

Wittengstein's *Language Games* and *Forms of Life* from a Social Constructivist Point of View

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Abstract In this paper our main objective is to interpret the major concepts in Wittgenstein's philosophy of mathematics, in particular, *language games* and *forms of life*, from a social constructivist point of view in an attempt to show that this philosophy is still very relevant in the way mathematics is being taught and practiced today. We start out with a brief discussion of *radical constructivism* followed by a rudimentary analysis of the basic tenets of *social constructivism*, the final blow against *absolutism* – the soulless landmark of mathematics as often construed by the uninitiated. We observe that, the social constructivist epistemology of mathematics reinstates mathematics, and rightfully so, as "...a branch of knowledge which is indissolubly connected with other knowledge, through the web of language" (Ernest 1999), and portrays mathematical knowledge as a process that should be considered in conjunction with its historical origins and within a social context. Consequently, like any other form of knowledge based on human opinion or judgment, mathematical knowledge has the possibility of losing its truth or necessity, as well. In the third section we discuss the main points expounded in Wittgenstein's two books, *Tractatus Logico-Philosophicus* and *Philosophical Investigations*, as well as in his "middle period" that is characterized by such works as *Philosophical Remarks*, *Philosophical Grammar*, and *Remarks on the Foundations of Mathematics*. We then briefly introduce the two main concepts in Wittgenstein's philosophy that will be used in this paper: forms of life and language games. In the fifth and final section, we emphasize the connections between social constructivism and Wittgenstein's philosophy of mathematics. Indeed, we argue that the apparent certainty and objectivity of mathematical knowledge, to paraphrase Ernest (Ernest 1998), rest on natural language. Moreover, mathematical symbolism is a refinement and extension of written language: the rules of logic which permeate the use of natural language afford the foundation upon which the objectivity of mathematics rests. Mathematical truths arise from the definitional truths of natural language, and are acquired by social interaction. Mathematical certainty rests on socially accepted rules of discourse embedded in our *forms of life*, a concept introduced by Wittgenstein (Wittgenstein, 1956). We argue that the social constructivist epistemology draws on Wittgenstein's (1956) account of mathematical certainty as based on linguistic rules of use and forms of life, and Lakatos' (1976) account of the social negotiation of mathematical concepts, results, and theories.

Keywords: *radical constructivism, social constructivism, forms of life, language games*

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1. Introduction

Constructivism, a movement associated with such figures as Immanuel Kant (1724-1804), John Dewey (1859-1952), and Jean Piaget (1896-1980), is an epistemological perspective which asserts that the concepts of science are mental constructs posited to elucidate our sensory encounters. The fundamental tenets of constructivist epistemology are:

- I) Knowledge is a construct rather than a compilation of empirical data
- II) There is no single valid methodology in science, but rather a diversity of functional and effective methods
- III) One cannot focus on an ontological reality, but instead on a constructed reality. Indeed, search for

ontological reality is entirely illogical, since to verify one has reached a definitive notion of Reality, one must already know what Reality consists of

IV) Knowledge and reality are products of their cultural context, that is, two independent cultures will likely form different observational methodologies.

Although the term *constructivist epistemology* was first used by Jean Piaget in his famous 1967 article *Logique et Connaissance Scientifique*¹ (Piaget 1967), one can trace constructivist ideas back to early Greek philosophers such as Heraclitus (c.535 BCE - c. 475 BCE), Protagoras (490 BCE - 420 BCE), and Socrates (c. 469 BCE – 399 BCE). Indeed, Heraclitus' adage *panta rhei* (everything flows),

¹ This article appeared in the *Encyclopédie de la Pléiade*. The title can be translated as Logic and Scientific Knowledge. See also Berger and Luckmann (1967).

Protagoras' claim that *man is the measure of all things*², and the Socratic maxim "I only know that I know nothing,"³ can clearly be interpreted as harbingers of the constructivist paradigm. This perspective was even more discernible in the works of Pyrrhonian skeptics⁴, who rejected the prospect of attaining truth either by sensory means or by reason, who, in fact, even considered the claim that nothing could be known to be dogmatic. See Bett (2000) or Svavarsson (2010) for more information.

Remaining dormant for a few centuries, constructivist epistemology was revitalized by the Italian philosopher, historian, and rhetorician, Giambattista Vico (1668–1744). Vico, best known for his book *La scienza nuova*⁵ - considered by many to be his masterpiece - was opposed to all kinds of reductionism, and hence to Cartesian synthesis.

