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Mathematical Study of Blood Flow in a Circular Tube of Varying Cross-section of Non-newtonian Biviscous Incompressible Fluid in the Permeable Wall

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Abstract The present paper, consider a pulsatile fluid flow of blood in a circular tube with permeable wall of varying cross-section has been investigatation of non- Newtonian biviscous incompressible fluid. The governing equations are solved by perturbation scheme. The results are depicts of wall shear stress and pressure drop have been discussed and shown graphically of suction and injection.

Keywords: *pulsatile motion, slip flow, biviscous fluid, leakage parameter, permeability parameter*

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1. Introduction

The control of the blood pressure has been possible by using porous effect in cases of cholesterol and related diseases. For describing the mechanics of red blood cell motion in narrow capillaries, we distinguish two situations according to the convenience with which the cells fit into the vessels. Womersley [1,2] considered the oscillatory flow in a cylindrical tube with uniform cross-section. Lee and Fung [3] studied the flow of blood through an artery with an axisymmetric stenosis taking blood as a Newtonian fluid. Bitoun Bellet [6] studied pulsatile flow of blood with reference to stenosis in microcirculation. Radhakrishnamacharya et al. [8] and Prasad et al. [9] studied the pulsatile flow of blood in circular tubes of varying cross-section with Suction/injection. But the non-Newtonian property is not taken into consideration in these studies. Nakayama and Sawada [13] studied the flow of a non- Newtonian fluid through an axisymmetric stenosis numerically. The pulsatile flow of a non-Newtonian biviscous fluid through a tube with varying cross-section and non- permeable walls in presence of external magnetic field has been analysed by Elnaby et al. [14]. Sanyal et al. [15] investigated the pulsatile flow of biviscous fluid through a tube of varying cross-section with suction/injection. But they considered no effect of slip velocity at the wall of the tube and so the effect of slip velocity has been neglected. Our main object in the present work is to study the pulsatile motion of blood in a circular tube of permeable wall and varying cross-section in presence of slip velocity at the tube wall. In this analysis, we assume that blood is a non- Newtonian biviscous fluid and the blood vessel is a straight, rigid circular tube of varying cross-section. Kumar at el. [18] founded computational technique for flow in blood vessels with porous effects and their using Galerkin finite element method. Gupta [19] investigated a performance and analysis of blood flow through carotid artery and their using finite element method. Gupta [20] made performance modeling and mechanical behaviour of blood vessel in the presence of magnetic effects and they are using finite difference method.

2. Mathematical Model

In the present communication of pulsatile motion for an incompressible non- Newtonian biviscous fluid in an axi-symmetric rigid circular tube of varying cross-section and permeable wall with slip flow is considered.

Then the radius of the tube $r = R(z)$ is given by

$$R(z) = R_0\left\{1+\varepsilon S\left(\frac{\varepsilon z}{R_0}\right)\right\}\ with\ S(0)=1, \tag{1}$$

The governing equation of the pulsatile flow of an incompressible non- Newtonian fluid obeying biviscosity model axisymmetric circular artery are given by:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial w}{\partial z}=0 \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r}+\upsilon_B(1+b^{-1})\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})\frac{\delta y}{\delta x}+\frac{\partial\ u}{\partial r^2}\right] \tag{3}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z} =-\frac{1}{\rho}\frac{\partial P}{\partial r}+\upsilon_B(1+b^{-1})\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v}{\partial r})\frac{\delta y}{\delta x}+\frac{\partial^2 v}{\partial r^2}\right]. \quad (4)$$

The normal component of the fluid velocity at the tube wall is given by

$$u-\frac{dR}{dz}w=v_s\left(1+\delta e^{\mathrm{int}}\right)\left\{1+\left(\frac{dR}{dz}\right)^2\right\}^{\frac{1}{2}} \quad at\ r=R(z), \quad (5)$$

where v_s is the steady state suction/injection velocity, δ is the ratio of the amplitudes of the oscillatory and steady parts of the suction/injection velocity and n is the frequency of the oscillation.

