American Journal of Educational Research, 2015, Vol. 3, No. 4, 535-540
Available online at http://pubs.sciepub.com/education/3/4/21

DOI:10.12691/education-3-4-21

Probability Concepts in Primary School

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Received March 24, 2015; Revised April 05, 2015; Accepted April 12, 2015

Abstract In recent years, many countries have tried to incorporate the probability concepts into the curriculum of primary school. The researchers disagree as to what the age of children dealing with probability contents should be. The aim of this study was to investigate the grade of understanding of the probability concepts in primary school students depending on their age and their gender. 404 pupils of the second to the sixth grade of elementary school participated in the study. It has been concluded that the majority of students was able to recognize different events and categorize them depending on their likelihood. The major difference in their abilities was noticed between the children of the second grade and those of the third grade whereas it has been experienced that girls performed better in all the tasks and in all the grades except for the fourth grade in which boys present a slightly better score.

Keywords: *teaching probabilities, primary school, probability concepts, probability tasks*

Cite This Article: Helen Tsakiridou, and Eleni Vavyla, "Probability Concepts in Primary School." *American Journal of Educational Research*, vol. 3, no. 4 (2015): 535-540. doi: 10.12691/education-3-4-21.

1. Introduction

Among the new development and trends in mathematics education internationally at primary level, is the introduction of probability. Nowadays, there is a need to work on the Probability Theory because we are obliged to make predictions and to take decisions under uncertain situations having to evaluate at the same time a big amount of information. Researchers from many countries (like Farnworth, 1991; Fischbein & Schnarch, 1997; Freudenthal, 1973; Gardner, 1989; Jones, 1995; Koshy, Ernest & Casey, 1999; Shaughnessy, 1992; Sobel & Maletsky, 1988), notice that probability is very important branch in mathematics education because:

- it provides opportunities for students to engage in interesting and purposeful learning activities,
- it is easily understood by students that are in different ages and have different capabilities as well as
- wherever it has been taught, the results were positive and enhanced the mathematical thinking (Skoumpourdi, 2004).

According to Fischbein (1984) the reason to introduce probability are "dealing" with uncertain situations, predicting, deciding among different probabilities (critical implementation), problem solving (deliberate action-taking) and developing the thinking ability different from the deterministic one.

Gal (2005), Franklin et al (2005) and Jones (2005) state that the reasons for including probability in schools are related to the usefulness of probability for daily life, its instrumental role in other disciplines, the need for a basic stochastic knowledge in many professions, and the important role of probability reasoning in decision making. Students will meet randomness not only in the mathematics classroom, but also in biological, economic, meteorological, political and social activities (games and sports) settings.

As evident, teaching contents in probability have numerous advantages, which other mathematical disciplines lack. Through dealing with the mentioned contents children learn to accept the fact that also negative situations can be encountered, which are not possible to be precisely predicted. The only thing to be done is to critically interpret all the possibilities and choose the one which is most likely to happen. In this way children gather experiences for real life situations, in which it is necessary to decide on the best option out of many on a daily basis. At the same time children have to accept the fact that some events are impossible to happen. So, it is necessary to act deliberately and solve the problem, whereby one should make use of his mode of thinking, different from the one applied at learning other mathematical disciplines. (HodnikCadez & Skrbe, 2011).

There was very intensive research conducted into the establishment of understanding the probability contents of the early school children in the 70-ies and 80-ies of the last century. These research works, which shall be presented in detail in continuation, were the key starting points of our research. It should be emphasized that the subject matter of understanding the probability contents with the youngest population is still topical, as it also being delved into nowadays. Let us shortly mention some such research works: Gelman and Glickman (2000) researched the importance of the demonstration and concrete experience with teaching probability contents and established that children better understood more difficult concepts if they actively participated in the corresponding demonstrations. Mills (2007) delved into the attitudes of teachers towards probability contents establishing their positive attitude to the statistics and probability contents, and their wish to be offered the possibility of suitable additional training. Ashline and Frantz (2009) dealt with the connection between proportionality and probability contents, while Click (2010) was engaged in probability games played at lessons. Van Dooren et al. (2003) was interested in pupils' misconceptions pertaining to the probability contents.