In his 1710 essay *De Antiquissima Italorum Sapientia* (On the Ancient Wisdom of Italians), he articulated the principle *verum esse ipsum factum* (the truth itself is made), that is, that truth is verified through creation or invention and not, as promoted by Cartesians through observation - a proposition that can certainly be interpreted as an early instance of constructivist epistemology (Bizzell and Herzberg 2000).

Indeed, by the same token, the British idealist philosopher George Berkeley (1685-1753), the Bishop of Cloyne, whose claim *esse est percipi* (to be is to be perceived) challenged metaphysics, can also be construed as one of the forefathers of constructivism.

In 1970s, the German-American philosopher Ernst von Glasersfeld (1917-2010), who referred to the above type of constructivism as *trivial constructivism*, introduced the idea of *radical constructivism*, based on two premises:

- Knowledge is not passively received but actively built up by the cognizing subject
- The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality (Glaserfeld 1989, 162).

The term *radical* was used primarily to emphasize the fact that from an epistemological perspective, any constructivism had to be radical in order not to revert back into some form of realism. For details see Glasersfeld (1983, 1989, 1990, and 1995).

2. Radical Constructivism In Philosophy of Mathematics: Social Constructivism

One of the most crucial and most disquieting issues in the modern philosophy of mathematics is the *absolutist*⁶ versus the *conceptual change (fallibilist)* dichotomy.

The absolutist philosophies, which date back to Plato, assert that mathematics is a compilation of absolute and certain knowledge, whereas the opposing *conceptual change* perspective contends that mathematics is a

corrigible, fallible and transmuting social product (Putnam 2000).

Absolutism makes two basic assumptions. First of all, it assumes that mathematical knowledge is, in principle, separable from other human activities – living possibly in a Platonic netherworld of ideas, casting shadows upon walls while waiting to be discovered. The second assumption is that mathematical knowledge, logic, and the mathematical truths obtained through their applications are absolutely valid and eternally infallible. This second assumption can be written as

- i) Certain established rules and axioms are true
- ii) If p is a statement that is proven to be true at time t_0 then p is true at time $t_0 + t$, for any $t \geq 0$.
- iii) Logical rules of inference preserve truth: If p is a true statement, and L is a logical rule of inference, then $L(p)$ is true.

While in science absolutist views, through the collective efforts of philosophers such as Karl Popper (1902-1994), Thomas Kuhn (1922-1996), Imre Lakatos (1922-1974), and Paul Feyerabend (1924-1994), have vanished into the archives of history as definitively and as deservedly as alchemy and astrology, among philosophers of mathematics the absolutist views nevertheless still prevail: mathematics is the epitome of certainty and mathematical truths are universal, and culture- and value-free. Its concepts are discovered, not invented.

There are two major objections to mathematical absolutism. First, as noted by Lakatos (1978), deductive logic, as the means of proof, cannot establish mathematical certainty for it inexorably leads to infinite regress - there is no way to elude the set of assumptions, however minimal, mathematical systems require. Second, even within an axiomatic system, mathematical theorems cannot be considered to be certain, for Gödel's Second Incompleteness Theorem demonstrates that consistency requires a larger set of assumptions than contained within any mathematical system. As we shall see, both of these predicaments can be prevailed over by the implementation of the social constructivist approach (See Section 5).

The *social constructivist* point of view in mathematics, developed by Paul Ernest, is rooted in the *radical constructivism* of Ernst von Glasersfeld. This point of view regards mathematics as a corrigible, and changing social construct, that is, as a cultural product fallible like any other form of knowledge. Presumed in this stance are two claims:

- The origins of mathematics are social or cultural
- The justification of mathematical knowledge rests on its quasi-empirical basis

Social constructivists argue that the absolutist philosophy of mathematics should be replaced by the conceptual change philosophy of mathematics built upon principles of radical constructivism that, nevertheless, does not deny the existence of the physical and social worlds. This requires the incorporation of two extremely natural and undemanding assumptions, namely,

- The assumption of physical reality: There is an enduring physical world, as our common-sense tells us
- The assumption of social reality: Any discussion, including this one, presupposes the existence of the human race and language (Ernest 1999)

²Known as the *homo-mensura* (man-measure) statement.

³In ancient Greek $\acute{\epsilon}\nu\ \acute{\omicron}\tilde{\iota}\tilde{\nu}\tilde{\alpha}\ \tilde{\sigma}\tilde{\iota}\ \acute{\omicron}\tilde{\upsilon}\tilde{\delta}\tilde{\epsilon}\nu\ \acute{\omicron}\tilde{\iota}\tilde{\nu}\tilde{\alpha}$. This was translated to Latin as *scio me nihil scire*.