The non-dimensional form is given below:

$$\alpha^2\frac{\partial\omega}{\partial t}+\varepsilon R_e\left\{\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial\omega}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial}{\partial r}\left(\frac{\omega}{r}\right)\right\} =(1+b^{-1})\left\{\varepsilon^2\frac{\partial^2\omega}{\partial z^2}+\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\frac{\omega}{r^2}-K\omega\right\} \quad (6)$$

$$\omega=\frac{1}{r}\left\{\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}(\psi)+\frac{\partial^2\psi}{\partial z^2}\right\} \quad (7)$$

$$\left.\begin{aligned}&\frac{\partial\psi}{\partial r}-\varepsilon^2\frac{dS}{dz}\frac{\partial\psi}{\partial z}=ru_0\\ &\psi=(1+\delta e^{\mathrm{int}}+K)\left[1-v_s\int_0^z G(\xi)d\xi\right]\end{aligned}\right\} at\quad r=S(z) \quad (8)$$

$$\psi=0,\frac{\partial\psi}{\partial z}=0,\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\psi}{\partial r})=0 \quad as\ r\to 0, \quad (9)$$

where R_e is the Reynolds number of entrance flow, α is Womersley's parameter, v_s is the leakage parameter and $G(z)=S(z)\left\{1+\varepsilon^2(\frac{dS}{dz})^2\right\}^{1/2}$ and $\delta=0$ and $v_s=0$.

3. Perturbation Technique

We assume that the pulsatile flow consists of the steady part and the oscillatory part of small amplitude of oscillation δ such that the terms of the order δ^2 can be neglected (ie $\delta\langle\langle 1$) and the expression of vortices and stream function are given by

$$\left.\begin{aligned}&\omega=(\omega_{00}+\delta e^{\mathrm{int}}\omega_{01})+\varepsilon(\omega_{10}+\delta e^{\mathrm{int}}\omega_{11})+O(\varepsilon^2,\delta^2)\\ &\psi=(\psi_{00}+\delta e^{\mathrm{int}}\psi_{01})+\varepsilon(\psi_{10}+\delta e^{\mathrm{int}}\psi_{11})+O(\varepsilon^2,\delta^2)\end{aligned}\right\} \quad (10)$$

The shear stress is given by

$$\tau'_w=\frac{\left[\sigma_{zr}\left\{1-\left(\frac{dR}{dz}\right)^2\right\}+\left(\sigma_{rr}-\sigma_{zz}\right)\frac{dR}{dz}\right]}{\left\{1+\left(\frac{dR}{dz}\right)^2\right\}}.$$

Then using the boundary conditions at $r=S(z)$ and equations (6) and (7), we obtain the dimensionless wall shear stress τ_w are given by:

$$\begin{aligned}\tau_w&=\frac{2\pi R_0{}^3}{\mu Q_s}\tau'_w\\ &=\left(\omega_{00}+\delta e^{it}\omega_{01}\right)+\varepsilon\left(\omega_{10}+\delta e^{it}\omega_{11}\right)\\ &\quad+o\left(\varepsilon^2,\delta^2\right)\ at\ r=S(z).\\ &=-\frac{8\{1-v_sF(z)\}}{S^3}\Bigg[1+\delta e^{it}\left(\frac{\alpha_1 SI_1(\alpha_1 S)}{4I_2(\alpha_1 S)}\right)\\ &\quad-\varepsilon R_{e_1}\Bigg\{\frac{(g_1+4g_2)}{24S^2}\left(2S^2+3g_0u_0\right)\\ &\quad-\frac{\delta e^{it}}{8\alpha_1{}^2S^6I_2(\alpha_1 S)}\Big\{K+T_1SI_0(\alpha_1 S)\\ &\quad-T_2S^2I_1(\alpha_1 S)-T_3S^3I_2(\alpha_1 S)+T_4S^4I_1(\alpha_1 S)\\ &\quad-8T_6S+T_7I_1(\alpha_1 S)\Big\}\Bigg\}\Bigg]+o\left(\varepsilon^2,\delta^2\right).\end{aligned} \quad (11)$$

