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Drawings and Tables as Cognitive Tools for Solving Non-Routine Word Problems in Primary School

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Abstract External representations play a central role in the process of word problem solving. This study aimed to shed light on teacher-provided representations as cognitive tools for primary students when working on non-routine word problems. Non-routine word problems are characterized by the fact that they cannot be solved by simply applying familiar routine calculations due to their demanding mathematical structure or complex situations described in the problem text. Since primary students often do not generate external representations, the present study examined the questions if providing students with a representation facilitates problem solving in general, and, more in detail, what type of representation (table or drawing) and what level of pre-structuring provided in the representation is most helpful. In an experimental design we studied a sample of 199 4th-graders who worked on non-routine word problems. The experimental design consisted of three tests: A pre-, a treatment-, and a transfer-test. In the pre-test, we measured participant's prior performance with non-routine word problems. In the treatment-test, an experimental group received problems accompanied by tables and drawings with different levels of pre-structuring to measure student's performance when external representations were provided for the problem solving process. A control group received no representations. In the transfer-test, participants worked on problems without provided representations to measure participant's performance after they were exposed to external tools in the treatment-test. Results indicate that providing drawings or tables did not facilitate problem solving in general, which was against our hypothesis. If a representation was provided, a drawing was more helpful than a table, which was in line with our assumptions. However, the drawing effect was depending on the problem type and the level of pre-structuring. Obviously, simply providing external representations was not sufficient to facilitate problem solving. This speaks of the necessity of an early training in diagram literacy.

Keywords: *external representations, non-routine word problems, primary school, problem solving, drawings, tables*

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1. Introduction

"A snail in a 24 m deep well wants to crawl up to the field. Each day it crawls 6 m up the side of the well and slides down half the distance it crawled in the daytime during the night when sleeping. The snail starts on Monday morning. On which day will the snail reach the top of the well?" To solve this famous riddle - which can be considered as non-routine word problem for primary school students - representation plays a central role. This study aims to shed light on teacher-provided representations as cognitive tools. Can they facilitate the solution process, and what is the right level of pre-structuring provided in the representations? In a theoretical part, we first define non-routine word problems. Second, we introduce a theoretical framework of external representations and their role in non-routine word problem solving. In an empirical part, we third formulate research questions and derive hypotheses about non-routine word problem solving with different types of provided external representations. Fourth, we present a study designed to test the hypotheses. Fifth, we report the results of the study with respect to the hypotheses, and sixth, we discuss the findings.

2. Non-routine Word Problems

In mathematics literature a distinction is made between routine and non-routine word problems [35,40]. Contrary to routine word problems, non-routine problems are characterized by the facts that a problem solver (1) does not immediately see how to solve the task [35], and he (2) cannot simply use well-trained algorithmic calculating procedures [40]. When working on a non-routine word problem, the student encounters a barrier. The given state cannot be transferred into the goal state by simply doing (which would be an exercise), but requires thinking [15]. This is in line with psychological definitions of problem-solving (for an overview see Funke [19], which makes non-routine word problems ideal tasks to foster problem solving in classroom [46]. Instead of applying algorithmic procedures, the problem solver has to re-structure existing knowledge [57], and make use of heuristic strategies [40] to develop and execute a solution. Among a variety of heuristic strategies [20,39,40,54], both using a drawing or a table can be considered as heuristic tools highly relevant for classroom practice [8].

3. Cognitive Processes and the Role of External Representation for Solving Non-routine Word Problems

Research on the cognitive processes involved in word problem solving mainly shows that the difficulty of solving mathematical word problems is not primarily executing the necessary operations, but understanding the problem for which an adequate mental model of the situation has to be constructed [30,35,55]. With regard to the snail problem presented above, the difficulty of the task does not primarily stem from executing multi-step addition and subtraction without mistakes, but from the need to correctly understand the whole situation described in the text and to transfer it into an appropriate mathematical model. Even when the distance and the direction of the upwards and downwards movements are (1) correctly understood, (2) appropriately transferred into a mathematical model and (3) correctly executed, an adequate mental model of the situation must take into consideration that on the last day the snail reaches the top of the well and does not slide back anymore. Hence, deriving and executing a mathematical model of 24m divided by 3m a day equals 8 days would lead to the wrong solution and reflect an incomplete mental model of the situation.

External representations can act as cognitive tools in the whole process of solving mathematical (non-)routine word problems [1,26,53]. When a problem solver makes use of external representations, he starts internal communication by producing and receiving signs alternately [44]. Moreover, he compares his mental model of the situation in a dynamic and iterative process with the information externalized in the representation [10], captures unstable mental representations and relieves working memory with its limited capacity [44].

The main focus of the present study was to examine the role of external representations for constructing a mental model. But how do students construct a mental model from word problem text, and in addition, from provided or self-constructed external representations like drawings and tables? The integrated text and picture comprehension (ITPC) model of Schnotz and Bannert [45] provides a theoretical framework on how a mental model is build when both text and pictures are processed. It integrates van Dijk and Kintsch's [49] model on text comprehension with models of picture comprehension within a cognitive architecture of a multiple memory system with multiple sensory channels [43]. In contrast to Mayer's cognitive theory of multimedia learning [34], where in a first step a verbal and a pictorial mental model is constructed and in a second step both models are integrated into one mental model, the ITPC model assumes that only one mental model is built directly. Schnotz and Bannert [45] generally distinguish between descriptive and depictive representations. Descriptions like natural language or propositional representations consist of symbols with an arbitrary structure that share no resemblance with the object they are representing. They are powerful in expressing abstract knowledge [43]. On the other hand, depictions like pictures, visual images, and mental models [28] consist of icons. These icons are signs that always have a spatial configuration and share resemblance or other structural commonalities with the object they are referring to. They are informationally complete [43]. This gives depictions a high computational and inferential efficiency [31].

