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Does Temperature Effects the Growth of Microcracks in a Broken Femur with a Prosthetic Device AMBI?

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Abstract In present paper we considered if temperature effects the growth of microcracks in a broken femur with AMBI. We studied three particularly points of fracture. We used theory of adaptive elasticity neglecting and accounting temperature and energy density theory. We showed for both cases: that after a long time femur locally at points of our interest: i) will be (quickly or normally or delayed) united. Our results are verified by clinical studies. We concluded that temperature plays no role to growth of microcracks.

Keywords: *broken femur with AMBI, particularly points of fracture, theory of adaptive elasticity, density energy theory, temperature, microcracks*

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1. Introduction

The purpose of this paper is to study if temperature effects the growth of microcracks in a broken femur with AMBI. For this reason we will use the theory of adaptive elasticity [1,2] neglecting and accounting temperature and energy density theory [3,4,5].

Macroscopically the bone has a volume V and a surface S. The volume V microscopically consists of microvolumes $\underline{\Delta V}$ which generally are not homogenous [4,5]. Expanding theory of adaptive elasticity [[1], p. 322] at microscopic area we assume:

i) A microvolume $\underline{\Delta V}$ of bone consists of an elastic microvolume ΔV (micromatrix bone) and of microporous (microcracks)

$$\underline{\Delta V} = \Delta \mathrm{V} + \Delta \mathrm{p} \quad (1)$$

where Δp is the volume of microcracks.

From the other hand Sih [[4], p.179] showed that every microvolume $\underline{\Delta V}$ contains an homogenous microvolume. Thus we suppose that an elastic microvolume ΔV given by (1) is homogenous microvolume.

ii) The mechanical properties of microvolume of bone $\underline{\Delta V}$ coincides with the mechanical properties of homogenous microvolume ΔV of micromatrix bone.

iii) The fraction of microvolume of the micromatrix bone Δξ is defined as [[2], p. 322]:

$$\Delta\xi = \Delta \mathrm{V} / \underline{\Delta V} = \rho / \gamma \quad (2)$$

where ρ is the density of microvolume ΔV, while γ is the density of material (bone) and assume to be constant. From the above it follows 0< Δξ <1.

v) The porosity that is the mean length of microcraks of microvolume $\underline{\Delta V}$ alters with mass added /removal to /from micro matrix bone and linearly depends from the history of microstrain [[1], p. 322]. The above is characterized by a parameter ê [2]:

$$\hat{e}(\mathrm{t}) = \Delta\xi(\mathrm{t}) - \Delta\xi_o \quad (3)$$

where $\Delta\xi_o$ is the initial fraction of the microvolume of micromatrix bone. With other words parameter ê is the change of the mean value of microcracks.

2. The Problem and Its Physical Approximation

Assume that someone breaks his /her left femur due to fall or traffic accident. Suppose that we deal with an intertrochanteric fracture type A1 and case 3 [[6], p.120; [7], p. 60]. The fracture starts from proximal femur and results to the last third of its diaphysis. We impose into injury bone a prosthetic device AMBI as indicates in Figure 1 and suggest to patient to stay at bed immobilization three months. We want to predict the situation of femur when patient start again walking, locally at three points of fracture area. The lasts have no contact with AMBI as indicated in Figure 2 and are the followings:

Γ: at middle distance between endosteal and periosteal surface and at the end of diaphysis of femur. The origin and end of the diaphysis of femur are defined as: nearest to joint knee and to proximal femur respectively.

Δ: at endosteal surface of femur and at 5/6 of the distance between origin and end of femur's diaphysis.

E: at periosteal surface of femur and at 2/3 of the distance between origin and end of femur's diaphysis.