The pressure drop are given by

$$\begin{aligned}\varepsilon\Delta p&=16\int_0^z\frac{\{1-v_sF(z)\}}{S^4}dz\\ &\quad+2\alpha_1{}^2\delta e^{it}\int_0^z\frac{\{1-v_sF(z)\}}{S^2I_2(\alpha_1 S)}I_0(\alpha_1 S)\,dz\\ &\quad-4\varepsilon R_{e_1}\left[\int_0^z\frac{\{1-v_sF(z)\}}{S^4}\left\{\begin{aligned}&(3g_1+4g_2)\\ &+\frac{g_0u_0}{2S^4}(g_1+4g_2)\\ &\times\left(5S^2+8g_0u_0\right)\end{aligned}\right\}dz\right.\\ &\quad+2\delta e^{it}\int_0^z\left[\begin{aligned}&\{1-v_sF(z)\}T_9\\ &-\frac{4v_s}{\alpha_1{}^2S}(g_1+4g_2)\end{aligned}\right]\left.\frac{dz}{S^4I_2(\alpha_1 S)}\right]\\ &\quad+o\left(\varepsilon^2,\delta^2\right),\end{aligned} \quad (12)$$

where

$$T_9=2I_0(\alpha_1 S)g_1+\left\{\begin{aligned}&2I_0(\alpha_1 S)-\alpha_1 SI_1(\alpha_1 S)\\ &+\frac{\alpha_1 SI_0(\alpha_1 S)}{I_2(\alpha_1 S)}\end{aligned}\right\}g_2 +\frac{T_8}{4\alpha_1{}^2S^5}.$$

4. Results and Discussion

The real part of dimensionless streamlines ψ is plotted for different values of slip parameter u_0 , Reynolds number R_e and upper limit of apparent viscosity b in

Figure 1 of the value of τ_w decreases in the converging region and increases in the diverging region of the tube. From Figure 2, Figure 3, it is seen that the similar results occur for a locally constricted tube. The effects of different parameters on the real part of dimensionless pressure drop Δp are indicated graphically through Figure 2. And Figure 3 depicts that of the Δp decreases with increase in u_0 and increases with increase in b for both suction and injection velocities.