According to the ITPC model, the construction process of a mental model is fed by external and internal sources. In line with the dual-coding approach of Paivio [37] the ITPC model distinguishes between a verbal and a pictorial channel to process external information. Descriptions are processed by a verbal channel, depictions by a pictorial channel. When reading a text of a word problem or gathering information from a table, the problem-solver (1) selects relevant information, (2) transfers it into a text-surface representation, (3) organizes it in propositional representations and (4) constructs a mental model by including prior knowledge (internal source) from long-term memory [45]. When using a picture as external source of information, (1) relevant pictorial information is selected, (2) transferred into a pictorial surface representation, and (3) directly organized and combined with prior knowledge in a mental model [45]. Inferences are made within the mental model and coded in propositions [43]. Many students seem to have difficulties constructing a mental model of a word problem on basis of the problem text alone [4,9,12,56]. Adding a depictive representation to the problem text, which constitutes another source of information processed via a second channel (pictorial channel), should enhance and facilitate mental model construction [43]. With respect to the snail problem, a drawing with an appropriate structure would depict the complete distance the snail has to crawl, and the upwards and downwards movements. Thereby the situation of the last day would be obvious to the problem solver. Although the information is in the text, the student is more likely to discover it in a picture.

However, results of empirical studies involving external pictorial representations for word problem solving are mixed. Some studies found positive effects [7,36,58], whereas others found drawings not to be of help [13,17,50].

Aside from the representation type (description or depiction), the question of the appropriate level of pre-structuring is of importance and not answered satisfactorily yet. On a scale, one extreme would be providing a ready-made representation, allowing the students to read-off the answer. The other extreme of the scale would be asking the problem solver to self-construct a representation. Between those two extreme positions, one can provide representations with different levels of pre-structuring, so that the problem solver has to edit it to a greater or lesser extent, e.g. for the snail problem providing a drawing depicting the complete distance the snail has to crawl, but only one upward and one downward movement. According to the generative theory of drawing construction [52], actively drawing a picture while reading a text fosters cognitive in-depth and meta-cognitive processing. However, students often do not spontaneously construct external representations [3,5,16,18], even if explicitly asked [6]. Moreover, self-generated drawings often do not map the structure of the problem, but fulfill decorative purposes only [14,25,51]. The authors conclude that making an appropriate drawing has to be trained. In a study of Fagnant and Vlassis [18], participants received word problems with provided external representation in a short intervention. In a re-use test the solution rates increased significantly. The authors concluded that even simply exposing students to a possible tool can help them in later problem solving with the same type of tasks. However, the study involved no control group which is a clear limitation. On contrary, other studies found no effects of a representational training [2,3,50].

4. Research Questions and Hypotheses

The principle idea of the present study was to provide two different types of representations to students for solving non-routine word problems: drawings (depictions) and tables (descriptions). Furthermore, the drawings and the tables differed in their level of pre-structuring.

The following research questions (RQ) were addressed:

RQ 1: Does providing an external representation facilitate and enhance the construction and use of an adequate mental model or should students self-generate external representations?

RQ 2: Which format of external representations – descriptions or depictions – enhances and facilitates the construction and use of an adequate mental model more?

RQ 3: Which level of pre-structuring is appropriate for fourth-grade students?

With respect to RQ 1 we assumed that provided external representations in form of a tables or a drawings facilitate and enhance the construction and use of an adequate mental model more than student’s self-generated external representations. Providing an external representation should result in a better performance of the students in the treatment-situation (hypothesis 1a) – which is working with external representations – as well as in a transfer-situation (hypothesis 1b) – which tests the effect of learning from external representations. From this point of view the treatment-test is considered as an intervention. Moreover, we assumed with respect to RQ 2 that a drawing should enhance and facilitate the construction and use of an adequate mental model more than a table (hypothesis 2). With respect to RQ 3 we assumed that the higher the level of pre-structuring, the better and more effective the construction and use of an adequate mental model should be (hypothesis 3). The enhancement should translate into higher solution rates and better understanding of the problem (effectivity). The facilitation should translate into less perceived difficulty, less perceived effort and shorter processing time (efficiency).

5. Materials and Method

5.1. Participants

The participants were 199 fourth-grade students from five primary schools in Germany. 103 students were female and 96 were male. Their average age was 9.21 years (SD = 0.435). The experiment was undertaken with the understanding and consent of each subject’s parents and, the responsible school authority.

5.2. Materials

5.2.1. Non-routine Word Problems

We used three different types of word problems adapted from Rasch [40]: combinatorics, comparison and motion problems. All three types can be considered as non-routine for primary school students. The combinatorics problems had the combinatorial logic of (n (n-1)) / 2), and the cover stories were about four children doing something together (e.g. shaking hands). The difficulty with that type of task was that primary students are not familiar with combinatorics and therefore cannot simply calculate the answer using a routine procedure. The comparison problems had the mathematical structure of a + b = c, with c and the difference of a and b given. Both the values of a and b were unknown. The scenarios described two children with a certain number of objects (e.g. building blocks). The difficulty was due to the fact that there are two conditions which students had to handle simultaneously. Usually, primary students are not familiar with algebraic solution procedures, making this type of task a non-routine word problem. An example of a motion problem is the snail riddle given in the introduction part of this paper. The difficulty with motion problems was to model the forward-and-backward movement correctly and to realize that on the last day e.g. the snail reaches the top of the well and does not slide back again that night. Figure 1 gives an example for each type of word problem.