As the most absorbing discussion on the understanding of the probability contents was conducted a long time ago among the scientists, such as, e.g. Piaget, Inhelder, Fischbein, Davies and others, we shall focus more precisely on their findings, and on some conclusions of the most recent research in this field, which are topical for our research. The opinions of various researchers about the abilities of children with regard to solving probability tasks differ a lot. Piaget as well as Inhelder (1951) state that a child in a concrete-operational period is neither able to differentiate between certain and random predictions nor formulate predictions, taking into account his experiences from previous similar situations. In their opinions a child first encounters the concept of probability at the level of his concrete operations, at which time he starts to differentiate between a certain and a possible event (Piaget, Inhelder, 1951; Goldberg, 1966) They also note that the systematic understanding of probability starts not earlier than between the ages of 9 and 12 years and even during that period children solve problems intuitively, and not on the basis of formal reasoning.

Many researchers contradicted their findings on abilities of children regarding their perception of the probability contents and argued the converse, among them Fischbein et al. (1984) and also Davies (1965) and Yost et al (1962) criticized Piaget's research mainly because it was based on a child's verbal abilities; they developed "the decision making method", which was not based on verbal abilities; using it children decide between two boxes (children draw out of the very bow, from which they believe to extract the chip of the desired color), but they do not need to use the expression "most probable", whereas in Piaget's research the box contained chips of two colors and children had to choose the color, which was more probable to be extracted (Yost et al., 1962).

Fischbein and his numerous colleagues also elaborated on teaching and learning the probability concepts, thereby concluding that it was possible to teach probability without any major efforts, which had a positive influence on the child's prejudices and misconceptions about the sequence of events and uncertain situations (Fischbein, Gazit, 1984; Fischbein, Pampu, Manzat, 1970). Among other things he found out that under certain conditions learning of probability concepts may have a negative impact (children taught probability topics performed worse at some tasks compared to those children who were not presented with these topics); nevertheless, Fischbein believes it would be possible to avoid this by presenting children with tasks including relationships calculations and probability estimations (Fischbein, Gazit, 1984).

Lately also Gurbuz et al. (2010) dealt with probability teaching and learning. They were trying to establish the effectiveness of the teaching approach based on pupils' active participation, whereby pupils were making numerous experiments pertaining to probability, and were discussing their findings among themselves in the follow-up. In the control group the pupils were deprived of this possibility at lessons (Gurbuz et al. 2010). In the research, in which 50 children participated, it was established that children who were provided with the teaching approach based on the discussion between pupils and teachers performed better than those children who were provided with lecturing lessons only (Gurbuz et al, 2010). Also Andrew (2009) stresses the importance of concrete experience, as he believes that pupils better understand probability contents if they perform experiments related to probability in advance. Thus, it is important that pupils gain experience also by drawing out, thus trying to determine the more likely event. Concepts in probability can be more readily understood if pupils are first exposed to probability via experiment. Performing probability experiments encourages pupils to develop understandings of probability grounded in real events, as opposed to merely computing answers based on formulae (Andrew, 2009). Andrew (2009) further states that pupils who have gained concrete experience in probability develop their understanding on this basis and wish to define the starting points to calculate probability of certain events (HodnikCadez & Skrbe, 2011).

Apart from researchers, many constitutions, like Unesco (1972) and Ceeb (1958), have recognized the important role that probability plays in our society and they suggest to include them in the primary school's curriculum.

Probability was introduced to the academic lesson series in the western world as a part of the "new mathematics" changes in the 1960s. They are one of the few things that have been preserved in the last forty years. But only in the late 1970s did the educational importance of probability in the primary and junior high school become apparent. This was due to the fact that until that time the grounds of their introduction was based more on practical and academic reasons rather than on some educational belief having to do with their importance as a part of the overall education. At exactly the same period (1975) when the book written by Piaget & Inhelder was translated into English, Fischbein (1975) published his significant work on intuition and probability that is based on research made over many years. Thus, at about 1975 probability teaching and learning began to interest the researchers (Truran, 2001).

The study of probability, in the 1970s, became initially an object of teaching in junior high schools, while later on it was also introduced in the fifth and sixth grade of the primary school. However, since the 1990s, probability seem to hold an integrated part in the mathematics curriculum from even the smaller grades in primary schools internationally.

Recent curriculum recommendations of the USA and Canada (NCTM, 2000), England (DfEE - Department for Education and Employment, 2001), Cyprus (CIE, 1997) and Greece (1987 and 1998) have recognized the importance of having all students develop an awareness of probability constructs and applications. Specifically, they mention that the students should understand and apply basic concepts of probability using the property vocabulary for them depending of course, their age.