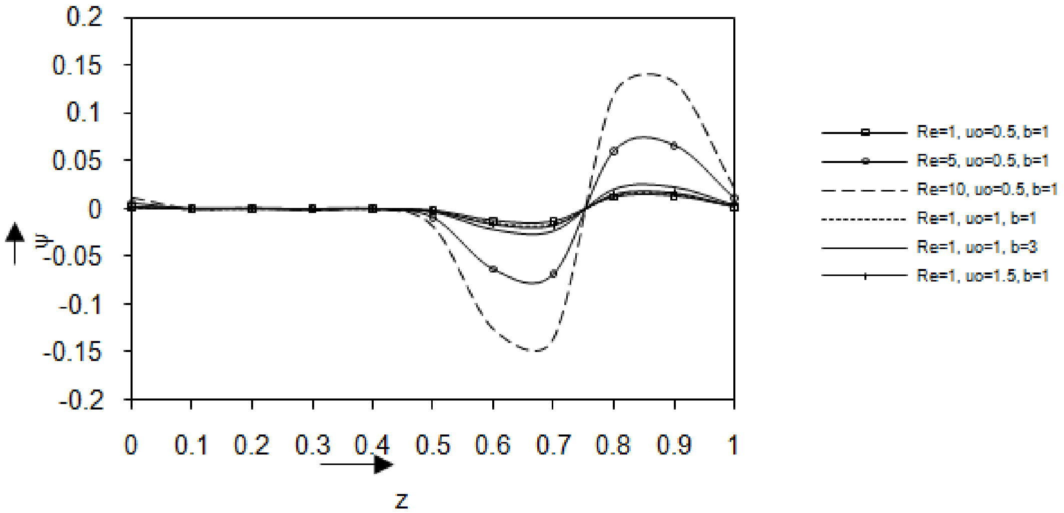

Figure 1. ψ vs z for sinusiodal tube with injection at the permeable wall

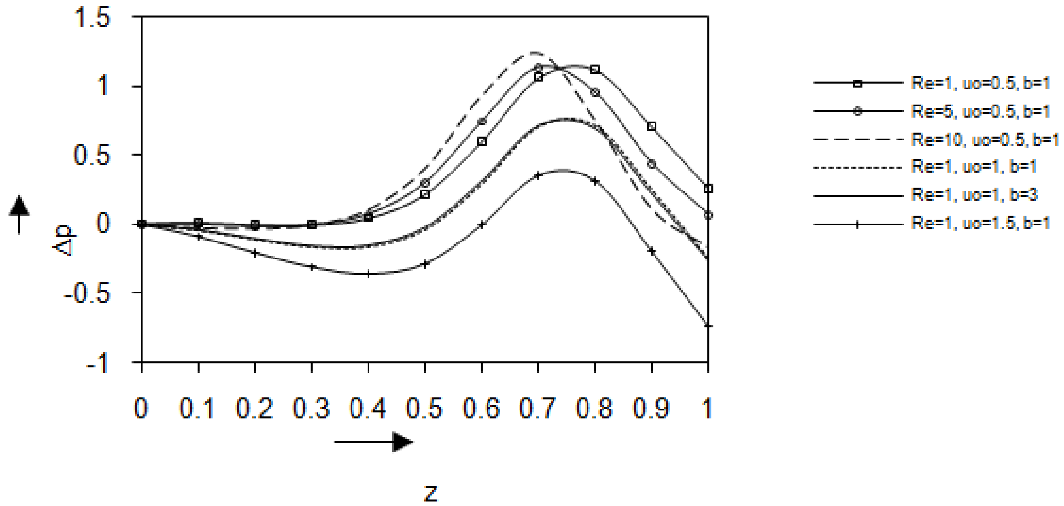

Figure 2. Δp vs z for sinusoidal tube with suction at the permeable wall

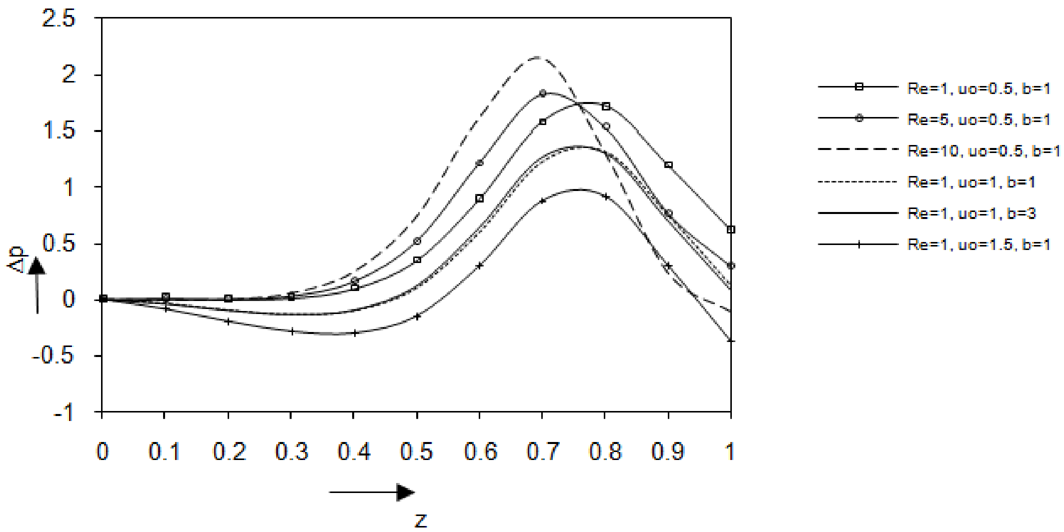

Figure 3. Δp vs z for sinusoidal tube with injection at the permeable wall

5. Conclusions and Applications

This investigation helps us to note that the influence of permeable parameter in the pressure drop is much significant and decreases rapidly with increases in slip parameter. It is also to be noted that this presentation help us to draw the flow characteristic of blood and the wall shear stress on the inner permeable wall of capillaries and small blood vessels where suction, injection and slip velocities arises and Reynolds number is very low. So, this investigation may be helpful in various fields of medical science.