The study involved 12 tasks altogether: Four combinatorics, four comparison and four motion problems. The scenarios and the numbers used varied between the four corresponding tasks, but the mathematical structures as well as the number ranges were identical. The tasks were compiled in four problem-sets each consisting of a randomly assigned combinatorics-, comparison- and motion problem. Each set built a booklet resulting in four booklets.

5.3. Experimental Design

The experimental design consisted of a one-factorial between-subjects manipulation and a three-factorial within-subjects manipulation.

5.3.1. Between-Subjects Manipulation

The between-subjects factor was group with “1 = experimental group” (EG), and “0 = control group” (CG). Participants in the EG (n = 159) were provided external representations, whereas CG’s participants (n = 40) did not receive any external representations to solve the tasks.

The experimental design consisted of three tests: (1) pre-, (2) treatment-, and (3) transfer-test. In the pre-test, participants were administered booklet 1 consisting of problem-set 1 without provided representations to measure participant’s prior performance. In the treatment-test, participants received booklets 2 and 3 consisting of problem-sets 2 and 3. In the EG the tasks were accompanied by external representations (intervention), the CG’s booklets contained no representations. In the transfer-test, participants worked on booklet 4 which contained problem-set 4 without provided representations to measure participant's performance after the intervention.

The between-subjects manipulation aimed at answering RQ 1: Does providing an external representation facilitate and enhance the construction and use of an adequate mental model or should students self-generate external representations? To analyze hypothesis 1a (working with external representations), the comparison of EG's and CG's performance in the treatment-test taking the pre-test performance into account was relevant. To analyze hypothesis 1b (learning from external representations), the comparison of EG's and CG's performance in the transfer-test after the intervention (treatment-test) taking the pre-test performance into account was relevant.

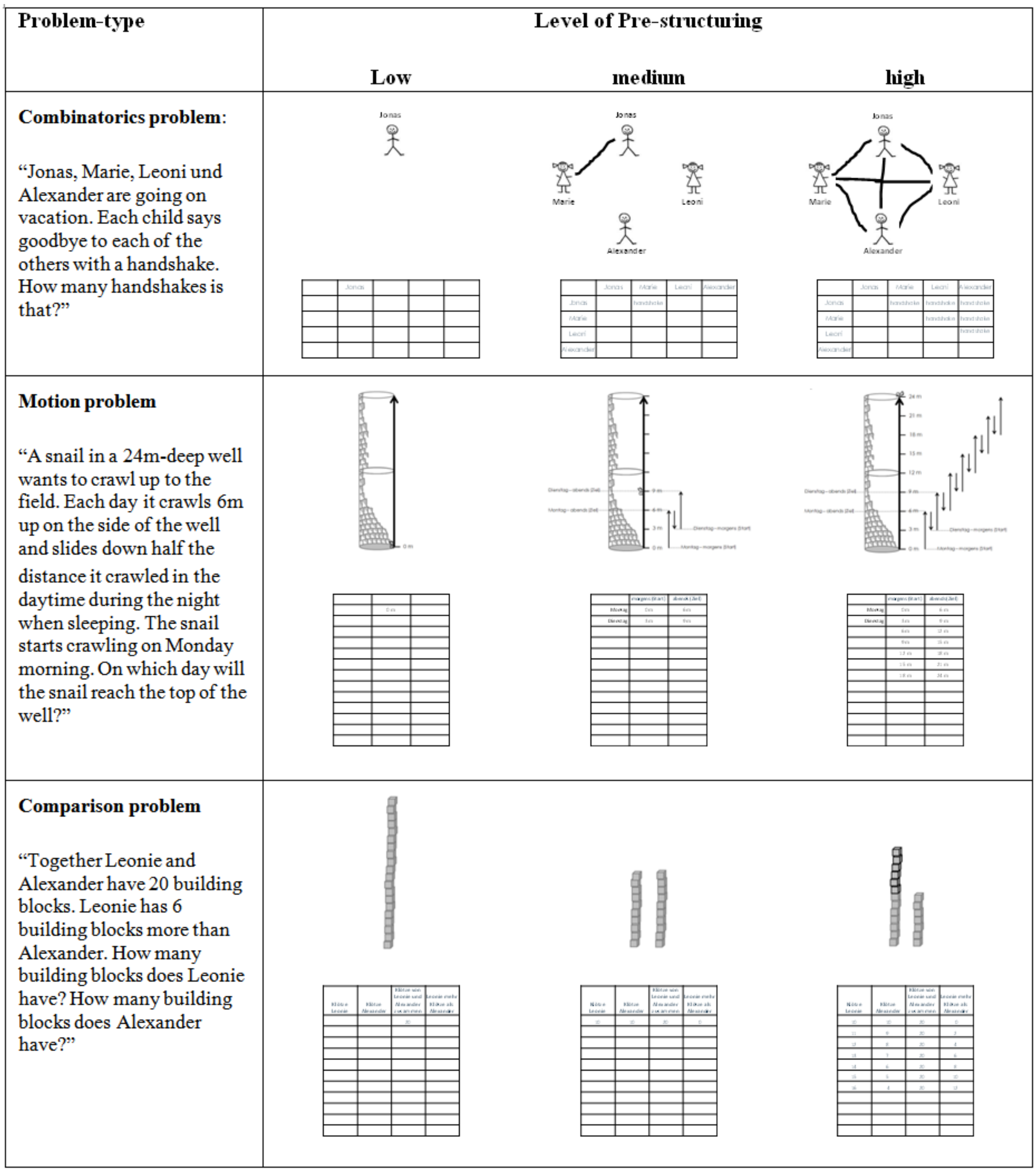

Figure 1. Types of word problems and the provided drawings and tables with different levels of pre-structuring

5.3.2. Within-Subjects Manipulation

The within-subjects manipulation consisted of the factors (1) representation type (RT), (2) level of pre-structuring (LPS) and (3) problem-type (PT). The factors RT and LPS manipulated the provided representations in the EG. The factor RT had two factor levels with "1 = table", and "2 = drawing". The drawings and tables were task-specific. The drawing and the corresponding table were informational equivalent [38], that is they provided the same information but in a different way. The factor LPS differentiated the representations with respect to the amount of provided structure with "1 = low", "2 = medium", and "3 = high". This resulted in three versions of each drawing and each table. The levels of pre-structuring were done by theoretical considerations taking into account the structure of the representation on the one hand and the processes operating on the structure of the representation on the other hand. Drawings and tables with a high level of pre-structuring not only provided the complete structure of the representation, but all necessary operations on the structure were executed already.