⁴Named after the sophist philosopher Pyrrho (c. 360 BCE - c. 270 BCE).

⁵The New Science.

⁶As exemplified by Platonism but also including Logicism, Formalism and Intuitionism.

With these added assumptions the principles of radical constructivism can now be extended to elaborate the epistemological basis of social constructivism in mathematics:

- The personal theories which result from the organization of the experiential world must *fit* the constraints imposed by physical and social reality
- They achieve this by a cycle of theory-prediction-test-failure-accommodation-new theory
- This gives rise to socially agreed theories of the world and social patterns and rules of language use
- Mathematics is the theory of form and structure that arises within language.

The social constructivist epistemology does effectively circumvent criticisms of solipsism and subjectivism (Goldin 1989, Kilpatrick 1987), since “the principles of radical constructivism are consistent with, and can be supplemented by assumptions of the existence of physical and social reality” (Ernest 1999).

For more details on absolutist versus conceptual change philosophies of mathematics see Thom (1973), Wilder (1981), Restivo (1988), and Ernest (1991, 1998, and 1999). For the implications of conceptual change approach on mathematics curriculum see Confrey (1981).

3. The Three Wittgensteins: *Tractatus Logico-Philosophicus* vs. the Middle Period vs. *Philosophical Investigations*

Ludwig Josef Johann Wittgenstein (1889 – 1951) was an Austrian born British philosopher, whom Russell deemed to be “perhaps the most perfect example... of genius as traditionally conceived, passionate, profound, intense, and dominating” (McGuinness 1988, 118, Edmonds and Eidinow 2002, 44).

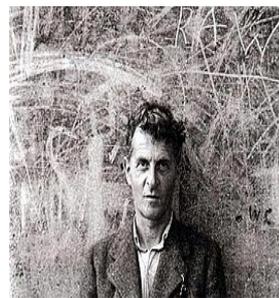
An “arresting combination of monk, mystic, and mechanic,” as elegantly put by the literary theorist Eagleton (Edmonds and Eidinow 2002, 22), Wittgenstein was a rather enigmatic, unfathomable character, at times deeply contemplative, at times utterly pugnacious, and almost always resplendent with inconsistencies and paradoxes. Born into one of Europe's most opulent families, he gave away his entire inheritance. Three of his brothers committed suicide, and he constantly pondered it, as well. Though Jewish, he often expressed anti-Semitic feelings (Redner, 2002). A professor of philosophy at the University of Cambridge from 1939 until 1947, he left academia on several occasions - at times to travel to and to live in isolated areas for extended periods⁷, at times to teach elementary school, and at times to serve as an ambulance driver (during the Second World War) - only to return each time. He described philosophy as “the only work that gives me real satisfaction” (Malcolm 1958, 84), yet, in his lifetime he published just one book of philosophy, the 75-page *Tractatus Logico-Philosophicus* (1921), written in the trenches during World War I, with the title so chosen to pay homage to *Tractatus Theologico-Politicus* of Baruch Spinoza (Sahlin 1990, 227). His *Philosophical Investigations* was not ready for

publication when he died in 1951. G.E.M. Anscombe translated the manuscript, which was posthumously published in 1953, and was later ranked as one of the most significant philosophical tour de forces of twentieth century philosophy. The two books also demarcate the two distinct periods in Wittgenstein's philosophy, namely, his early period, epitomized by the *Tractatus Logico-Philosophicus*, and the later, articulated in the *Philosophical Investigations*.

Although Wittgenstein worked primarily in logic and the philosophy of language, his contributions to the philosophy of mathematics were quitesubstantial and noteworthy (Dummett 1959, Gerrard 1991 and 1996, Diamond 1996, Floyd 2000 and 2005). Indeed, Wittgenstein, who devoted the majority of his writings from 1929 to 1944 to mathematics, himself said that his

... chief contribution has been in the philosophy of mathematics (Monk 1990, 40).

It is customary to distinguish three periods in Wittgenstein's philosophy of mathematics: The early period characterized by the concise treatise *Tractatus Logico-Philosophicus*, the middle period exemplified by such works as *Philosophical Remarks*, *Philosophical Grammar*, and *Remarks on the Foundations of Mathematics*, and the late period embodied by *Philosophical Investigations* (Malcolm (1958), Monk (1990), Bartley (1994), Glock (1996) or Edmonds and Eidinow (2002).