Figure 1. Prosthetic device AMBI in a left broken femur

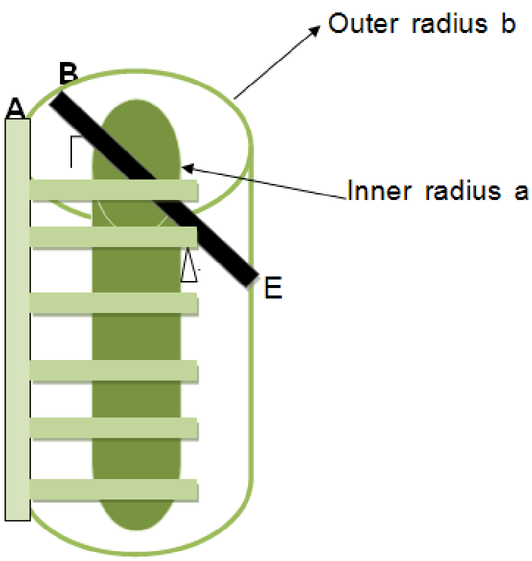

Figure 2. A cylinder length L, with inner and outer radii a and b respectively and with AMBI device. The points Γ, Δ and E belong to fracture area (defined with intense black) but have no contact with AMBI

At t=0 the last third of the diaphysis of femur was separated into two pieces due to fracture. Consequently all points that belong to fracture area ABEZ had êo=+∞ and the same goes for points of our interest Γ, Δ and E. At t >0 we impose a prosthetic device AMBI into broken femur as indicated in Figure 1 and Figure2. We want to predict ê(t) after long time period at points Γ, Δ and E.

3. The Mathematic Modeling and the Solution of Our Problem

Femur's diaphysis is modeled as a hollow circular cylinder with length L, inner and outer radii a and b respectively. These radii corresponds to endosteal and periosteal surfaces of bone and assume to be constant due to internal remodeling [8]. Since we deal with microscopic area, we use the density energy theory [3,4,5].

The equations of the above theory in cylindrical coordinates are:

i) the stress relations between macroscopic and microscopicarea [[4], p. 182]

$$
\begin{aligned}
&\tau_{rr} = \sigma_{rr} + \rho(d^2\breve{u}_r / dt^2)(dV/dA)_r\\
&\tau_{r\theta} = \sigma_{r\theta} + \rho(d^2\breve{u}_\theta / dt^2)(dV/dA)_r\\
&\tau_{rz} = \sigma_{rz} + \rho(d^2\breve{u}_z / dt^2)_z(dV/dA)_r\\
&\tau_{\theta r} = \sigma_{\theta r} + \rho(d^2\breve{u}_r / dt^2)(dV/dA)_\theta\\
&\tau_{\theta\theta} = \sigma_{\theta\theta} + \rho(d^2\breve{u}_\theta / dt^2)(dV/dA)_\theta \qquad (4)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6\text{-}7\text{-}8\text{-}9}\\
&\tau_{\theta z} = \sigma_{\theta z} + (d^2\breve{u}_z / dt^2)(dV/dA)_\theta\\
&\tau_{zr} = \sigma_{zr} + \rho_r(d^2\breve{u}_r / dt^2)(dV/dA)_z\\
&\tau_{\theta r} = \sigma_{\theta r} + \rho(d^2\breve{u}_r / dt^2)(dV/dA)_\theta\\
&\text{and } \tau_{zz} = \sigma_{zz} + \rho(d^2\breve{u}_z / dt^2)(dV/dA)_z
\end{aligned}
$$

where ŭi τ_{ij}, σ_{ij} and (dV/dA) are respectively: the macroscopic displacement, the stress on microvolume, the stress at macrovolume and the change of volume with surface [4,5]

ii) the macrostress - equations [[5], p.182]:

$$
\begin{aligned}
&\sigma_{rr,r} + \sigma_{r\theta,\theta}/r + \sigma_{rz,z} + \sigma_{rr} - \sigma_{\theta\theta} = \gamma(d^2\breve{u}_r / dt^2)\\
&\sigma_{r\theta,r} + \sigma_{\theta\theta,\theta}/r + \sigma_{\theta z,z} + 2\sigma r\theta = \gamma(d^2\breve{u}_\theta / dt^2) \qquad (5)_{1\text{-}2\text{-}3}\\
&\text{and}\\
&\sigma_{rz,r} + \sigma_{\theta z,z} + 2\sigma_{r\theta} + \sigma_{zz,z} + \sigma_{rz}/r = \gamma(d^2\breve{u}_z / dt^2)
\end{aligned}
$$

