of Joint Construction. He says he is interested in the pupils' formulations, he points out that the task is demanding and he announces a joint proceeding ("let's try"). He asks a pupil to read his version aloud. The pupil reads and the conversation proceeds:

P: Where the right-angle is on a right-angled triangle, C squared can be calculated with the formula: A squared plus B squared equals C squared.

T: Good. What do you all say to that? ... Short comment, IVO.

P: Well, you should write in words somehow, not an equation.

How could the teacher react at this point in order to support the cumulative work on the formulation? He could say, for example, "*Yes, indeed, you were asked to formulate in words... Florian has already made a start ... Florian said, ... How could the sentence go on?*". The teacher in the IKEA class actually says:

T: Mmm [yes]. ... But it contains good elements, **you started really well. Continue** ... another version. -Eh- (BLUCIN).

The teacher mentions "good elements" but does not specify what he finds good in Florian's formulation. He asks the pupils to continue, which could mean to take up Florian's suggestion. Instead, the teacher asks for "another version" and thus stops the pupils revising and developing the first formulation. The pupil Blucin gives a further suggestion, and the conversation continues:

P: Eh - the square of the one side plus the square of the other side is the square of the third side.

T: Very interesting. What do you think? ... (MIKE)

P: Well () **much easier to understand than the other one**. So () - anyone who doesn't - anyone who doesn't have much idea would maybe understand it better.

[Omission: Misunderstanding between pupil and teacher, see transcript in Appendix 1]

T: Good. And is it right?

The teacher finds the new suggestion "interesting". A pupil (Mike) compares the new suggestion with the first formulation. This can be evaluated as an attempt at co-construction. After a misunderstanding between Mike and the teacher is clarified, the teacher asks very openly: "And is it right?" This question draws attention away from the formulation of the statement (the real purpose of the discussion) and focuses on the content of the statement (technical level). The pupil Mike doesn't follow the teacher's train of thought. He remains on the language level:

P: Well - I just wouldn't have written the square of the first part, I would have written the first side. (In contrast) the one from FLORIAN with the right-angled triangle would have been - umm - been good, **so a combination would almost have been the best** of the two.

Mike's purpose is to formulate the sentence. He makes a constructive proposal, which is to combine both formulations put forward until then. Implicitly he hints at the importance of one of the elements of understanding of the theorem, which is the precondition 'right-angled triangle'. Mike has recognized that this element of understanding was included only in the first formulation, the one from Florian. To support a co-constructive and cumulative procedure, the teacher could at this point take up Mike's attempt to consider all the elements of understanding by co-construction. Instead, he ignores this effort and switches again from the linguistic to the technical level. He does take up the precondition 'right-angled triangle' and asks:

T: Does the statement only apply to a right-angled triangle?

At this point, a conversation follows about the element of understanding 'right-angled triangle' which I will not elaborate on in this article. Some utterances later, *a pupil* draws the attention back to the purpose of formulating the theorem in words.

P: Well, I think (BLUCIN) (), he doesn't even mention that it has to be a triangle, or... It could just as well be a (pentagon) and (you just take) three of its sides.

This utterance offers an ideal opportunity to return to the linguistic level, resume the formulation work and finally make clear that the element of understanding 'right-angled triangle' is a precondition of the statement and must certainly be included in any formulation. Instead, the teacher points out that the formulation to which the pupil refers was good simply because it is expressed in words.

T: Good, but it is formulated completely in words, so from that point of view it's - umm - good. Let's keep that - mmm - in mind- PHILIPP.

The teacher then calls up another pupil who reads out a further version in which the element of understanding 'right-angled triangle', the precondition of the assertion, does not occur either. Neither is this element of understanding explicitly taken up at any point in the course of this lesson phase as a necessary element of the theorem. In spite of this, towards the end of the phase a pupil proposes a formulation, which the teacher finds "*not bad*". This pupil formulates: "*In a right-angled triangle the area of the Katheten squared is the same as the*

⁷ T = Teacher, P = Pupil; (...) = Speech in brackets is hard to understand; ... = long pause in the flow of speech; - = short pause in flow of speech.

hypotenuse squared.” Nevertheless, the teacher does not take this formulation up. He says:

T: Good. That’s not a bad way of putting it. -Eh- we’ll write down a conventional version. Let’s say – uh you could call it the teacher’s version. And I’ll dictate it.