Ludwig Wittgenstein⁸

The aim of the *Tractatus*, as clarified in 4.113, was to reveal the relationship between language and the world, that is to say, to identify the association between language and reality and to define the limits of science.

It was Bertrand Russell who as a logical atomist⁹ pioneered the rigorous use of the techniques of logic to elucidate the relationship between language and the world. According to logical atomists all words stood for objects. So, for instance, for a logical atomist the word “computer” stands for the object computer. But then what object does “iron man” signify?

Let us look at Russell's famous example, the phrase “The King of France is bald.” This is an utterly coherent construction but what does “the King of France” stand for? Russell construed that to think of “the King of France” behaving like a name was causing us to be confused by

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http://en.wikipedia.org/wiki/File:Ludwig_Wittgenstein_by_Ben_Richard_s.jpg

⁹Russell was the originator and the principal exponent of *logical atomism*, the philosophical belief that the world consists of ultimate logical facts (*atoms*) that cannot be broken down any further. This was also the view of Wittgenstein of *Tractatus* who later rejected it in his *Philosophical Investigations*.

⁷ In western Ireland, in Iceland, and in Norway, where in 1913 he built himself a wooden house.

language. He posited that this sentence, in fact, was formed of three logical statements:

- i) There is a King of France.
- ii) There is only one King of France.
- iii) Whatever is King of France is bald.

If *Fx* stands for “*x* is the King of France” and *Gx* stands for “*x* is bald,” then the sentence can be written as follows:

$$\exists x[Fx \& (y)(Fy \rightarrow y = x) \& Gx]$$

There is an *x* such that *x* is the King of France (first statement) and if *y* is the King of France then *x* and *y* must be the same (second statement), and *x* is bald (the third statement).

The Wittgenstein of *Tractatus* was working in the same intellectual universe of Russell’s logical atomism. Like Russell, early Wittgenstein believed that everyday language obscured its underlying logical structure. He argued that language had a core logical structure, a structure that established the limits of what can be said meaningfully, and ergo, the limits of what can be thought. In fact, he wrote in the preface:

The book will, therefore, draw a limit to thinking, or rather—not to thinking, but to the expression of thoughts; for, in order to draw a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought) (Bartley, 1990, 160ff.)

Much of philosophy, Wittgenstein claimed, involved attempts to verbalize what in fact could not be verbalized, and that by implication should be unthinkable:

What can we say at all can be said clearly. Anything beyond that—religion, ethics, aesthetics, the mystical—cannot be discussed. They are not in themselves nonsensical, but any statement about them must be (Bartley 1990, 40–44).

Tractatus was devoted to explaining what a meaningful proposition was - what was asserted when a sentence was used meaningfully. It comprised propositions numbered from one to seven, with various sub-levels denoted 1, 1.1, 1.1.1, ... (Klagge, 2001, p. 185):

1. *Die Welt ist alles, was der Fall ist.* The world is all that is the case.
2. *Was der Fall ist, die Tatsache, ist das Bestehen von Sachverhalten.* What is the case—a fact—is the existence of states of affairs.
3. *Das logische Bild der Tatsachen ist der Gedanke.* A logical picture of facts is a thought.
4. *Der Gedanke ist der sinnvolle Satz.* A thought is a proposition with a sense.
5. *Der Satz ist eine Wahrheitsfunktion der Elementarsätze.* A proposition is a truth-function of elementary propositions.
6. *Die allgemeine Form der Wahrheitsfunktion ist: $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$.* Dies ist die allgemeine Form des Satzes. The general form of a truth-function is: $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$. This is the general form of a proposition.
7. *Wovon man nicht sprechen kann, darüber muß man schweigen.* What we cannot speak about we must pass over in silence.