References

[1] Womersley, JR.. Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known, *Journal of physiology,* 127 (1955) 553-563.

[2] Womersley, JR.. Oscillatory motion of a viscous liquid in a thin-walled elastic tube. I. The linear approximation for long waves, *Phil. Mag.*, 46 (1955) 199-221.

[3] Lee, JS., Fung, YC.. Flow in locally constricted tubes at low Reynolds number, *ASME J. Appl. Mech.,* 37 (1970) 9-16.

[4] Rao, AR., Devanathan, R.. Pulsatile flow in tubes of varying cross-section, *Z.A.M.P.,* 24 (1973) 203-213.

[5] Schneck, DJ., Ostrach, S.. Pulsatile blood flow in a channel of small exponential divergence-I. The linear approximation for low mean Reynolds number, *Journal of Fluids Engineering.,* 16 (1975) 353-360.

[6] Bitoun, JP., Bellet, D.. Blood flow through a stenosis in micro-circulation, *Biorheology,* 23 (1986) 51-61.

[7] Manton, MJ.. Low Reynolds number flow in slowly varying axisymmetric tubes, *Journal of Fluid Mech.,* 49 (1971) 451-459.

[8] Radhakrishnamacharya, G., Chandra, P., Kaimal, MR.. A hydro dynamical study of flow in renal tubule, *Bull. Math. Biol.,* 43 (1981) 151-163.

[9] Chandra, P., Prasad, JS.. Pulsatile flow in circular tubes of varying cross-section with suction/injunction, *J. Austral. Math. Soc.,* 35 (1994) 336-381.

[10] Chow, JCF.. Blood flow theory, effective viscosity and effects of particle distribution, *Bull. Math. Biol.,* 37 (1975) 472-488.

[11] Hill, CD., Bedford, A.. A model for erythrocyte sedimentation, *Biorheology,* 18 (1981) 255.

[12] Srivastava, LM., Agarwal, RP.. Oscillating flow of a conducting fluid with a suspension of spheical particles, *J. Appl. Mech.,* 47 (1980) 196.

[13] Nakayama, M., Sawada, TJ.. Numerical study on the flow of a non- Newtonian fluid through an axisymmetric stenosis, *Biomech. Eng.,* 110 (1988) 137.

[14] Elnaby, MA., Eldabe, NT M., Abou Zied, MY., Sanyal, DC.: Mathematical analysis on M.H.D. pulsatile flow of a non-Newtonian fluid through a tube with varying cross-section, *J. of Inst. of Math. and Comp. Sci.,* 20 (2007) 29-42.

[15] Sanyal, DC., Das, K., Debnath, S.. Pulsatile flow of biviscous fluid through a tube of varying cross-section, *International Journal of computational Intelligence and Healthcare Informatics,* 1 (2008) 1-8.

[16] Raoufpanah, A., Rad, M., Borujerdi AN.. Effects of slip condition on the characteristic of flow in ice melting process, *IJE Transactions B: Applications* 18 (2005), 1-9.

[17] Das.K.. Heat transfer peristaltic transport with slip condition in an asymmetric porous channel, *IJE Transactions B: Applications, 24*(3)(2011), 293-307.

[18] Kumar, Anil, Varshney, C.L. and Sharma, G.C. (2005) Computational technique for flow in blood vessels with porous effects. Applied Mathematics and Mechanics, (2005)26, 63-72.

[19] Anil Kumar Gupta (2011). Performance and analysis of blood flow through carotid artery, International Journal of Engineering and Business Management vol. 3(4) pp 1-6.

[20] Anil Kumar Gupta (2013). Performance modeling and mechanical behaviour of blood vessel in the presence of magnetic effects, African Journal of Basic & Applied Sciences 5 (3): 149-155.