Because of this emphasis on probability in the school curriculum, there has been considerable research into student's probabilistic thinking. The current tendency even for primary school level is towards a data-orientated teaching of probability, where students are expected to perform experiments or simulations, formulate questions or predictions, collect and analyze data from these experiments, propose and justify conclusions and predictions that are based on data (Carmen & Carmen, 2012).

Based on the views that distinguished scientists maintain and the suggestions that a number of organisms have made, it becomes apparent that the introduction of probability in primary education is necessary as well as feasible. Of course, an important condition for the introduction of the probability concepts in primary school is the use of the appropriate probabilistic expressions and the suitable vocabulary for the different age groups of children (Skoumpourdi & Kalavassis, 2003).

The purpose of the present study was to investigate the level of understanding the probability concepts in primary school students and to establish any potential statistically significant differences among different age groups of children and between genders at solving individual tasks.

2. Method

2.1. Participants

The sample to establish the ability of children to solve probability tasks was composed of 404 children of three elementary schools in the city of Florina in Greece. The children participating in the research were in the age group of 7-11 years old. Thus, 77 were second grade students, 81 were in the third grade, 83 were fourth grade students, 81 were in the fifth grade and 82 were in the sixth grade. Also, the proportion of participants was almost equal regarding gender, as 52.2% of the children were boys. The greatest deviation between the genders was with the second grade students (58.4%, 41.6%) in favor of the boys.

2.2. Instruments

In order to establish the ability of children to solve probability tasks, a Test was used which comprised 7 tasks. In the majority of these tasks, it was required to circle the solution. In the third and the fourth task children were supposed to continue the sentence and in the sixth task, a justification of both answers was required. Objectivity was achieved through provision of standard instructions and anonymity.

In the first two tasks of the test, children should differentiate among certain, possible and impossible events and compare various probabilities of events at the level of their graphic presentation. The tasks were different for younger children due to their misunderstanding of the meaning of “Certain”, “Possible” and “Impossible”. Thus, concepts as “Always”, “Sometimes” and “Never” were used in the second, the third and the fourth grade of primary school.

In the third and the fourth task, students had to focus on the event which is more likely to happen. In the younger grades, phrases such as “it is more possible” or “there are more possibilities” were replaced with the phrases “it is more certain” or “it will happen more times”.

In the fifth task, students were required to circle the correct answer, while three different situations had been explained and the questions were related with how possible some events were.

In the sixth and the seventh task, students had to choose between two boxes, the one of which it was more possible to extract a white or a black ball -or for the younger grades, the box of which they could extract more times a white or a black ball. Of course, in all the questions, the number of the white and black balls was different.

2.3. Data Collection

Data was collected by the Test which is used. The instructions for the students were uniform but because of the early age of the students of the second grade, some extra instructions were given, especially in the part of the understanding of the questions and where exactly they had to put their answer.

All the pupils were tested in their classrooms during their lesson. The time which they were allowed to perform their tasks was 20 minutes and only the students of the second grade surpassed their time.

3. Results

The results of children of different age groups at solving the tasks of the Test are presented below.

All the tasks relating to the children’s differentiating among certain, possible and impossible events were correctly solved by more than half of the respondents, which is well evident from Figure 1. In addition, many of the tasks were correctly solved by more than the 75% of the students of all the grades.

In the first task, we observe that the lowest score belongs to younger participants (75.1%) and as the age grows, the score is getting higher as well. Students of the third grade obtained 85.2% and children of the fourth grade have a better score (95.2%) than the kids of the fifth grade (92.1%) and the kids of the sixth grade (94.6%). A statistically significant difference is observed in this task between the grades ($F4, 399 = 24.907$, $p < 0.001$) according to the technique of One Way ANOVA which was used in order to examine possible differences among the score of the five grades of the elementary school.

In the second task, children of the second grade obtain the lowest score (78.7%) and they are followed by the students of the third grade (90.6%). The highest score belongs to the sixth grade students (97.1%). Students of the fourth grade (94%) performed better than children of the fifth grade (93.6%). A statistically significant difference is experienced in this task ($F4, 399 = 19.295$, $p < 0.001$) as well.