1	Die Welt ist alles, was der Fall ist.
1.1	Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge.
1.11	Die Welt ist durch die Tatsachen bestimmt und dadurch, dass es alle Tatsachen sind.
1.111	Denn, die Gesamtheit der Tatsachen bestimmt, was der Fall ist und auch, was alles nicht der Fall ist.
1.1111	Die Tatsachen im logischen Raum sind die Welt.
1.11111	Die Welt zerfällt in Tatsachen.
1.111111	Man kann der Fall sein oder nicht der Fall sein und alles übrige gleich bleiben.
1.1111111	Was der Fall ist, die Tatsache, ist das Bestehen von Sachverhalten.
1.11111111	Der Sachverhalt ist eine Verbindung von Gegenständen. (Sachen, Dingen.)
1.111111111	Es ist dem Ding wesentlich, der Bestandteil eines Sachverhalts sein zu können.
1.1111111111	In der Logik ist nichts zufällig: Wenn das Ding im Sachverhalt vorkommen kann, so muss die Möglichkeit des Sachverhalts im Ding bereits präjudiziert sein.
1.11111111111	Es erscheint gleichsam als Zufall, wenn dem Ding, das allein für sich bestehen könnte, nachträglich eine Sachlage passen würde.
	Wenn die Dinge in Sachverhalten vorkommen können, so muss dies schon in ihnen liegen. (Etwas Logisches kann nicht vor-möglich sein. Die Logik handelt von jeder Möglichkeit und alle Möglichkeiten sind ihre Tatsachen.)

* Die Darstellung als Nennens der einzelnen Sätze drückt die logische Gewichtung der Sätze an, den Nachdruck, der auf ihnen in meiner Darstellung liegt. Die Sätze 1.1, 1.1.1, 1.1.1.1, etc., sind Bemerkungen zum Satz 1.1, 1.1.1, etc. Bemerkungen zum Satz 1.1.1, 1.1.1.1, etc. sind so weiter.
30

First page of *Tractatus*¹¹

Wittgenstein proclaimed that the only genuine propositions, namely, propositions we can use to make assertions about reality, were empirical propositions, that is, propositions that could be used correctly or incorrectly to depict fragments of the world. Such propositions would be true if they agreed with reality and false otherwise (4.022, 4.25, 4.062, 2.222). Thus, the truth value of an empirical proposition was a function of the world.

Accordingly, mathematical propositions are not real propositions and mathematical truth is purely syntactical and non-referential in nature. Unlike genuine propositions, tautologies and contradictions (and Wittgenstein claimed that all mathematical proofs and all logical inferences, no

Tractatus Logico-Philosophicus

By
LUDWIG WITGENSTEIN

With an Introduction by
BERTRAND RUSSELL, F.R.S.



NEW YORK
HARCOURT, BRACE & COMPANY, INC.
LONDON: KEGAN PAUL, TRENCH, TRUBNER & CO., LTD.
1922

Title page, first English-language edition, 1922¹⁰

¹⁰http://en.wikipedia.org/wiki/File:Tractatus_title_page_1922_Harcourt.png

¹¹http://en.wikipedia.org/wiki/File:Tractatus_first_page_1992_Harcourt.png

matter how intricate, are merely tautologies) “have no ‘subject-matter’” (6.124), and “say nothing about the world”(4.461). Mathematical propositions are “pseudo-propositions” (6.2) whose truth merely demonstrates the equivalence of two expressions (6.2323): mathematical pseudo-propositions are equations which indicate that two expressions are equivalent in meaning or that they are interchangeable. Thus, the truth value of a mathematical proposition is a function of the idiosyncratic symbols and the formal system that encompasses them.

The logical positivists, who claimed that meaningful statements had either to be analytic¹² or open to observation, of course, adopted *Tractatus* as their Bible (Edmonds and Eidinow, 158).

The middle period in Wittgenstein’s philosophy of mathematics is characterized by *Philosophical Remarks* (1929-1930), *Philosophical Grammar* (1931-1933), and *Remarks on the Foundations of Mathematics* (1937-1944).

One of the most crucial and most pivotal aspects of this period is the (social constructivist) claim that “we make mathematics” by inventing purely formal mathematical calculi (Waismann 1979, 34, footnote 1). While doing mathematics, we are not *discovering* preexisting truths

that were already there without one knowing (Wittgenstein 1974, 481).

We use *stipulated* axioms (Wittgenstein, 1975 section 202) and syntactical rules of transformation to *invent* mathematical truth and mathematical falsity (Wittgenstein 1975 Section 122).

That mathematical propositions are pseudo-propositions and that the propositions of a mathematical calculus do not refer to anything is still prevalent in the middle period:

Numbers are not represented by proxies; numbers are there (Waismann 1979, 34, footnote 1).

Thus, this period is characterized by the principle that mathematics is a human invention. Mathematical objects do not exist independently. Mathematics is a product of human activity.

One cannot discover any connection between parts of mathematics or logic that was already there without one knowing (Wittgenstein 481)

The entirety of mathematics consists of the symbols, propositions, axioms and rules of inference and transformation.

