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Does Temperature Effects the Growth of Cracks in a Tibia due to Distance – running?

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Abstract In present paper we investigated if temperature plays any role to the growth of cracks in a tibia due to distance-running. We used modified theory of adaptive elasticity taking into account the temperature. We compared our results with that of the corresponding problem neglecting temperature and we concluded that temperature effects the bone disease.

Keywords: *temperature growth of cracks, tibia distance running, modified adaptive elasticity*

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1. Introduction

It is well known that bone fracture due to many factors: as age [1-14], microstructure [14-24], bone density [2,3,8,9,10,14,17,24,25] and loading mode [14,23,24,25,26,27,28]. From the other hand very few known studies investigated the effect of temperature in bone disease [23,25].

The purpose of this paper is to study if temperature plays role to growth of cracks in a tibia due to distance-running. For that reason we will base upon the theory of adaptive elasticity [29] assuming that rate remodeling equation depends also from temperature.

2. The Problem and Its Physical Approximation

i) The internal remodeling of tibia due to distance running neglecting temperature:

In earliest paper we studied the internal remodeling of tibia due to distance-running neglecting temperature [30]. We modeled tibia as a hollow circular cylinder with constant inner and outer radii a and b respectively and we assumed that it was under a constant axial load $G_{f.}$ We follow a process analytically described in [30] and we used the rate remodeling equation [29]:

$$de(t)/dt = A(e) + A_T(e)(E_{rr} + E_{\theta\theta}) + A_A(e)E_{zz} \quad (1)$$

where e(t) is the change of the volume fraction of the bone, from its reference state, that is the change of the mean length of its cracks, while A(e), $A_T(e)$, $A_A(e)$ are rate remodeling coefficients. At continuity we imposed [30]:

$$\begin{aligned} &A(e) = c_2 e^2 + c_1 e + co \\ &A_T(e) = \alpha_\tau + e\alpha_\tau \\ &A_T(e) = \alpha_A g e\alpha_A \\ &\lambda_1(e) = \Lambda_1 + e\Lambda_1 \\ &\lambda_2(e) = \Lambda_2 + e\Lambda_2 \\ &\mu_1(e) = M_1 + eM_1 \\ &\mu_2(e) = M2 + eM_2 \end{aligned} \quad (2)_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6\text{-}7}$$

and we concluded to:

$$de/dt = \alpha(\hat{e}^2 - 2\beta\hat{e} + \gamma) \quad (3)$$

where:

$$\begin{aligned} &\alpha = c_2 \\ &\beta = -c_1/2c_2 \\ &\gamma = \frac{[c_o + [\Lambda_1\alpha_T - (\Lambda_2 + M_2)\alpha_A]G_f B}{[\pi(b^2 - a^2)F]c_2} \\ &\text{and } F = (\Lambda_2 + M_2)(\Lambda_1 + 2M_1) - \Lambda_1^{\,2}. \end{aligned} \quad (4)_{1\text{-}2\text{-}3\text{-}4}$$

In $(4)_3$ B is the weight of athlete assumed to be constant during the period of training.

The solutions of (1) satisfying initial condition [29]:

$$e(0) = e_o \quad (5)$$

were:

$$e_1 = \beta + \sqrt{(\beta^2 - \gamma)} \text{ and } e_2 = \beta - \sqrt{(\beta^2 - \gamma)} \quad (6)_{1\text{-}2}$$

Since e(t) is defined as the mean length of the cracks of the bone, it must be $e_1>0$ and $e_2>0$. Therefore the acceptable solutions for are in Table 1.

Table 1. The acceptable solutions of earliest problem [30]

Case:	The solution and the physical sense
$0<e_2<e_o<e_1<1-\xi_o$ and $\alpha>0$	lime(t) = $e_2<e_o$. Tibia has been more porous (weaken)
$0<e_2<e_1<e_o<1-\xi_o$ and $\alpha>0$	lime(t) = $e_1<e_o$. Tibia has been more porous (weaken).
$0<e_2<e_o<1-\xi_o<e_1$ and $\alpha>0$	lime(t) = $e_2<e_o$. Tibia has been more porous (weaken).

ii) The internal remodeling of tibia due to distance running accounting temperature.

We use the modified rate remodeling equation:

$$de(t)/dt = A(e)+A_T(e)(E_{rr}+E_{\theta\theta}) + A_A(e)E_{zz}+B(e)\theta \quad (7)$$

where B(e) is an unknown rate coefficient depends from temperature of bone. We follow exactly the same process as in [30] and in addition with (2) we impose:

$$B(\hat{e}) = \delta + \delta\hat{e} \quad (8)$$

where $\delta \neq 0$ is an unknown rate remodeling equation depending upon temperature. We again conclude to

$$d\hat{e}(t)/dt = \alpha_1(\hat{e}^2 - 2\beta_1\hat{e} + \gamma_1) \quad (9)$$

where:

$$\alpha_1 = c_2,\ \beta_1 = -(c_1+2\delta)/2c_2 \text{ and}$$
$$\gamma_1 = \frac{[c_o+[\Lambda_1\alpha_T-(\Lambda_2+M_2)\alpha_A]G_f B}{[\pi(b^2-a^2)F+\delta]c_2}. \quad (10)_{1\text{-}2\text{-}3}$$

The solutions of present problem are:

$$\hat{e}_1 = \beta_1 + \sqrt{\left(\beta_1^2-\gamma_1\right)} \text{ and}$$
$$\hat{e}_2 = \beta_1 - \sqrt{\left(\beta_1^2-\gamma_1\right)} \quad (11)_{1\text{-}2}$$

and the acceptable solutions are again in Table 1. with the only difference that instead of $(6)_{1\text{-}2}$ we replace $(11)_{1\text{-}2}$. The atrophia arised initially in tibia at continuity will be worse and after a long time period will progressively result to "stress fracture" [31-57].