In the third task, children of the fourth grade made it better (82.5%) whereas some small differences are observed among the third, the fourth, the fifth and the sixth grade as the statistically significant difference is observed to a lesser extent ($F4, 399 = 3.025$, $p = 0.018 < 0.05$).

In the fourth task, students of the fourth grade (97%) made it better than the children of the fifth grade (89.5%) and the children of the sixth grade (93.9%). Second grade students (76.6%) and third grade students (85.8%) have the worst score. According to the technique of One Way ANOVA, a statistically significant difference is observed ($F4, 399 = 6.077$, $p < 0.001$).

In the fifth task, a great deviation between the second and the sixth grade is observed, as students of the second grade obtain the lowest score (65.5%) and the students of the sixth grade have the highest score (92.9%). For that reason, the statistically difference is very important ($F4, 399 = 43.806$, $p < 0.001$).

In the sixth task, all the students of all the grades obtain the lowest score, as very few children answered the second question. Also, in this task, we observe that children of the fifth grade have the highest score (65.4%) whereas children of the second grade obtain the lowest score (50.6%). In this task, a statistically significant difference is experienced ($F4, 399 = 11.738$, $p < 0.001$).

In the seventh task, we observe a great deviation between the five grades and a statistically significant difference ($F4, 399 = 22.661$, $p < 0.001$). It is noticeable that there is not important difference between the scores of the third (68.5%) and the fourth grade (68.1%) but there is an important difference between the kids of the fifth (75.1%) and the sixth grade (84.1%).

Upon examination of the whole Test, it can be established that children of all age groups were able to solve probability tasks as the average performance of all the respondents is 79.5%. A statistically significant difference is observed in the final score ($F4, 399 = 38.802$, $p < 0.001$). Specifically, based on the post hoc LSD test (0.05 level) there are statistically significant differences in the final score among the five grades. The second and the third grade (67.0% and 76.6%) present some important statistically differences with the rest grades. The fourth grade (81.9%) displays some statistically significant differences with the second ($p < 0.001$), the third ($p = 0.003 < 0.005$) and the sixth grade ($p = 0.003 < 0.005$) and not with the fifth grade. Respectively, the fifth grade (84.1%) presents statistically significant differences with the second and the third grade ($p < 0.001$) and not with the fourth and the sixth grade. Also, the sixth grade (87.2%) displays some statistically important differences with the second ($p < 0.001$), the third ($p < 0.001$), and the fourth grade ($p = 0.003 < 0.005$) and not with the fifth grade.

The influence of gender on solving probability tasks is noticed as the girls performed better on each of the seven tasks of the Test. Statistically significant differences among the gender and the tasks are observed in the first task ($t = 2.497$, $df = 402$, $p = 0.013 < 0.05$), the fourth task ($t = 3.527$, $df = 402$, $p < 0.001$), the fifth task ($t = 2.204$, $df = 402$, $p = 0.028 < 0.05$) and in the final score ($t = 3.128$, $df = 402$, $p = 0.002 < 0.005$). In addition, girls performed better in all the grades except for the fourth grade in which boys present a slightly better score.