The teacher asks the pupils to differentiate between “my version” (the pupils’ own version) and “teacher’s version” (see Figure 1). He then dictates the “teacher’s version”: “*In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the Katheten*”. As a result, all pupils have two versions in their notebooks: first, in most cases, an incomplete and formulaic version of their own and second, under the heading “teacher’s version”, the version dictated by the teacher. Co-construction and cumulative work on the formulation have scarcely taken place and the pupils have not been supported in revising their own drafts.

Incidents in the further procedure of the lessons indicate that the pupils have not learnt to formulate the theorem of Pythagoras in words. At the beginning of the third lesson, for example, pupils are asked to repeat the statement and some pupils again do not include the precondition, the element of understanding ‘right-angled triangle’, in their wordings.

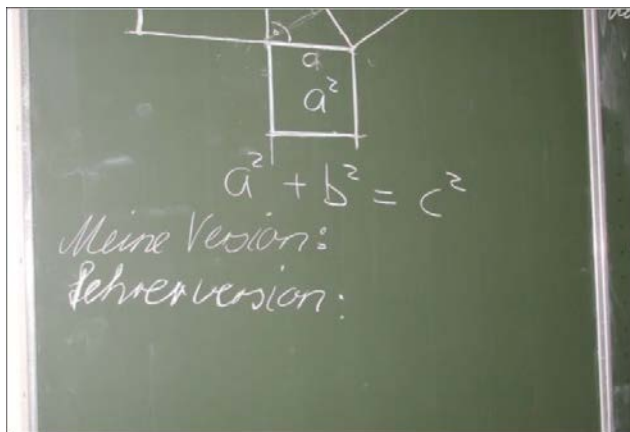


Figure 1. Blackboard IKEA class

3.2. The Piepenbrink Class

3.2.1. Macro-Scaffolding

The order of the lesson phases in the Piepenbrink class conforms more to Gibbons’s model than in the IKEA class. I distinguish three phases in the following order: 1. Building the Field, 2. Joint Construction and 3. Joint Writing (instead of Independent Writing). Just as in the other example, there is no distinct phase of Modeling the Genre.

In the **Building the field** phase this teacher, too, uses a practical example to facilitate access to the problem: He tells the story of farmer Piepenbrink and two other farmers. The problem that will lead to the theorem of Pythagoras is connected with the sizes of the farmers’ fields.⁸ The pupils work on the problem in groups of three. In this class, too,

⁸ The farmers can exchange two small square fields for an adjacent, large square field. Farmer Piepenbrink’s two small squares together equal the area of the large square, but this is not true for the other two farmers. The sizes of the fields are dependent on the triangles between the fields. Thus, attention is drawn to the special properties of a right-angled triangle.

a whole-class discussion follows. To start the discussion the teacher puts a slide onto the overhead projector showing “Piepenbrink’s situation” as he calls it. It shows an iconic technical presentation of the theorem of Pythagoras, but the title of the theorem is not given. Instead, the class deals with a condition (“Farmer Piepenbrink’s situation”) and then works on a statement concerning the sizes of the fields. Towards the end of the discussion the teacher explicitly emphasizes the importance of the right angle in the triangle, i.e. he specifically points out an element of understanding of the theorem, namely the precondition (see section one). At the end of the lesson phase, the teacher also explicitly introduces the technical terms “hypotenuse” and “Katheten”, supported by a drawing and writing on the blackboard. At this point, the teacher translates between technical language, informal language and academic language: “*So we don’t always have to say this side and that side*”, and “*terms have been agreed to distinguish between the sides that are next to the right-angle and the side that does not touch the right-angle, the one opposite.*” Furthermore, the teacher translates from Greek into German, which is also an example of language observation: “*if you know Greek you’ll know that hypo means under and hypotenuse is the subtending line or the line beneath.*”

As proposed by Gibbons, the phase of **Joint Construction** follows next. The teacher introduces the phase by asking: “*Now, what would be the assertion here?*” The joint work on formulations during this phase is analyzed on the micro-level (section 2.2.2).