Students could read-off the answer without the necessity to further manipulate the drawing or table. Drawings and tables with a medium level of pre-structuring provided the complete structure but only one first operation on the structure. The low pre-structured versions provided the structure (comparison and motion problems) or even a part of the structure (combinatorics problems) but no operations. Figure 1 gives an overview of the representations. The factor PT differentiated the word problems in “1 = combinatorics”, “2 = comparison”, and “3 = motion”. The within-subjects manipulation aimed at answering RQ 2 and RQ 3. To analyze hypothesis 2, EG’s performance with provided tables was compared to the performance with provided drawings. To analyze hypothesis 3, EG’s performance with representations providing a high level of pre-structuring was compared to the performance when representations with a medium and low level were provided.

The within-subjects manipulation was conducted in a multi-matrix design. Theoretically, the 2 x 3 x 3 factorial within-subjects manipulations resulted in 18 within-subjects conditions. In order not to overburden students and to minimize the time the teachers had to provide to the experimenters, each participant received only six out of 18 conditions. Each EG’s participant received drawings as well as tables. In one test-booklet the three tasks were accompanied by drawings and the instruction “You can use the drawing to solve the problem”. In the other booklet the corresponding tasks were accompanied by tables and a corresponding instruction. The first task in a booklet always was accompanied by a representation providing a high level of pre-structuring. The second task was presented with a representation providing a medium level, and the third problem with a representation providing a low level.

To control for order effects of the type of representation, one half of EG’s participants started with the booklet containing drawings. The other half started with the booklet containing tables. This resulted in two treatments in the EG. To control for order effects of the problem-sets, half of EG’s as well as CG’s participants first received set 2 and continued with set 3, whereas the other half started with set 3 and continued with set 2. This resulted in four treatments (2 x 2) in the EG, and two treatments in the CG. The order of the tasks in a booklet was completely counterbalanced across subjects resulting in six task-sequences to (1) control for order effects of the tasks, and to (2) combine each problem with each level of pre-structuring. In consequence, the four treatments in the EG had to be multiplied by six task-sequences resulting in 24 treatments in the EG. Theoretically, in the CG the two treatments had to be multiplied by six task-sequences as well. However, due to the small number of CG’s participants (n = 40) we used only six out of twelve possible CG treatments. CG’s treatments consisted of all six task-sequences. Half of these treatments started with problem-set 2 and continued with set 3, whereas the other half started with set 3 and continued with set 2. Figure. 2 gives an overview of the resulting 30 treatments.

5.4. Dependent Variables

We measured five dependent variables for each of the 12 tasks: (1) solution rate, (2) understanding of the problem, (3) perceived difficulty, (4) invested effort, and (5) processing time. Solution rate was measured dichotomous (“0 = wrong answer or no answer”, “1 = correct answer”). Understanding of the problem was rated by three trained raters on a 4-point Likert-scale from “0 points = completely wrong or no understanding” to “3 points = completely correct understanding”. Using rational task analysis [42], we defined typical error types in which more or less understanding of the participants was expressed. We tested rater’s agreement on approximately 5 % of the solutions (n = 122). Cohen’s Kappa was .827. Perceived difficulty was measured on a 4-point Likert-scale with “1 = very easy” to “4 = very difficult” directly after the participants had finished the task. Cronbach’s α was .780 for the EG and .864 for the CG. Invested effort was measured on a 4-point Likert-scale with “1 = no effort at all” to “4 = very much effort”. Cronbach’s α was .894 for the EG and .945 for the CG. To measure processing time, each participant used an electronic pen (“livescribe echo smartpen”) which recorded a video of the participant’s individual writing process for each booklet. Using the continuous time stamps in the video material, raters coded the time when the participant started to read the problem text and the time when he provided his final answer for each subject and task. Rater’s agreement was tested on approximately 5 % of the material (n = 116). The ICC for the start point was .984 and .991 for the end-point.

				booklet 2 level of pre-structuring					booklet 3 level of pre-structuring		
Treatment				high	medium	low			high	medium	low
EG	1	drawing	set 1	CO	C	M	table	set 2	CO	C	M
	2			CO	M	C			CO	M	C
	3			C	CO	M			C	CO	M
	4			C	M	CO			C	M	CO
	5			M	CO	C			M	CO	C
	6			M	C	CO			M	C	CO
	7		set 2	CO	C	M		set 1	CO	C	M
	8			CO	M	C			CO	M	C
	9			C	CO	M			C	CO	M
	10			C	M	CO			C	M	CO
	11			M	CO	C			M	CO	C
	12			M	C	CO			M	C	CO
	13	table	set 1	CO	C	M	drawing	set 2	CO	C	M
	14			CO	M	C			CO	M	C
	15			C	CO	M			C	CO	M
	16			C	M	CO			C	M	CO
	17			M	CO	C			M	CO	C
	18			M	C	CO			M	C	CO
	19		set 2	CO	C	M		set 1	CO	C	M
	20			CO	M	C			CO	M	C
	21			C	CO	M			C	CO	M
	22			C	M	CO			C	M	CO
	23			M	CO	C			M	CO	C
	24			M	C	CO			M	C	CO
CG	25		set 1	CO	C	M		set 2	CO	C	M
	26			CO	M	C			CO	M	C
	27			C	CO	M			C	CO	M
	28		set 2	C	M	CO		set 1	C	M	CO
	29			M	CO	C			M	CO	C
	30			M	C	CO			M	C	CO