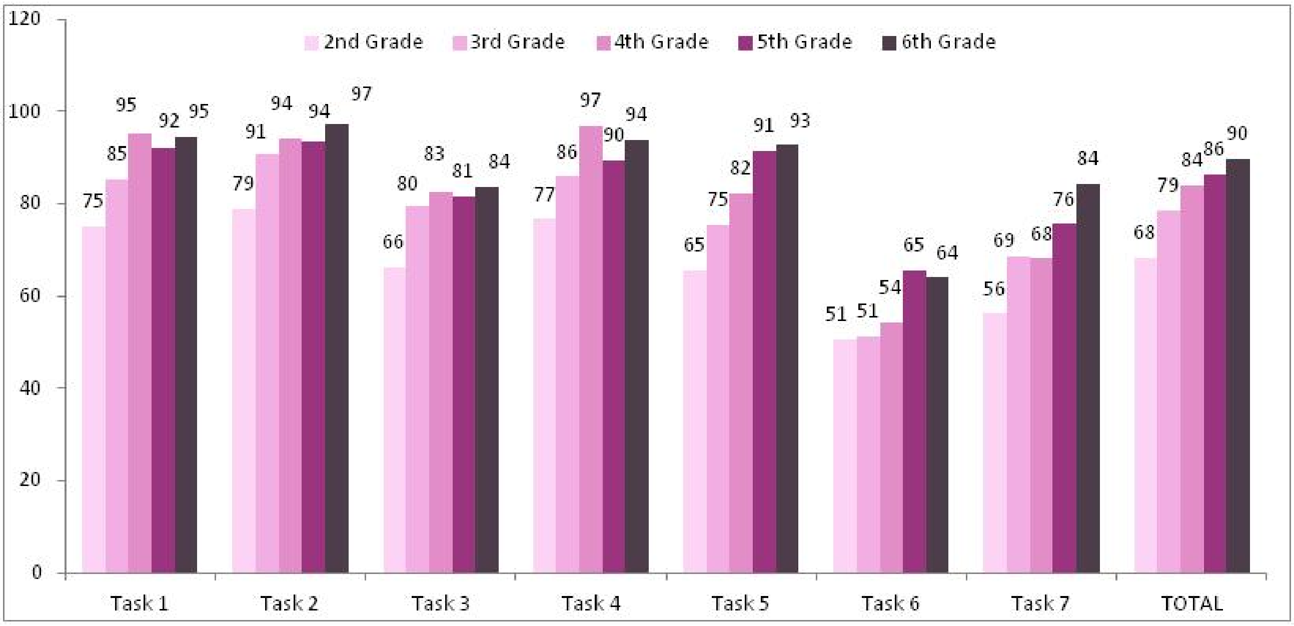

Figure 1. The percentages of correctly solved tasks

4. Discussion and Conclusion

In the research, it was established that the majority of children of second to sixth grade of the elementary school was able to compare the probability of various events. Also, some statistically significant differences have been experienced between the different ages of the students and the gender at solving probability tasks.

It can be concluded that our results are similar to the ones of the researchers, such as Fischbein et al. (1984) and Davies (1965), who believe that children in elementary school are able to solve certain probability tasks. In addition, these results agree with the outcomes of the research of HodnikCadez & Skrbe (2011) who support that children of the younger grades of the primary school were able to recognize different events and categorize them depending on their likelihood.

In all the mentioned activities children predicted and assessed the likelihood of an event. The situations differ among themselves, they are related to everyday life, to common language, they are presented in different ways and offer children many possibilities for discussion, assessment and arguing the likelihood of an event. Alongside the vocabulary development and familiarity with recording conventions, all of the different types of activity offered to primary aged children in mathematics lessons were supposed to bring with them some aspect of a mathematical perspective on the relationship between possibilities and probabilities. That is, after all, the main point of introducing probability into mathematics classes.

On some tasks, younger children performed better than the older ones, the difference being certainly due to two different knowledge tests, as the youngest three age groups took rather different knowledge tests for reasons of their luck of comprehension the appropriate vocabulary.

In addition, it was observed that some mistakes which have been made by the students of all the grades of elementary school are a result of the misunderstanding of the questions and the exercises. For that reason, there is no doubt that when the instructions are well explained, the score of the students is higher.

It may be an argument that children were required to solve some simple exercises and differentiate among some simple events. On the basis of his research Threlfall (2004) believes probability contents should be explored only when children are able to deal with complex situations and not only simple ones, for which he expresses his doubt whether they prove mathematical understanding and thinking of children. The author concludes, that children should be taught probability in higher grades of primary school, whereas in the lower grades only those children should be presented with probability tasks that are able to deduce the complexity of simple situations from probability. Although, as HodnikCadez & Skrbe (2011) mentioned, probability contents could be introduced within the context of mathematical literacy in the early school period, with the emphasis being on a child’s active participation in the discussions on situations that are possible, impossible, certain or even equally probable. Furthermore, young children are taught the probability “alphabet”, because, as already Fischbein (1985) found out, the probability concepts and techniques need to be integrated in mathematics lessons as early as at the primary level, and not only in higher grades or even in high school, when the mindset of a human is already developed.

After all, these results prove the main point of introducing probability into mathematics classes, as students do have the ability of understanding the probability concepts, compare the probability of various events and thus, choose the best option among different situations which demand probabilistic thought. Last but not least, all this contributes to developing those competences that modern human should possess in order to adapt rapidly to the world of today, which is of unpredictable situations and various challenges, and in order to foster the critical attitude towards “numerical information” disseminated by media almost every day (Howson & Kahane, 1986).

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