The later Wittgenstein, namely the Wittgenstein of *Philosophical Investigations* (*Philosophische Untersuchungen*), repudiated much of what was expressed in the *Tractatus Logico-Philosophicus*. In *Philosophical Investigations*, language was no longer a considered to be delineation but an implement. The meaning of a term cannot be determined from what it stands for; we should, rather, investigate how it is actually used.

Whereas the *Tractatus* had been an attempt to set out a logically perfect language, in *Philosophical Investigations* Wittgenstein emphasized the fact human language is more complex than the naïve representations that attempt to explain or simulate it by means of a formal system (Remark 23). Consequently, he argued, it would be erroneous to see language as being in any way analogous to formal logic.



Cover of *Philosophical Investigations*, Oxford, B. Blackwell 1953¹³

Philosophical Investigations was unique in its style, in that it treated philosophy as an activity, rather along the lines of Socratic *maieutics*. The following *gedanken* experiment is an example:

...think of the following use of language: I send someone shopping. I give him a slip marked 'five red apples'. He takes the slip to the shopkeeper, who opens the drawer marked 'apples', then he looks up the word 'red' in a table and finds a color sample opposite it; then he says the series of cardinal numbers—I assume that he knows them by heart—up to the word 'five' and for each number he takes an apple of the same color as the sample out of the drawer.—It is in this and similar ways that one operates with words—"But how does he know where and how he is to look up the word 'red' and what he is to do with the word 'five'?" Well, I assume that he 'acts' as I have described. Explanations come to an end somewhere. But what is the meaning of the word 'five'? No such thing was in question here, only how the word 'five' is used. (Wittgenstein 1956, Part 1, Section 1)

Wittgenstein of *Philosophical Investigations* held that language was not enslaved to the world of objects. Human beings were the masters of language not the world. We chose the rules and we determined what it meant to follow the rules.

One might say that *Tractatus* is modernist in its formalism while the *Investigations* anticipates certain postmodernist themes (Peters & Marshall 1999a, 1999b, 2001).

There is disagreement on whether later Wittgenstein follows from middle Wittgenstein as is claimed by Wrigley (1993) or Rodych (2000) or if it is significantly different than that as claimed by Gerrard (1991) or Floyd (2005). As far as his philosophy of mathematics is concerned, we claim that there is at least one persistent thesis in both (in fact in all three) periods. This, the most enduring constant in Wittgenstein’s philosophy of mathematics is the claim that mathematics is a human invention. Just like the early and middle Wittgenstein, the late Wittgenstein also claims we “invent mathematics” (Wittgenstein 1956, I, 168; II, 38, V, 5, 9, 11) and thus

... the mathematician is not a discoverer, he is an inventor (Wittgenstein 1956, Appendix II, 2).

¹² That is its truth or falsity can be assessed by examining the meaning of the words or symbols utilized.

¹³ http://www.google.com/search?rlz=1T4GGIC_enUS229US237&q=philosophical+investigations+wittgenstein&um=1&ie=UTF-8&hl=en&tbn=isch&source=og&sa=N&tab=wi&biw=1024&bih=558&sei=WQTRTqnRCpLksQLc-MC7Dg

4. Language Games and Forms of Life

Wittgenstein's metalanguage in *Philosophical Investigations* included such terms as *meaning as use*, *rule-following*, *language games*, *family resemblance*, *private language*, *grammar*, and *forms of life*. Of these we discuss, albeit briefly, family resemblance, language games and forms of life.

4.1. Family Resemblance and Language Games

Let us start out by depicting what Wittgenstein meant by *family resemblance* (*Familienähnlichkeit*)¹⁴. Some words, which at first glance look as if they perform similar functions, actually operate to distinct set of rules. It is, to use Wittgenstein's own example, like peeking into the cabin of a locomotive and seeing handles that all look more or less alike.

But one is the handle of a crank which can be moved continuously (it regulates the opening of a valve); another is the handle of a switch, which has only two effective positions, it is either off or on; a third is the handle of a brake-lever, the harder one pulls on it, the harder it breaks; a fourth, the handle of the pump: it has an effect only so long as it is moved to and fro (Wittgenstein 1953, 7).

If we examine how language is actually used, we will notice that most terms have not just one use but a multiplicity of uses, and that these various applications do not necessarily have a single component in common. The example Wittgenstein gives for that is the term "game." This word could be referring to sports, to kids playing, to a competitive game, to an individual game, to a cooperative game, to a game of skill, or to game of chance, etc. There is nothing that unites all games. There is no "essence" of game. Wittgenstein called such terms *family resemblance concepts*. They are like a family, some members of which might have the distinctive family baldness, or eye color, so on, but there is not a single characteristic possessed by all.