The second phase results in a joint formulation, which the teacher writes down on the blackboard and the pupils write into their notebooks. Therefore, I am calling this writing phase **Joint Writing**, rather than Independent Writing. The teacher and the pupils write: “*In a right-angled triangle the areas of the squares on the Katheten equal the area of the square on the hypotenuse*”. The title on the blackboard is simply “*Statement*”. Pythagoras’s theorem as such is only introduced in the third lesson, which is not analyzed in this article.

3.2.2. Extracts from the Sequence Analysis: Discourse Practices and Classroom Interaction

In the Piepenbrink class, the phase of Joint Construction lasts seven minutes and 28 seconds. During this phase, elements of dialogic teaching and academic discourse practices of language observation can be observed.

In the transcript (Appendix 2) all utterances that support the **cumulative** course of the co-constructive formulation are in **bold** letters (cf. also an analysis of a shorter sequence of this phase [18, p. 157 *et seq*]). The following different ways in which the teacher stimulates the cumulative course of the discussion were identified:

1. The teacher begins with a formulation, a pupil continues (or vice versa). Example: “*so, if we start a triangle – a right-angle – can we continue it? [...] If, Simon!*” (line 18 *et seq*).

2. The teacher explicitly refers to earlier utterances and formulations. Example: “*Daniel formulated it just now*” (line 8).

3. The teacher offers a pupil (Julia) a second go at formulating (line 35). In an interview, which was conducted immediately after the lesson the teacher said

that Julia was a weak pupil and that he had given her a second opportunity on purpose.

4. The teacher requests precise formulations. Example: “*Can we formulate it even better?*” (line 30 *et seq.*).

5. The teacher draws attention to the importance and meaning of words. Example: “*The word together is good, right?*” (line 48).

The examples given above and the whole transcript (Appendix 2) make it evident that two of the three academic discourse practices of language observation introduced in section one occur in the Piepenbrink class: text revision on the level of linguistic expression (1) and explanation of and reasoning behind proposals of formulations (2). Another observation of an expedient procedure, from the perspective of mathematical didactics (see section one), is that the teacher explicitly differentiates between the formulation of the precondition and the assertion.

The third academic discourse practice of language observation – translation between linguistic registers (3) – is partially realized. Some steps can be observed with respect to the transition between informal language use and academic language use. For example, the teacher asks the pupils to attempt context-reduced formulations to achieve abstraction (in this case abstraction from the actual drawing of the triangle and the squares): “*Could we try to formulate it without the drawing?*” (line 64 *et seq.*), and he explains, “*Whether I draw it or whether I mean the length is something different*” (line 82). The teacher also supports the use of condensed language – for example, “*right-angled triangle*” instead of “*a triangle with a right-angle*” (line 36 *et seq.*). He explains, “*That sounds better, and it's better German, certainly better*”. In this case, the teacher could have explicitly emphasized the importance of a technical term (right-angled triangle) instead of calling on pupils’ implicit or even intuitive knowledge of language (“*sounds better*”). Overall, there is no explicit differentiation between the registers of academic language and mathematical technical language. The teacher’s exclamation “*We should end up with a proper German sentence, shouldn't we*” (line 19) particularly calls attention to this problem. After all, the purpose of the lesson is to end up with *technical* representations of the theorem, including the verbal one (see section one).

Nevertheless, incidents in the further course of the lessons indicate that the pupils have learnt to formulate the theorem of Pythagoras in words in the Piepenbrink class. At the beginning of the third lesson, a repetition of the sentence is requested (just as in the IKEA class) and (unlike in the IKEA class) the complete theorem, including the precondition, is formulated by a pupil in words, without reading. In addition, further data from the Pythagoras study indicate that the pupils of the Piepenbrink class have gained, in comparison with other classes, an above-average understanding of Pythagoras’s theorem.

4. Discussion

The analysis of lessons from the Pythagoras study has provided insights into the realization of teaching practices of academic language support in mathematics lessons on the macro-level and on the micro-level. The analysis has

shown *how* such teaching practices can support the pupils’ acquisition of a subject-specific genre, the mathematical theorem, namely the theorem of Pythagoras. The findings indicate that teaching practices of academic language support are useful to foster not only pupils’ linguistic learning and development, but also their comprehension of mathematical content. I will finish with two points for discussion.