CO = combinatorics, C = comparison, M = motion

Figure 2. Treatments of the within-subjects manipulation

5.5. Procedure

Data were collected in the classroom at four sessions. There were three weeks between the sessions. Teachers provided 90 minutes of lesson time each session. The first 45 minutes each session the participants worked on the booklet containing the word problems. A time span of 45 minutes was sufficient even for the slowest student to complete the booklet. The experimenter did not answer content related questions of the participants nor did he provide any assistance with the tasks. At sessions 2, 3, and 4 the remaining 45 minutes were used to administer additional instruments. At session 2, participants completed subscales of standardized test to measure reading skills [32], and calculation skills [23]. At session 3, participants were administered the CPM Raven test [41] to measure general cognitive abilities. At session 4, participants answered a short questionnaire (attitudes towards word problems, demographics).

5.6. Statistical Analyses

To answer RQ 1, we conducted a mixed-ANOVA with two within-subjects factors and one between-subjects factor. Within-subjects factors were (1) measurement time with "1 = pre-", "2 = treatment-", and "3 = transfer-test, and (2) problem type with "1 = combinatorics", "2 = comparison", and "3 = motion". Between-subjects factor was group with "1 = EG", and "0 = CG". Mauchly's test for sphericity was performed for all within-factors. If sphericity assumption was not met, degrees of freedom were corrected by Huynh and Feldt method. A generalized estimating equations model (GEE) of Liang and Zeger [33] was used for the dichotomous variable solution rate with the same factors. For MT the contrasts "pre- vs. treatment-test" (contrast 1) and "pre- vs. transfer-test" (contrast 2) were conducted. With respect to hypothesis 1a, we expected a significant interaction effect of measurement time*group for contrast 1. With respect to the hypothesis 1b, we expected a significant interaction effect of measurement time*group for contrast.

To answer RQ 2, EG's treatment-test data were analysed. We conducted a GEE model with three within-subjects factors (1) form of representation with "1 = table", and "2 = drawing", (2) level of pre-structuring with "1 = low", "2 = medium", and "3 = high", and (3) problem type with "1 = combinatorics", "2 = comparison", and "3 = motion" for each dependent variable. The GEE model allowed modelling all interaction effects despite the incomplete measurement series due to the multimatrix-design of the within-subjects manipulation.

6. Results

6.1. Research Question 1

Basis of the analyses were students who participated in all four sessions (EG: n = 144; CG: n = 37). The participants of the two groups did not differ with respect to their calculation skills, reading skills, age, and sex. However, the two groups differed significantly with respect to their general cognitive abilities: $t(177) = 2.157$, $p = .032$. Participants in the EG had a somewhat higher IQ ($M = 108.24$, $SD = 10.74$) as the students in the CG ($M = 103.92$, $SD = 10.76$).

With respect to the solution rates, the GEE model provided a significant main effect for measurement time, a significant main effect for problem-type and a significant interaction effect for measurement time*problem type (see Table 1). Contrary to our hypothesis, there was no significant interaction of measurement time*group, suggesting that providing external representations had no effect on the solution rates in both the situations of working with and learning from external representations.

Table 1. GEE model effects for solution rates

Factor	Df	Wald-χ^2	p
(A) measurement time	2	14.894	.001
(B) problem type	2	36.607	< .001
(C) group	1	1.315	.251
A x B	4	33.932	< .001
A x C	2	2.507	.286
B x C	2	1.134	.567
A x B x C	4	8.335	.080

The mixed ANOVA models for the dependent variables understanding of the problem, perceived difficulty, invested effort, and processing time all revealed significant main effects for measurement time, problem-type, and significant interaction effects for measurement time*problem-type (see Table 2). Contrary to our hypothesis, no significant interactions of measurement time*group were found in the analyses. Providing an external representation did not increase the understanding of the problem nor did it decrease perceived difficulty, invested effort and processing time. Table 3 provides an overview of the mean values of all dependent variables for the EG and the CG at the three test-situations.

Table 2. Mixed-ANOVA model effects for understanding of the problem, perceived difficulty, invested effort, and processing time

Factor	*Df*	*Df error*	*F*	*p*	partial η^2
Understanding of the problem					
(A) measurement time	1,933	345,926	18,323	< .001	.093
(B) problem type	2	179	52,103	< .001	.225
(C) group	1	179	1,768	.185	.010
A x B	3,739	669,346	13,292	< .001	.069
A x C	1,933	345,926	0.716	.485	.004
B x C	2	179	1,043	.353	.006
A x B x C	3,739	669,346	0.792	.523	.004
Perceived Difficulty					
(A) measurement time	2	242	24,997	< .001	.171
(B) problem type	2	242	39,757	< .001	.247
(C) group	1	121	1,808	.181	.015
A x B	3,590	434,434	18,953	< .001	.135
A x C	2	242	0.405	.668	.003
B x C	2	242	1,376	.255	.011
A x B x C	3,590	434,434	0.811	.518	.007
Invested Effort					
(A) measurement time	1,845	236,222	31,241	< .001	.196
(B) problem type	2	256	37,891	< .001	.228
(C) group	1	128	0.016	.900	.000
A x B	3,777	483,472	14,016	< .001	.099
A x C	1,845	236,222	0.293	.729	.002
B x C	2	256	0.540	.584	.004
A x B x C	3,777	483,472	1,172	.322	.009
Processing Time					
(A) measurement time	1,444	98,188	44,173	< .001	.394
(B) problem type	1,663	113,102	43,150	< .001	.388
(C) group	1	68	0.497	.483	.007
A x B	2,792	189,858	3,967	.011	.055
A x C	1,444	98,188	0.086	.856	.001
B x C	1,663	113,102	0.981	.378	.014
A x B x C	2,792	189,858	3,847	.012	.054