iii) the microstress equations [[5], p.182]:

$$
\begin{aligned}
&\tau_{rr,r} + \tau_{r\theta,\theta}/r + \tau_{rz,z} + \tau_{rr} - \tau_{\theta\theta} = \rho(d^2\acute{u}_r / dt^2)\\
&\tau_{r\theta,r} + \tau_{\theta\theta,\theta}/r + \tau\theta z,z + 2\sigma r\theta = \rho(d^2\acute{u}_\theta / dt^2) \qquad (6)_{1\text{-}2\text{-}3}\\
&\tau_{rz,r} + \tau_{\theta z,z} + 2\tau_{r\theta} + \tau_{zz,z} + \tau_{rz}/r = \rho(d^2{}_z / \acute{u}dt^2)
\end{aligned}
$$

iv) the microstrain-microdisplacement relations [[5], p.179):

$$
\begin{aligned}
&e_{rr} = 2\partial\acute{u}_r / \partial_r e_{\theta\theta} = 2(\partial\acute{u}_\theta / \partial_\theta + \acute{u}_r)/r\\
&e_{zz} = 2\partial\acute{u}_z / \partial_z e_{r\theta} = \partial\acute{u}_\theta / \partial_r - \acute{u}_\theta / r e_{rz} \qquad (7)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}\\
&= \partial\acute{u}_{r/}\partial_z + \partial\acute{u}_z / \partial_r e_{\theta z} = \partial\acute{u}_\theta / \partial_z + \partial\acute{u}_{z/\theta}
\end{aligned}
$$

v) the microstress - microstrain relations:

$$
\begin{aligned}
&\tau_{rr} = (\lambda_2 + 2\mu_2)e_{rr} + \lambda_2 e_{\theta\theta} + \lambda_1 e_{zz}\\
&\tau_{\theta\theta} = \lambda_2 e_{rr} + (\lambda_2 + 2\mu_2)e_{\theta\theta} + \lambda_1 e_{zz}\\
&\tau_{zz} = \lambda_1(e_{rr} + e_{\theta\theta}) + (\lambda_1 + 2\mu_1)e_{zz} \qquad (8)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}\\
&\tau_{r\theta} = 2\mu_{2r} e_{r\theta} \tau_{rz} = 2\mu_A e_{rz}\\
&\text{and } \tau_{\theta z} = 2\mu_A e_{\theta z}
\end{aligned}
$$

where: $\lambda_1 = v_A E_A E_T / (1 - v_T) E_A - 2v^2 A E_T$

$$
\begin{aligned}
&\lambda_1 = v_A E_A E_T / (1 - v_T) E_A - 2\nu^2 A E_T\\
&2\mu_1 = (1 - v_T) E^2{}_A - v_A E_A E_T / (1 - v_T) E_A\\
&\quad - 2\nu^2 A E_T \qquad (9)_{1\text{-}2\text{-}3\text{-}4}\\
&\lambda_2 = v_T E_T E_A + \nu^2 A E^2{}_T / (1 - v_T) E_A\\
&\quad - 2\nu^2 A E_T (1 + v_T)\\
&2\mu_2 = E_T / (1 + v_T)
\end{aligned}
$$

where E_A, E_T and v_A, v_T are Young's modulus and Poisson ratio in transverse and axial direction at macroscopic area.

Finally rate remodeling equation [2] at microscopic area without /and accounting temperature are respectively:

$$d\hat{e}/dt = A(\hat{e}) + A_T(\hat{e})\,(e_{rr} + e_{\theta\theta}) + A_A(\hat{e})e_{zz}$$
$$\text{and } d\hat{e}(t)/dt = A(\hat{e}) + A_T(\hat{e})(e_{rr} + e_{\theta\theta}) + A_A(\hat{e})e_{zz} + B(\hat{e})\theta \quad (10)_{1\text{-}2}$$

where $A_T(\hat{e})$, $A_A(\hat{e})$ are rate remodeling coefficients in transverse and axial direction respectively while $B(\hat{e})$ is a rate remodeling coefficient depends from temperature.