1. Teaching practices of academic – and technical – language support are a challenge for the training of teachers of all school subjects. First, teachers have to become experts in their subject-specific genres. The examples from the Pythagoras study make clear that this is a precondition especially for teaching practices of technical language support. It is probably no coincidence that in both the classes analyzed in this article, the lesson phase ‘Modeling the Genre’ proposed by Gibbons [11] was not observed. However, the course of discussions in the classrooms indicates that the metalinguistic discussion of the requirements of a mathematical theorem, as proposed by Gibbons, would have been helpful to many of the pupils (for example, a discussion of the requirement for ‘precision’, technical terms and of the necessary formulation of precondition and assertion). According to Gibbons, “if teachers have an explicit understanding themselves of what an effective piece of writing in a particular genre looks like, it is much easier to make this explicit to learners” [[11], p. 113]. At some points, analysis of these lessons on Pythagoras’s theorem gives rise to doubts that the teachers have this necessary explicit understanding. Moreover, these teachers are certainly not alone in this. It is clearly an area of teacher training and further training that requires our attention.

2. There is no specific target group – all pupils profit from teaching practices of academic – and technical – language support. As pointed out in the introduction to this article, in educational science the idea of academic language teaching is discussed above all with regard to pupils from multilingual migrant families. There is no doubt that the educational disadvantages of this group are a challenge for the development of schooling and teaching. Nevertheless, teaching practices of academic language support should not be limited to selected target groups. The analyses from classes in a German Gymnasium and a Swiss higher-track-school reported in this article have made this abundantly clear. Interestingly, in some respects these classes are the exact opposite of the classes filmed in the BiLe video study (cf. footnote 3). The pupils come from socially privileged families, their parents have a high level of formal education, and German is the only language spoken in the families. It is, however, evident that these pupils, despite their assumed familiarity with academic discourse practices at home, also need support to acquire linguistic function and learn the various uses of language [13].

5. Conclusions

Comparing the classes analyzed in this article with the classes filmed in the BiLe video study, I conclude that the important difference is not the social background of the pupils. It is the difference in teaching practices. All the expert teachers filmed in the BiLe video study have long

years of experience in teaching multilingual pupils from migrant families. For them, the reasons for intertwining linguistic and content teaching and learning in all school subjects are self-evident. All of them have, in cooperation with their colleagues, developed strategies of teaching academic language. Due to widespread migration and diversification processes, classes everywhere are becoming progressively multilingual and socially diverse [5]. In addition, the analyses in this article show that expertise in teaching practices of academic and especially technical language support is essential in the classrooms of all schools for all pupils, regardless of their social and linguistic backgrounds. For these reasons, teaching practices of academic and technical language support are an issue for further research in mainstream classrooms of all school subjects.

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Appendix 1: transcript IKEA class, third phase: Joint Construction

Appendix 2: transcript Piepenbrink class, second phase: Joint Construction

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Teaching practices of academic language support in mathematics lessons.

Appendix 1: IKEA class, third phase: Joint Construction (7 minutes, 40 seconds)