Whether a representation was provided or not, the solution rates in both groups were higher in the treatment- (*M* = 0.36, *SD* = 0.26) and the transfer-test (*M* = 0.34, *SD* = 0.29) then in the pre-test (*M* = 0.26, *SD* = 0.27). Understanding of the problem significantly ($p < .001$) increased from *M* = 1.45 (*SD* = 0.73) in the pre-test to *M* = 1.83 (*SD* = 0.62) in the treatment-, and *M* = 1.71 (*SD* = 0.74) in the transfer-test. Perceived difficulty significantly ($p < .001$) decreased from *M* = 2.89 (*SD* = 0.54) to *M* = 2.02 (*SD* = 0.49), and *M* = 1.91 (*SD* = 0.57) respectively. Invested effort significantly ($p < .001$) decreased from *M* = 2.68 (*SD* = 0.66) to *M* = 2.40 (*SD* = 0.68), respectively *M* = 2.19 (*SD* = 0.83). Processing time significantly ($p < .001$) decreased from *M* = 308.7 seconds (*SD* = 144.2) to *M* = 178.6 seconds (*SD* = 64.6), respectively *M* = 166.6 seconds (*SD* = 66.1). However, for all dependent variables there was an interaction of measurement time*problem-type. To shed light on this interaction, we conducted analyses for contrast 1 and contrast 2 for each problem-type separately, resulting in six comparisons for each dependent variable. Significance level was Bonferroni-corrected at $p < .008$. The analyses revealed the same pattern for all dependent variables. We found the significant effect of measurement time for the combinatorics and motion problems, but not for comparison problems. With respect to processing time, the effect of measurement time was true for all problem-types, but in particular for the motion problems (two-way interaction). Looking at contrast 1, the processing time for the motion problems decreased dramatically (Δ 251.0 seconds) in the EG, but only moderately in the CG (Δ 119.9 seconds). With the comparison problem we found the opposite pattern (three-way interaction): Δ 17.3 seconds in the EG and Δ 110.8 seconds in the CG. Obviously, providing a representation for the motion problems led to shorter processing time while providing a representation to the comparison problem made processing time even longer as if no representation was provided.

6.2. Research Question 2

6.2.1. Drawing or Table?

The GEE model revealed a significant main effect for representation type for all dependent variables (see Table 4). In line with hypothesis 2a, providing a drawing led to higher solution rates and better understanding of the problems, less perceived difficulty, less invested effort and a shorter processing time than providing a table. Table 5 gives an overview of the mean values. However, with respect to solution rates there was a significant interaction with the level of pre-structuring (A*B): If the level of pre-structuring was low, providing a drawing (*M* = 0.33, *SD* = 0.47) did not result in different solution rates than providing a table (*M* = 0.35, *SD* = 0.48, *p* = .558). Figure 3 visualizes the interaction effect. Further there was a significant interaction with the problem-type (A*C): Providing a drawing led to higher solution rates than providing a table only for the combinatorics problem (*M* = 0.42, *SD* = 0.49 vs. *M* = 0.22, *SD* = 0.42, $p < .001$). With the comparison (*M* = 0.39, *SD* = 0.49 vs. *M* = 0.40, *SD* = 0.49, *p* = .952) and the motion problem (*M* = 0.38, *SD* = 0.49 vs. *M* = 0.36, *SD* = 0.48, *p* = .737), providing a drawing resulted in similar solution rates as providing a table.

Table 3. Mean values of all dependent variables of experimental group and control group participants in the Pre-, Treatment- and Transfer-Test

	Pre-Test				*Treatment-Test*				*Transfer-Test*			
	Experimental Group		Control Group		Experimental Group		Control Group		Experimental Group		Control Group	
Dependent Variables	*M*	(*SD*)	*M*	(*SD*)	*M*	(*SD*)	*M*	(*SD*)	*M*	(*SD*)	*M*	(*SD*)
Solution rates [a]	0.25	(0.26)	0.29	(0.29)	0.36	(0.25)	0.37	(0.28)	0.32	(0.29)	0.39	(0.27)
Understanding of the problem [b]	1.41	(0.71)	1.58	(0.80)	1.82	(0.60)	1.88	(0.69)	1.67	(0.74)	1.87	(0.74)
Perceived difficulty [c]	2.30	(0.52)	2.17	(0.58)	2.03	(0.48)	1.94	(0.52)	1.92	(0.58)	1.67	(0.58)
Invested effort [d]	2.72	(0.65)	2.68	(0.76)	2.43	(0.65)	2.43	(0.80)	2.28	(0.77)	2.17	(0.96)
processing time (in sec) [e]	304.3	(138.9)	322.2	(161.5)	171.2	(54.9)	201.6	(85.1)	166.3	(67.9)	167.8	(61.4)

[a] mean value between 0 und 1, [b] rating-scale from 0 = completly wrong understanding to 3 = completly correct understanding, [c] self-rating of the students on a scale ranging from 1 = very easy to 4 = very difficult, n = 102 (EG) and n = 21 (CG), [d] self-rating of the students on a scale ranging from 1 = no effort at all to 4 = very much effort, n = 106 (EG) und n = 24 (CG), [e] n = 57 (EG) und n = 16 (CG)