The boundary conditions of our problem are:

i) at point Γ:

$$\tau_{rr} = \tau_{\theta\theta} = \tau_{rz} = \tau_{\theta z} = \tau_{r\theta} = \tau_{zz} = 0$$
$$\text{at } r = (a+b)/2,\ 0 \le \theta = \theta_o \text{ and } z = L \quad (11)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$

ii) at point Δ:

$$\tau_{rr} = \tau_{\theta\theta} = \tau_{rz} = \tau_{\theta z} = \tau_{r\theta} = \tau_{zz} = 0$$
$$\text{at } r = a,\ 0 \le \theta = \theta_1 \text{ and } z = 5L/6 \quad (12)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$

iii) at point Z:

$$\tau_{rr} = 0,\ \tau_{\theta\theta} = \tau_{rz} = \tau_{\theta z} = \tau_{r\theta} = \tau_{zz} = 0$$
$$\text{at } r = b,\ 0 \le \theta = \theta_2 \text{ and } z = 2L/3 \quad (13)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$

Our problem has a unique solution **[[5],** p.186] and assume that microdisplacements are of the form:

$$\breve{u}_r = A(t)r + B(t)/r\breve{u}_\theta = 0$$
$$\text{and } \breve{u}_z = C(t)z \quad (14)_{1\text{-}2\text{-}3}$$

where A(t), B(t), C(t) are unknowns. Then (7) are written as:

$$e_{rr} = 2A(t) - 2B(t)/r^2$$
$$e_{\theta\theta} = 2A(t) + 2B(t)/r^2$$
$$e_{zz} = 2C(t),\ e_{r\theta} = 0,\ e_{rz} = 0 \quad (15)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$
$$\text{and } e_{\theta z} = 0.$$

Therefore microstress - microstrains relations (8) because of (12) take the forms:

$$\tau_{rr} = 4(\lambda_2 + \mu_2)A(t) - 4\mu_2 B(t)/r^2 + 2\lambda_1 C(t),$$
$$\tau_{\theta\theta} = 4(\lambda_2 + \mu_2)A(t) + 4\mu_2 B(t)/r^2 + 2\lambda_1 C(t) \quad (16)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6}$$
$$\tau_{zz} = 4\lambda_1 A(t) + 2(\lambda_1 + 2\mu_1)C(t),$$
$$\tau_{r\theta} = 0,\ \tau_{rz} = 0 \text{ and } \tau_{\theta z} = 0.$$

Applying the boundary conditions (11)-(12)-(13) into (16) it is possible to obtain unknown functions A(t), B(t) and C(t):

i) at point Γ:

$$A_1(t) = 0, B_1(t) = 0 \text{ and } C_1(t) = 0 \quad (17)_{1\text{-}2\text{-}3}$$

ii) at point Δ:

$$A_2(t) = 0,\ B_2(t) = 0 \text{ and } C_2(t) = 0 \quad (18)_{1\text{-}2\text{-}3}$$

iii) at point E

$$A_3(t) = 0,\ B_3(t) = 0 \text{ and } C_3(t) = 0. \quad (19)_{1\text{-}2\text{-}3}$$

Employing (17)-(18)-(19) into (14) it is possible to calculate microdisplacements for points Γ, Δ and E. Also (15) due to (17)-(18)- (19) become:

i) at point Γ:

$$e_{rr}{}^1(t) = 0, e_{\theta\theta}{}^1(t) = 0 \text{ and } e_{zz}{}^1(t) = 0 \quad (20)_{1\text{-}2\text{-}3}$$

ii) at point Δ:

$$e_{rr}{}^2(t) = 0, e_{\theta\theta}{}^2(t) = 0 \text{ and } e_{zz}{}^2(t) = 0 \quad (21)_{1\text{-}2\text{-}3}$$

iii) at point E:

$$e_{rr}{}^3(t) = 0, e_{\theta\theta}{}^3(t) = 0 \text{ and } e_{zz}{}^3(t) = 0. \quad (22)_{1\text{-}2\text{-}3}$$