T = Teacher, P = Pupil

(...) = Speech in brackets is hard to understand

... = long pause in the flow of speech

- = short pause in flow of speech

- T Right, I'll be very interested to hear your formulations. It's quite a challenge, to formulate something like that, ... a formulation to reflect the sentence we wrote on the board. ...So, ... let's try. ... FLORIAN, what did you write?
- P Where the right-angle is on a right-angled triangle, C squared can be calculated with the formula: A squared plus B squared equals C squared.
- T Good. What do you all say to that?... Short comment, IVO.
- P Well, you should write in words somehow, not an equation.
- T Mmm [yes]. ... But it contains good elements, you started really well. Continue ... another version. -Eh- (BLUCIN).
- P -Eh- the square of one side plus the square of the other side is the square of the third side.
- T Very interesting. What do you think? ... (MIKE).
- P Well () much easier to understand than the other one. So ()- anyone who doesn't – anyone who doesn't have much idea would maybe understand it better.
- T You don't find it understandable.
- P Well not this one here, that is more understandable than the one before, I think – well for somebody looking it up in a book, for instance, or in a – umm – big () of formulas.
- T Good. And is it right?
- P Well – I just wouldn't have written the square of the first part, I would have written the first side. (In contrast) the one from FLORIAN with the right-angled triangle would have been - umm- been good, so a combination would almost have been the best of the two.
- T Does the statement only apply to a right-angled triangle? ... Yes.
- P No.
- T Always applies, in every triangle? ... Aha, yes.
- P If you have an equilateral triangle - so one where the sides are all the same length, then it surely won't work.
- T Aha. Interesting. ... So it's not true in an equilateral triangle, is it, all three squares are likely to be the same size. ... Yes?
- P Actually, only with a right-angled triangle.
- T What do you think, THIERRY.
- P Yes. ()
- T Are you starting to have doubts?
- P Yes ... with an equilateral
- T Uh, yes?
- P Well, I think (BLUCIN) (), he doesn't even mention that it has to be a triangle, or... It could just as well be a (pentagon) and (you just take) three of its sides.
- T Good, but it is formulated completely in words, so from that point of view it's – umm - good. Let's keep that - mmm – in mind - PHILIPP.
- P Well, I wrote: the height squared plus the width squared equals the diagonal squared.
- T Aha. You say the height of the triangle, which is the height of the triangle?
- P -Uh- the, the... – well left. The ri- well the one with the right-angled, that goes up here. Yes. It could be both, it doesn't matter which one it is, just – just the one that is at the right-angled – the one by the right-angle.
- T And this one here?
- P -Umm - yes, it's also – it's – that's the other one.
- T So this side this is the height, on side B this is the height, (because it stands at a right-angle and goes to the corner). But on side C the height is this one here.
- P Yes. But I was referring to the – to a cube – like – or to a cuboid.
- T Or to a rectangle!
- P Yes.
- T Yes?
- P (This side is the) hypotenuse, the longest side of the triangle.
- T What is it called?
- P Hypotenuse, the longest side of the triangle.
- T Exactly. And what are the others called?
- P -Uh- they're A, B ... just A and B.
- T Mmm. ... Who can remember?
- P The *Katheten*.
- T Aha. So with the aid of these special words perhaps it will be easier to formulate the sentence.
- P (Yes, but if somebody doesn't know) ()

- T But these are mathematically defined terms, now, that's what we are assuming. So, who can find a formulation with the hypotenuses and squares? ... With hypotenuses and *Katheten*, yes.
- P Well: in a right-angled triangle the *Katheten* squared are the same size, the same size area as the hypotenuse squared.
- T Good. That's not a bad way of putting it. -Eh- we'll write down a conventional version. Let's say -uh you could call it the teacher's version. And I'll dictate it, just write it beneath your version. In a right-angled triangle ... in a right-angled triangle ... the square on the hypotenuse ... How do you spell hypotenuse?
- P With a Y.
- T Yes. And?
- Ps TH.
- P TH.
- T No. Just one H, heh. Hypotenuse has an H at the beginning, but not one after the T. ... So, the square on the hypotenuse is equal to the sum of the squares of the *Katheten*. ... Equal to the sum of the squares of the *Katheten*.

Teaching practices of academic language support in mathematics lessons.

Appendix 2: Piepenbrink class, second phase: Joint Construction (7 minutes, 28 seconds)

T = Teacher, P = Pupil

(...) = Speech in brackets is hard to understand

... = long pause in the flow of speech

- = short pause in flow of speech

Bold: Statements that contribute to the cumulative course of the exchange.