Wittgenstein introduced the term *language-game* (*Sprachspiel*) to designate forms of language simpler than the entirety of a language itself,

consisting of language and the actions into which it is woven (Wittgenstein 1953, 7)

and connected by family resemblance.

The classic example of a language-game is the so-called *builder's language* introduced in Section 2 of *Philosophical Investigations*:

The language is meant to serve for communication between a builder A and an assistant B. A is building with building-stones: there are blocks, pillars, slabs and beams. B has to pass the stones, in the order in which A needs them. For this purpose, they use a language consisting of the words "block", "pillar", "slab", "beam". A calls them out; B brings the stone which he has learnt to bring at such-and-such a call (Wittgenstein 1953, 2).

Later the words "this", "there" and numerals "a, b, c, d" are added. An example of its use: builder A says "d — slab — there" and points, and builder B counts four slabs,

"a, b, c, d..." and moves them to the place pointed to by A. The builder's language is an activity, it is a language but in a simpler form. This language-game resembles the simple forms of language taught to children, and Wittgenstein asks that we conceive of it as "a complete primitive language" for a tribe of builders (Kopytko, 2006).

Wittgenstein also gives the example of "Water!" which can be used as an exclamation, an order, a request, or as an answer to a question. The meaning the word has depends on the language-game in which it is used; the word "water" has no meaning apart from its use within a language-game.

Wittgenstein applied his concept of language games to sentences as well. He showed that independent use of a sentence does not yet say anything; it needs to be within some context of use. As an example he used the sentence "Moses did not exist" (§79) which can be used so as to say that no person or historical figure fits the set of descriptions attributed to the person that goes by the name of "Moses". But it can also mean that the leader of the Israelites was not called Moses. Or that there cannot have been anyone who accomplished all that the Bible relates of Moses, and so on.

Clearly, in order to participate in a language game, one must accept certain rules so that one can use terms within a social discourse. Meaning is, accordingly, established from social patterns. The rules and form of the game may change but are learned through participation in the game. For more details see Harris (1988).

4.2. Forms of Life

Wittgenstein used the term *forms of life* (*Lebensform*) to indicate all constituents - sociological, historical, cultural, psychological, etc., that comprise the milieu within which a given language has meaning (Schatzki 1996). Indeed, the concept of language games was intended

to bring into prominence the fact that the speaking of language is part of an activity, or a form of life (Wittgenstein 1953, 23).

When using a language, agreement is required

... not only in definitions but also in judgments (Wittgenstein 1953, 242).

and this is

... not agreement in opinions but in forms of life (Wittgenstein 1953, 241).

Forms of life are transformative and are dependent on culture, context, and history. However, they are

... the common behavior of mankind (Wittgenstein 1953, 206).

and they constitute

... the system of reference by means of which we interpret an unknown language (Wittgenstein 1953, 206).

Since language is governed by rules, it is essentially community based; it is embedded in our practice, in our forms of life. Rules have to be interpreted; there has to be consensus on what is permissible and what is not. Thus the idea of a private language – a language that only one person can understand – is incoherent.

This concept, in fact, paves the way to yet another espousal of constructivism by Wittgenstein. For, in order for the Cartesian "Cogito ergo sum" to have a meaning, there has to be prior acceptance of what is to count as thinking, and how the concept of "thought" is to be used.

¹⁴ Wittgenstein picked the idea and the term from Nietzsche, who had used it in reference to language families (Sluga H., *Family Resemblance*, Grazer Philosophische Studien 71 (2006) 1).

It is therefore impossible for *cogito* to be the starting point of what we can know. With this, Wittgenstein was overturning hundred years of philosophy by eliminating the Sisyphean burden of search for a primordial certainty.

5. Social Constructivism and Wittgenstein

The social constructivist thesis, as pronounced by Ernest, is that “mathematics is a social construction, a cultural product, fallible like any other branch of knowledge” and that “the justification of mathematical knowledge rests on its quasi-empirical basis.” The fact that the second claim, in addition, to Lakatos (1976, 1978), 3 and Hersh (1980), Kitcher (1983), and Tymoczko (1986), can also be attributed to Wittgenstein (1956), is the point of our paper. Indeed, the social constructivist epistemology, following Ernest (1999) “draws on Wittgenstein’s (1953) account of mathematical certainty as based on linguistic rules of use and ‘forms of life’, and Lakatos’ (1976) account of the social negotiation of mathematical concepts, results and theories.”