At continuity we distinguish the following cases:

i) Internal remodeling of femur does not depends upon temperature:

Then substituting (20), (21), (22) into $(10)_1$ it follows:

i) at point Γ:

$$d\hat{e}/dt = A(\hat{e}) \quad (23)$$

ii) at point Δ:

$$d\hat{e}/dt = A(\hat{e}) \quad (24)$$

iii) at point E:

$$d\hat{e}/dt = A(\hat{e}). \quad (25)$$

At continuity we impose [9,10]

$$A(\hat{e}) = c_2\hat{e}^2 + c_1\hat{e} + c_o. \quad (26)$$

Therefore (23)-(24)-(25) conclude to the following form:

$$d\hat{e}/dt = \alpha^{(i)}(\hat{e}^2 - 2\beta^{(i)}\hat{e} + \gamma^{(i)}),\ i = 1,2,3 \quad (27)$$

where:

i) at point Γ:

$$\alpha^{(1)} = c_2, \beta^{(1)} = -c_1/2c_2 \text{ and } \gamma^{(1)} = c_o/c_2 \quad (28)_{1\text{-}2\text{-}3}$$

ii) at point Δ:

$$\alpha^{(2)} = c_2, \beta^{(2)} = -c_1/2c_2 \text{ and } \gamma^{(2)} = c_o/c_2 \quad (29)_{1\text{-}2\text{-}3}$$

iii) at point E:

$$\alpha^{(3)} = c_2, \beta^{(3)} = -c_1/2c_2 \text{ and } \gamma^{(3)} = c_o/c_2. \quad (30)_{1\text{-}2\text{-}3}$$

The solution of (27) must satisfy the following condition

$$0 \le \hat{e}^{(i)}(t) < +\infty, i = 1,2,3 \quad (31)$$

since the mean length of microcracks can not be negative.

From the other hand (27) satisfies the initial condition:

$$\hat{e}^{(i)}(0) = +\infty, i = 1,2,3 \quad (32)$$

because as we stated earlier points Γ, Δ and E belong to fracture area. We defined $\Delta^{(i)} = 4\beta^{(i)2} - 4\alpha^{(i)}\gamma^{(i)}$ and distinguish the following cases [10]:

1) $\Delta^{(i)}<0$. The solution of (27) that satisfies (32) is:

$$\hat{e}^{(i)}(t)=\beta^{(i)}+\sqrt{\left(\gamma^{(i)}-\beta^{(i)2}\right)}\,\mathrm{tg}\left[\alpha^{(i)}\sqrt{\left(\gamma^{(i)}-\beta^{(i)2}\right)}t+\pi/2\right], \quad (33)$$

$i=1,2,3$

i) If $\alpha^{(i)}<0$, then for $t\to-\pi/2\alpha^{(i)}\sqrt{(\gamma^{(i)}-\beta^{(i)2})}$, then it results that $\hat{e}^{(i)}(t)=\beta^{(i)}<+\infty$. We distinguish the following subcases: i_a) If $\beta^{(i)}<0$ the solution has no physical sense, since contradicts to (31). $i_{b)}$ If $\beta^{(i)}=0$ then when the patient get of the bed immobilization, femur at points Γ, Δ and E will be under osteopetrosis. $I_{c)}$ Finally if $0<\beta^{(i)}<+\infty$, then when patient get of bed immobilization, femur at points Γ, Δ, E will be united. In addition if value of parameter $\alpha^{(i)}$ is great, then value of $t_1=-\pi/2\alpha^{(i)}\sqrt{(\gamma^{(i)}-\beta^{(i)2})}$ is small. The last means that femur at points of our interest will be quickly united. If value of parameter $\alpha^{(i)}$ is small, then the value of $t_1=-\pi/2\alpha^{(i)}\sqrt{(\gamma^{(i)}-\beta^{(i)2})}$ is great. The last means that femur at points of our interest will be normally united.

ii) If $\alpha^{(i)}>0$, then for $t\to+\infty$ it follows that $\lim\hat{e}^{(i)}(t)\to+\infty$. The last means that femur at points Γ, Δ and E, may be under a delayed union (healing).

iii) If $\alpha^{(i)}=0$ then $\hat{e}^{(i)}(t)=+\infty$. This is the worst case because it means that the femur at points of our interest has not a union (healing).