- T Umm, again the question, what is the statement here? **We said** the combined areas are the same size as the lower area. Under what conditions? **DANIEL formulated it just now.**
- P ()
- T Yes, squares, that's right, good. **First, squares, what else, KATHRIN?**
- P **(That the angle) ()**
- T **That the angle** is a right-angle – **how could we - um - how could we formulate that in one expression? ... You start, JULIA.**
- P Um- if-
- T **If is a good start**, it introduces the – the condition, **if what? Right. So if what?**
- P The () ...if in a triangle a right-angled – um – no – no... Can that-
- T I think so. **In mathematics we always take – we always approach the answer in stages**, so it's not a problem. So, if we start a triangle – a right-angl- eh- right-angle- **can we continue it?** We should end up with a proper German sentence, shouldn't we? ... **If, SIMON!**
- P **If** a right- well, if a triangle has a right-angle and ... you draw squares on the two sides -
- T Mmm!
- P If the- ... if the areas of the two squares are the same - um – well, has the same area as the square of the hypotenuse, **or something.**
- T **Or something. Or something, okay!** Can we leave, **if a square has a right-angle, you said, then** – and you draw the squares over- on the sides, hm? **Did you say something?**
- P (Over the squares... -)
- T No, if a triangle has a right-angle, **you're right, so -**
- Ps {laughter}
- T If a triangle ...has a right-angle, and you draw the squares on the sides, the squares have the same area as the square on the hypotenuse. Uh, so – **can we formulate it (simp-) maybe- even better, ALICE?**
- P I would say, if a triangle has a right-angle –um – (and you) draw squares on the sides and on the hypotenuse, then the areas of the two squares on the ... sides are the same size – well, together they're the same size as the – the area of the square on the hypotenuse.
- T Mmm, **JULIA, (last-) second try!** (JULIA the second)
- P Umm- a right-angled triangle (I think) well: If you have a triangle with a right-angle ()
- T **Good suggestion**, hm? That sounds better, and it's better German, **certainly better. So if you – if – what did you say again?**
- P **If you** – if you have a right-angle -
- T {laughs aloud}
- Ps {laughter}
- P A right-angled triangle, and ... a square on all the sides. So – so draw a square on each side - (on each side draw a square -)
- T Okay, if- okay, **if you draw a square on every side of a right-angled triangle,**
- P **then the - umm – the squares on the – the areas of the squares on the sides – on the sides are exactly the same size as the area of the square on the hypotenuse.**

- P **together!**
- T **Together! The word together has – um – is good, right?** They aren't the same size. Of course they're not the same size, clearly! Everybody thinks ... () that one word, together, right, then the areas of the squares on the sides together are the same size as the area of the square (on the) hypotenuse, HOLGER?
- P () a right-angled triangle, on all three sides a square () - umm – a square that – umm- corresponds to the three base areas, now that is a – a right-angled triangle, a square -
- T **You are irritated by draws?**
- P No!
- T What (inserts instead of draws), no?
- P I - (it's just because it's shorter)!
- T Yes!
- P I (keep coming back to it), **because the others always say**, how do you draw a square on the sides.
- T Mmm-
- P You could draw a smaller square, it's still a square.
- T Yes, do you need – do you have to draw the squares or can you just imagine them? So, the drawing is not the important thing, **now could we try to formulate it without the drawing? In a right-angled triangle -**
- P I think ()
- T Yes -
- P That doesn't have (to be) the corresponding side.
- T Yes.
- P It could also be - umm – you could draw a square on the side but one that's not the same length.
- T Yes, yes you could. Mmm.
- P ()
- T So, how do you do it? ... **In a right-angled triangle, what can we say? ...** It's clear what we mean when we say draw a square on the side, ... isn't it? It's not clear? **Could we do it better, does anyone have a suggestion, how we could do it better?**
- P Well, the first side squared, plus the second side squared is the hypotenuse squared.
- T What do the rest of you think?
- P It's right!
- T The length, the length of the first side squared, ... the length!
- P Yes ()
- T Not necessarily, I mean, whether I draw it or whether the length (I mean) is something different, yes?
- P Yes but the side is ()
- T The side is a segment, the side is a segment, that's what the side is here, yes? That is the side, this segment.
- Ps {general murmuring}
- T Is it the same if I say, this square here, or if I say forty square meters? ... The same if I say this segment here or if I say two meters?
- P **There's a difference.**
- T **There is a difference, isn't there?** To that extent, if you square the lengths of the sides, it's right, **which do you prefer, the formulation with the ar- with the square areas or this side square, side square?**
- P **Well, you (could) say both,** () side squared with one of those brackets.
- T **Mmm, add both, not a bad idea, yes?** So how could we say - umm - this ... in a right-angled triangle, the areas of the – **what was the last thing we said?** Of the squares ... if we can agree, that means, on the sides means, the whole length, the squares on the sides, then I think it would be clear. If you say, a square on the sides, then it's ambiguous (below). The squares on the sides, so the areas of the squares on the sides together, that wasn't it, as big as the area of – umm – the square on the hypotenuse. **Okay, we don't have – that is a sentence, that is a statement.**