Table 4. GEE model's main and interaction effects for all dependent variables

		Solution Rates (n_S = 159) n_F = 931)		Understanding of the Problem (n_S = 157) (n_F = 933)		Perceived Difficulty (n_S = 159) (n_F = 880)		Invested Effort (n_S = 158) (n_F = 887)		Processing Time (n_S = 155) (n_F = 799)	
Factor	*Df*	Wald-χ^2	*p*	Wald-χ^2	*p*	Wald-χ^2	*p*	Wald-χ^2	*p*	Wald-χ^2	*p*
(A) Representation Type	1	7.662	.006	12.865	< .001	11.159	.001	9.336	.002	27.262	< .001
(B) Level of Pre-Structuring	2	10.830	.004	17.276	< .001	19.015	< .001	6.668	.036	10.222	.006
(C) Problem Type	2	4.976	.083	17.155	< .001	97.917	< .001	57.703	< .001	144.632	< .001
A x B	2	7.878	.019	4.312	.116	0.012	.994	1.518	.468	9.848	.007
A x C	2	15.471	< .001	0.932	.627	1.444	.486	1.469	.480	7.200	.027
B x C	4	11.218	.024	5.189	.268	3.287	.511	12.611	.013	8.591	.072
A x B x C	4	6.597	.159	7.107	.130	3.530	.473	2.715	.607	9.451	.051

Note: n_P = number of subjects, n_F = number of cases.

To be able to evaluate the efficiency of the drawings and tables, perceived difficulty, invested effort and processing time in a second step was analysed for those problems only which the participants solved correctly (n = 337). GEE model revealed no significant main effect of the type of representation for perceived difficulty (Wald-$\chi^2(1) = 0.406$, $p = .524$), and invested effort (Wald-$\chi^2(1) = 0.201$, $p = .654$), but for processing time (Wald-$\chi^2(1) = 7.242$, $p = .007$): When a drawing was provided, participants worked $M = 162$ seconds ($SD = 97$) on the problems compared to $M = 203$ seconds ($SD = 113$) when a table was provided.

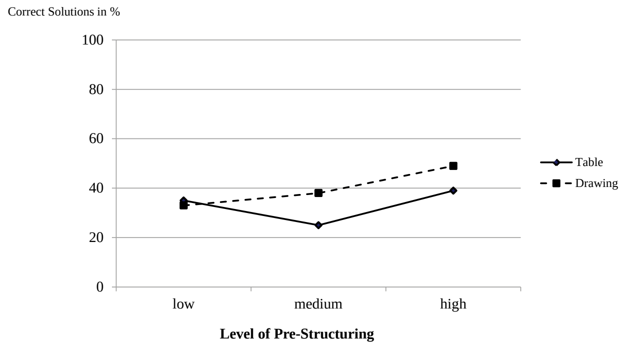

Figure 3. Interaction of type of representation*level of pre-structuring for solution rates

6.2.2. How much Pre-structuring?

The GEE model revealed a significant main effect for level of pre-structuring for all dependent variables (see Table 4). In line with H2b, providing representations with more pre-structuring resulted in higher solution rates, better understanding of the problem, and less perceived difficulty than providing representations with less pre-structuring. However, contrary to our assumption invested effort did not change significantly with an increasing level of pre-structuring taking a Bonferroni-corrected significance level of $p < .017$ into account, and processing time was significantly longer when a high level of pre-structuring was provided than when a medium ($p = .004$) or a low level ($p = .006$) was presented. Table 6 gives an overview of the mean values. With respect to solution rates the effect of the pre-structuring could be observed only for drawings, but not for tables. Figure 3 visualizes this interaction effect (A*B). For the table, solution rates did not differ significantly between the levels of pre-structuring. For the drawing, the low level ($M = 0.33$, $SD = 0.47$) differed significantly ($p = .001$) from the high level ($M = 0.49$, $SD = 0.50$). Moreover, there was a significant interaction effect of level of pre-structuring*problem-type" (B*C) for solution rates. Providing more pre-structuring did result in higher solution rates with the comparison and motion problems, but not with the combinatorics problems. Figure 4 depicts this interaction effect.

To be able to evaluate the efficiency of the different levels of pre-structuring, perceived difficulty, invested effort and processing time was analysed for those problems only which the participants solved correctly (n = 337). GEE model revealed a significant main effect for level of pre-structuring only for perceived difficulty (Wald-$\chi 2(2) = 7.645$, $p = .022$): When a low level of pre-structuring was provided, participants perceived a higher difficulty with $M = 2.02$ ($SD = 0.81$) compared to $M = 1.80$ ($SD = 0.78$) when a high level was provided ($p = .027$).

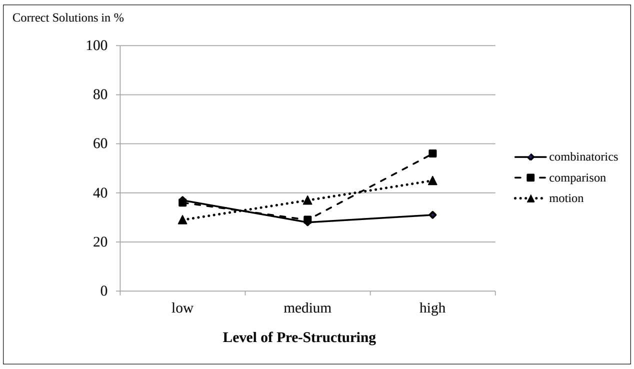

Figure 4. Interaction of level of pre-structuring*type of word problem for solution rates