3. Comparison of the Solutions of Earliest and Present Problem

If c_1=0 then from $(4)_2$ and $(10)_2$ we obtain respectively:

$$\beta = 0 \text{ and } \beta_1 = -\delta/c_2. \quad (12)_{1\text{-}2}$$

Then the solution $(6)_2$ has no physical sense since $e_2<0$. Also from $(6)_1$ it is possible to conclude that $\gamma< 0$. We impose

$$A = [c_o+[\Lambda_1\alpha_T-(\Lambda_2+M_2)\alpha_A]G_f B \text{ and}$$
$$\Gamma = \pi(b^2-a^2)F. \quad (13)_{1\text{-}2}$$

Therefore $(4)_3$ and $(10)_3$ due to the above can be written as respectively:

$$\gamma = A/\Gamma c_2 < 0 \text{ and } \gamma_1 = A/(\Gamma+\delta)c_2 \quad (14)_{1\text{-}2}$$

Since $\Gamma>0$ [30], from $(4)_1$ and Table 1. it follows that: $A<0$.

We distinguish the following cases:

i) If $\delta<0$, then from $(12)_2$ it is possible to obtain that $\beta_1>0$. Since $\hat{e}_2>0$ from $(11)_2$ it implies that $\gamma_1<0$. Therefore $(14)_2$ gives $(\Gamma+\delta)c_2>0$. Consequently $0<(\Gamma+\delta)c_2<\Gamma c_2$ and it follows that: $1/(\Gamma+\delta)c_2>1/\Gamma c_2>0$. Therefore: $-A/(\Gamma+\delta)c_2>-A/\Gamma c_2>0$ and due to $(14)_{1\text{-}2}$ it is possible to obtain $-\gamma_1>-\gamma$. Finally the combination of $(6)_1$,$(10)_1$, $(12)_2$ and $(14)_{1\text{-}2}$ gives that:

$$\hat{e}_1 = \beta_1 + [\beta_1^2-\gamma_1]^{1/2} > (-\gamma_1)^{1/2} > (-\gamma)^{1/2} = e_1 \quad (15)$$

that is the porosity of tibia accounting temperature is greater than that neglecting temperature. With other words the mean length of cracks at our case seem to be greater than the corresponding mean length of cracks of the case neglecting temperature [30].

ii) If $\delta>0$, then from $(12)_2$ it results that $\beta_1<0$. Then the solution $(10)_2$ has no physical sense since $\hat{e}_2<0$. Also from $(10)_1$ it is possible to obtain that $\gamma_1<0$. Therefore from $(14)_2$ it results that $(\Gamma+\delta)c_2>0$. Consequently $(\Gamma+\delta)c_2>\Gamma c_2>0$ and it follows that $0<1/(\Gamma+\delta)c_2<1/\Gamma c_2$. Therefore $0<-A/(\Gamma+\delta)c_2<-A/\Gamma c_2$ and due to $(14)_{1\text{-}2}$ it is possible to conclude that $0<-\gamma_1<-\gamma$.

Since $\beta_1<0$ it follows that $-2\beta_1(-\gamma)^{1/2}>0$. The last implies that: $\beta_1^2-2\beta_1(-\gamma)^{1/2}-\gamma>\beta_1^2-\gamma>0$ and consequently: $(-\gamma)^{1/2}-\beta_1>(\beta_1^2-\gamma)^{1/2}>(\beta_1^2-\gamma_1)^{1/2}>0$. Taking into account $(4)_1$, $(10)_1$ it is possible to conclude:

$$e_1 = (-\gamma)^{1/2} > \left(\beta_1^2-\gamma\right)^{1/2} + \beta_1 > \left(\beta_1^2-\gamma_1\right)^{1/2} + \beta_1 = \hat{e}_1 \quad (16)$$

that is the porosity of tibia neglecting temperature is greater than that accounting it. With other words the mean length of cracks of the case neglecting temperature [30] seem to be greater than the case of present problem.

4. Discussion and Conclusion

Our theoretical results come to accordance with previous studies [25,58,59]. Particularly the dependence between fatigue fracture of bone and temperature has been investigated by Carter and Hayes [58]. Also Carter et., al., [25] dealed with fatigue tests to failure of bone specimens at four temperature levels (21-45°C). The test results demonstrated highly significant correlation between fatigue life and temperature. In addition Yan et., al., [59] investigated the depence of temperature on the fracture toughness of compact bone and resulted that there is a revengelly analogous relation. Finally Murcia et., al., [60] investigated if temperature effects the fracture resistance in cyprinus fishes. The results showed that there was a significant reduction in tear resistance with decreasing temperature and the lowest resistance to fracture was obtained at -150°C.

Therefore we conclude that temperature plays role to the growth of cracks in a tibia due to distance running.

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