2) $\Delta^{(i)}=0$. The solution of (27) is:

$$e^{(i)}(t)=\beta^{(i)}{}_1-1/\left(\alpha^{(i)}t+K\right),\ i=1,2,3 \quad (34)$$

i) If K = 0, then employing initial condition (32) into (34) it results that $e^{(i)}(t)=-\infty$. The last contradicts to (32) and there is no solution. ii) If $K\neq0$, then employing initial condition (32) into (34) it possible to obtain $K\beta^{(i)}{}_1-1=+\infty$. Specially if $K>0$ then $\beta^{(i)}{}_1=+\infty$, that is $e^{(i)}(t)=+\infty$ and we coincide with case 1. iii). From the other hand if $K<0$ then $\beta^{(i)}{}_1=-\infty$, that is $e^{(i)}(t)=-\infty$ and we coincide with case i).

3) $\Delta^{(i)}>0$. Then the solution of (27) that satisfies (32) is:

$$e^{(i)}(t)=\frac{\left[e^{(i)}{}_1-Ke^{(i)}{}_2\exp\left[\alpha^{(i)}(e^{(i)}{}_1-e^{(i)}{}_2)t\right]\right]}{1-K\exp\left[\alpha^{(i)}(e^{(i)}{}_1-e^{(i)}{}_2)t\right]} \quad (35)$$

where:

$$e^{(i)}{}_1=\beta^{(i)}+\sqrt{\left(\beta^{(i)^2}-\gamma^{(i)}\right)}$$
$$\text{and } e^{(i)}{}_2=\beta^{(i)}-\sqrt{\left(\beta^{(i)^2}-\gamma^{(i)}\right)},\ i=1,2,3. \quad (36)_{1\text{-}2}$$

From the above it follows that:

$$e^{(i)}{}_1>e^{(i)}{}_2,\ i=1,2,3. \quad (37)$$

Employing the initial condition (32) into (37) it follows:

$$e^{(i)}{}_1-e^{(i)}{}_2K\ /1-K=-\infty. \quad (38)$$

From the above we conclude that K=1 and $e^{(i)}{}_1-e^{(i)}{}_2<0$. The last contradicts to (37). Therefore (35) has no solution.

The solutions for finite and infinite t are in Table 1. and Table 2. respectively. Accordingly to all that results when patient get of the bed immobilization, his (hers) femur at points of our interest will be: quickly, or normally, or delay united, or it will under osteopetrosis, or will be not united.

ii) Internal remodeling depends upon temperature:

Then $(10)_2$ because of (20), (21), (22) for all points Γ, Δ and E is written as:

$$d\hat{e}/dt=A(\hat{e})+B(\hat{e})\theta. \quad (39)$$

Finally substituting (26) and

$$B(\hat{e})=d+d\hat{e} \quad (40)$$

into (39) we again result to (27), where:

i) at point Γ:

$$\alpha^{(1)}=c_2,\ \beta^{(1)}=-(2d+c_1)/2c_2 \text{ and } \gamma^{(1)}=(c_o+d)/c_2 \quad (41)_{1\text{-}2\text{-}3}$$

ii) at point Δ:

$$\alpha^{(2)}=c_2,\ \beta^{(2)}=-(2d+c_1)/2c_2 \text{ and } \gamma^{(2)}=(c_o+d)/c_2 \quad (42)_{1\text{-}2\text{-}3}$$

iii) at point E:

$$\alpha^{(3)}=c_2,\ \beta^{(3)}=-(2d+c_1)/2c_2 \text{ and } \gamma^{(3)}=(c_o+d)/c_2. \quad (43)_{1\text{-}2\text{-}3}$$