7. Summary and Discussion

Contrary to hypothesis 1a and hypothesis 1b, providing tables and drawings yielded no treatment-effect (*working with* external representations) and no transfer-effect (*learning from* external representations). Obviously, the provided tables and drawings did not enhance and facilitate the construction and use of an adequate mental model. Only one partial finding was in line with our assumptions: Processing time for the motion problems decreased significantly more in the EG than in the CG, suggesting that the provided representation made cognitive processes more efficient for that type of problem. However, students seemed to have difficulties in externalizing their cognitive processes to external representations. This is in line with results of studies observing children's solution processes in non-routine word problem solving. According to these studies, students often do not spontaneously self-generate external representations [5,16,18,22,27], even if explicitly requested to do so or make use of ready-made drawings [13]. Furthermore, if representations are self-generated, they often do not reflect the structure of the problem [14,25]. The present study showed that even provided representations with different levels of pre-structuring – which should invite problem-solvers to edit the drawings and tables – were ignored or the children had difficulties in using them appropriately. A qualitative analysis of the solution processes revealed that only in 54 % of the cases participants edited the provided representations, and even more dramatically, only in 11 % of the cases the tables and drawings were edited as intended by the authors. In contrast to Fagnant and Flassis [18], who reported a positive effect of simply exposing students to possible representations, in the present study providing representations was obviously not enough.

When a representation was provided, the drawings led to higher solution rates than the tables for the combinatorics problem and for a medium and a high level of pre-structuring. This was in line with hypothesis 2. Apparently, for the combinatorics tasks the drawing enhanced mental model construction and use, whereas the table did not. With its matrix structure, the table obviously distracted participant's cognitive processes or conflicted with their preferred strategy of pairwise listing the combinations. As a qualitative analysis of the solution processes revealed, CG students showed that type of solution strategy very frequently and successfully. However, contrary to our assumption, for the comparison and motion problem as well as for the low level of pre-structuring both types of representations seemed to be equally (in)effective. Because students often did not (appropriately) use the provided representations, especially when a low level of pre-structuring was given, we assume that the drawings could not develop their full potential. However, providing a drawing made the solution processes more efficient: Participants arrived more quickly at the correct answer when a drawing was provided than when a table was given. This was in line with the ITPC model of Schnotz and Bannert [45].

Table 5. Mean values of all dependent variables for type of representation

	Type of Representation					
	Table			Drawing		
Dependent Variables	*n*	*M*	*SD*	*n*	*M*	*SD*
Solution Rates [a]	470	0.33	0.47	461	0.40	0.49
Understanding of the problem [b]	465	1.73	1.18	468	1.96	1.11
Perceived Difficulty [c]	437	2.12	0.92	443	1.97	0.87
Invested Effort [d]	444	2.48	0.96	443	2.33	0.96
Processing Time (in seconds)	399	199	132	400	163	110

[a] mean value between 0 und 1, [b] rating-scale from 0 = completly wrong understanding to 3 = completly correct understanding, [c] self-rating of the students on a scale ranging from 1 = very easy to 4 = very difficult, [d] self-rating of the students on a scale ranging from 1 = no effort at all to 4 = very much effort

Table 6. Mean values of all dependent variables for level of pre-structuring

	Level of Pre-Structuring								
	low			medium			high		
Dependent Variables	*n*	*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*
Solution Rates [a]	309	0.34	0.47	311	0.31	0.47	311	0.44	0.49
Understanding of the problem [b]	311	1.72	1.18	311	1.77	1.14	311	2.04	1.11
Perceived Difficulty [c]	289	2.19	0.90	292	2.05	0.92	299	1.89	0.84
Invested Effort [d]	292	2.46	0.95	298	2.45	0.98	297	2.30	0.94
Processing Time (in seconds)	269	176	112	268	169	137	262	198	117

[a] mean value between 0 und 1, [b] rating-scale from 0 = completly wrong understanding to 3 = completly correct understanding, [c] self-rating of the students on a scale ranging from 1 = very easy to 4 = very difficult, [d] self-rating of the students on a scale ranging from 1 = no effort at all to 4 = very much effort.

Providing a high level of pre-structuring led to higher solution rates than providing a medium level which was in line with hypothesis 3. But contrary to our assumptions, it made no difference if a low or high level was provided. It seemed that both the high and the low level were effective for problem solving. Given a low level of pre-structuring, participants had more freedom to develop the provided representation according to their own strategies, whereas a medium level restricted the number of possible (cognitive) operations and procedures without providing enough structure to read off the solution at the same time. With respect to efficacy, providing a low level of pre-structuring even seemed to be superior. Although participants who provided the correct solution reported less perceived difficulty when a high level of pre-structuring was provided compared to a low level, they showed a longer processing time, which was contrary to our expectations. A possible explanation is that more pre-structure meant more information to process. Providing a highly pre-structured representation reduced construction effort at the expense of interpretational effort [10,21,59]. From a cognitive load perspective [47,48], exposing students with a high level of prior knowledge to highly pre-structured tables or drawings can result in an expertise reversal effect [29] making the task more difficult.

8. Conclusions

As a conclusion, we agree with other author's propositions of an early teaching program fostering diagram literacy [21,51]. Apparently, primary students need to be taught how to appropriately use drawings and tables as tools to externalize cognitive load and to reason with them. Subsequent studies should not only provide representations, but integrate a short training session on how to use the prefabricated tables and drawings and additionally give feedback on the student's solutions during an intervention phase. Several training studies with primary students showed promising results [11,24]. Furthermore, future studies could use eye-tracking procedures to analyze primary student's processes of text-picture-integration while solving non-routine word problems more thoroughly.

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Conflict of Interests

The authors have no competing interests.

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