For Wittgenstein, mathematics is a type of language game, for, as we shall show below, the formation of mathematical knowledge depends profoundly and organically upon dialogue that reflects the dialectical logic of academic discourse. Wittgenstein distinguishes mere “sign-games” from mathematical language games by the following criterion:

It is essential to mathematics that its signs are also employed in *mufti*. It is the use outside mathematics and so the meaning of the signs, that makes the sign-game into mathematics (Wittgenstein 1953 V, 2).

Put another way,

Concepts which occur in necessary propositions must also occur and have a meaning in non-necessary ones (Wittgenstein 1953 V, 41)

a point also made in *Tractatus* (6.211).

The reason for this to be given as a necessary condition of a mathematical game is Wittgenstein’s interest in the use of natural and formal languages in diverse forms of life (Wittgenstein 1953). This prompts him to emphasize that mathematics plays diverse applied roles in many forms of human activity, such as sciences.

As a natural consequence of its identification as a language game, mathematics ought to possess certain attributes, namely, rules, behavioral patterns, and linguistic usage that must be adhered to. As in any language, these syntactic rules would be crucial to sustain communication among participants.

The structure of these syntactic rules and their modes of acceptance evolve within linguistic and social practices. This, in turn, implies that consensus regarding the acceptance of mathematical proofs and consequently establishing theories arises from a shared language, a set of established guidelines, which are dependent upon social conditions. An excellent example of this dependence is provided by the proof of the Four Color Theorem.

Interpreting mathematics as a language game also helps establish the social nature of mathematics. Language is crucial to social constructivism, as knowledge grows through language

... the social institution of language . . . justifies and necessitates the admission of the social into philosophy at some point or other (Ernest 1998, 131).

Mathematics has many conventions which are, in essence, social agreements on definitions, assumptions and rules, that is to say, mathematical knowledge is a social phenomenon that includes language, negotiation, conversation and group acceptance. The conjectures, proofs and theories arise from a communal endeavor that includes both informal mathematics¹⁵ and the history of mathematics.

As a result,

Social constructivism accounts for both the 'objective' and 'subjective' knowledge in mathematics and describes the mechanisms underlying the genesis . . . of knowledge socially (Ernest 1998, 136).

Consequently, objective knowledge in mathematics is that which is accepted and affirmed by the mathematical community. This in turn implies that the actual objective of a proof is to convince the mathematical community to accept a claimed premise.

To this end, a proof is presented to a body of mathematicians. The proof is then carefully parsed, analyzed, and then accepted or rejected depending on the nonexistence or existence of perceptible flaws. If it is rejected, then a new and improved version is presented. The cycle continues in a similar fashion until there is agreement. Mathematical knowledge is, thus, tentative, and is incessantly analyzed. The process is indeed incessant because the guiding assumptions are based upon human agreements that are capable of changing.

This account of a social constructivist epistemology for mathematics overcomes the two problems that we identified with absolutism in Section 2 – infinite regress and consistency.

Since the concepts of mathematics are derived by abstraction from direct experience of the physical world through negotiations within the mathematical community, mathematics is organically and inseparably coalesced with other sciences through language. Mathematical knowledge - propositions, theorems, concepts, forms of mathematical expressions - is constructed in the minds of individual mathematicians, participating in language games, in other words,

... mathematics is constructed by the mathematician and is not a preexisting realm that is discovered (Ernest 1998, 75).

Consequently, the unreasonable effectiveness of mathematics is a direct result of the fact that it is built into language that is to say, to paraphrase Ernest, it derives from the empirical and linguistic origins and functions of mathematics.

The apparent certainty and objectivity of mathematical knowledge rests on the fact that mathematical symbolism is a sophisticated extension of written language - the rules of logic and consistency which pervade natural language form the crux of the objectivity of mathematics. In other words, as Wittgenstein noted, mathematical truths arise from the definitional truths of natural language, acquired by social interaction

The truths of mathematics are defined by implicit social agreement - shared patterns of behavior - on what

¹⁵The role of informal mathematics is indeed a vital aspect of this argument – the idea is to first establish informal theories then explain them through formal means. It is through this process that mathematical knowledge is polished, and if necessary, reformulated. Consequently, the potential falsification of formal mathematics ultimately depends on informal mathematics.

constitute acceptable mathematical concepts, relationships between them, and methods of deriving new truths from old. Mathematical certainty rests on socially accepted rules of discourse embedded in our forms of life (Wittgenstein, 1956).

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