Table 1. The solutions for finite t

Case $\Delta^{(i)}=4\beta^{(i)2}-4\alpha^{(i)}\gamma^{(i)})<0$	**The solution for $t\to-\pi/2\alpha^{(i)}\sqrt{(\gamma^{(i)}-\beta^{(i)2})}$**	**The physical meaning of solutions.**
$\alpha^{(i)}<0$ $0<\beta^{(i)}<+\infty$, and $\alpha^{(i)}$ is great	$\lim e(t)=\beta^{(i)}>0$	Femur at points Γ, Δ and E is quickly united..
$\alpha^{(i)}<0$ $0<\beta^{(i)}<+\infty$, and $\alpha^{(i)}$ is small	$\lim e(t)=\beta^{(i)}>0$	Femur at points Γ, Δ and E is normally united.
$\alpha^{(i)}<0$ and $\beta^{(i)}=0$	$\lim e(t)=\beta^{(i)}=0$	Femur at points Γ, Δ and E is under osteopetrosis
$\alpha^{(i)}=0$	$e(t)=+\infty$	Femur at points Γ, Δ and E is not united

Table 2. The solutions for infinite t

Case	**The solution for $t\to+\infty$**	**The physical sense of solution**
$\Delta^{(i)}=4\beta^{(i)2}-4\alpha^{(i)}\gamma^{(i)}<0$ and $\alpha^{(i)}>0$,	$\lim e(t)=+\infty$	Femur at points Γ, Δ and E is delayed united.
$\Delta^{(i)}=4\beta^{(i)2}-4\alpha^{(i)}\gamma(i)=0$, K>0	$e(t)=+\infty$	Femur at points Γ, Δ and E is not united.

Table 3. The acceptable solutions of our problem

Case	**The solution:**	**The physical meaning of solutions.**
$\mathbf{\Delta^{(i)} = 4\beta^{(i)2} - 4\alpha^{(i)}\gamma^{(i)} < 0}$ $\alpha^{(i)} < 0$ $0 < \beta^{(i)} < +\infty$, and $\alpha^{(i)}$ is great	$t \to -\pi/2\alpha^{(i)}\sqrt{(\gamma^{(i)} - \beta^{(i)2})} \to$ lime(t)= $\beta^{(i)} > 0$	Femur at points Γ, Δ and E is quickly united.
$\alpha^{(i)} < 0$ $0 < \beta^{(i)} < +\infty$, and $\alpha^{(i)}$ is small	$t \to -\pi/2\alpha^{(i)}\sqrt{(\gamma^{(i)} - \beta^{(i)2})} \to$ lime(t) = $\beta^{(i)} > 0$	Femur at points Γ, Δ and E is normally united.
$\mathbf{\Delta^{(i)} = 4\beta^{(i)2} - 4\alpha^{(i)}\gamma^{(i)} < 0}$ and $\alpha^{(i)} > 0$,	$t \to +\infty \to$ lime(t)=$+\infty$	Femur at points Γ, Δ and E is delayed united.

The solution of (27) satisfying initial condition (32) are given by (33), (34) and (35). The acceptable solutions are the same as at previous case and are given by Table 1 and Table 2.

4. Discussion and Conclusion

Our theoretical results for both cases: neglecting and accounting temperature are verified by clinical studies. Papasimos [[6], p. 92] and [11] studied 40 cases of broken femur with AMBI. All fractures were united (100%) in a period ranged from 45 days until six months. Particularly 24 fractures were united in 45 days (quickly union 60%) 13 of them in 3 months (normally union 32%) and rests in 6 months (delay union7.5%). The mean time of union for all cases was 3.15 months and corresponds to normal union [[11], p.93].

Also other examples of quick union to humans diaphysis and to animals methaphysis has been reffered [12,13,14]. The acceptable solutions are in Table 3. From the above we result that temperature plays no role to growth of microcracks in a broken femur with prosthetic device AMBI.

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