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In the Issue of Complex Plane Motions of the Robot

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Abstract Two methods (classical and direct) for calculating an elliptical path for the kinematic mechanisms is considered briefly. A new method is proposed for such calculations. It is simpler, convenient for further calculations and can be applied to arbitrary curves in some cases. This method can serve as a basis for building simple robots with complex trajectories.

Keywords: *ellipse, calculation of trajectory, kinematic mechanisms, plane differentiable curves*

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1. Introduction

Many standards such as the life cycle of a product, company management, etc. assert that the right decisions to be applied in the early stages, which guarantees places without high costs in the following stages. Until now, the first author of the article used the new methods of valuation solely for the design or CAD or CAM [6,8,9] but the quality of product design starts much earlier in the development phase of the kinematic mechanisms by team of authors. One of the major tasks of this phase is to calculate the trajectory of motion of the mechanism. Rational calculation of the trajectory of motion actuators automatic machines is important for efficient production. Calculate the trajectories of the kinematic mechanisms studied and necessary mathematical theory in papers [1,2,5,7,10,12,13]. The calculations of the motion trajectory of the kinematic mechanisms are often yields a system of parametric equations of a circle or ellipse

$$\begin{cases} x = a\cos t + h\sin t \\ y = g\cos t + b\sin t \end{cases} \quad (1)$$

where $a, b, c, d, t \in \mathbf{R}$.

2. Transformation of the Circle

It is proposed to consider the parametric system as a transformation of the circle. Canonical equation of the transformed circle can be converted to determine the inverse transformation of the matrix $T = \begin{pmatrix} a & h \\ g & b \end{pmatrix}$. Matrix T^{-1} is equal to $T^{-1} = \begin{pmatrix} \frac{b}{ab-gh} & \frac{-h}{ab-gh} \\ \frac{-g}{ab-gh} & \frac{a}{ab-gh} \end{pmatrix}$ [Efimov 1967].

Consequently the canonical equation of a circle is as follows:

$$\left(\frac{bx}{ab-gh} - \frac{hy}{ab-gh}\right)^2 + \left(\frac{ay}{ab-gh} - \frac{gx}{ab-gh}\right)^2 - 1^2 = 0$$

$$\frac{b^2x^2}{(ab-gh)^2} - \frac{2bhxy}{(ab-gh)^2} + \frac{h^2y^2}{(ab-gh)^2} + \frac{a^2y^2}{(ab-gh)^2} - \frac{2agxy}{(ab-gh)^2} + \frac{g^2x^2}{(ab-gh)^2} - 1^2 = 0,$$

$$\frac{b^2+g^2}{(ab-gh)^2}x^2 - 2\frac{(bh+ag)}{(ab-gh)^2}xy + \frac{a^2+h^2}{(ab-gh)^2}y^2 - 1^2 = 0. \quad (2)$$

It is known from analytic geometry [Efimov 2009], how to get the angle of rotation of the quadratic form of the canonical equation of the second order line. If the general equation written in the form $Ax^2 + 2Bxy + Cy^2 + Dx + Ey + F = 0$, the angle of rotation can be found is $\tan 2\alpha = \frac{2B}{A-C}$.

We obtain from (1) the values of the coefficients: $A = \frac{b^2+g^2}{(ab-gh)^2}$; $B = -\frac{(bh+ag)}{(ab-gh)^2}$; $C = \frac{a^2+h^2}{(ab-gh)^2}$; $D = 0$; $E = 0$; $f = -1^2$. The angle of rotation of the resulting curve will be equal to

$$\tan 2\alpha = \frac{-\frac{2(bh+ag)}{(ab-gh)^2}}{\frac{b^2+g^2}{(ab-gh)^2} - \frac{a^2+h^2}{(ab-gh)^2}},$$

$$\tan 2\alpha = \frac{2(bh+ag)}{(a^2+h^2)-(b^2+g^2)}. \quad (3)$$

We solve the equation (1) from the characteristic equation $Pz = \lambda z$ [3], where $z = \begin{pmatrix} x \\ y \end{pmatrix}$. It is obtain solutions.

$$\lambda_{1,2} = \frac{A+C \pm \sqrt{(A+C)^2 - 4(AC - B^2)}}{2},$$

$$\lambda_{1\,2} = \frac{\frac{b^2+g^2}{(ab-gh)^2} + \frac{a^2+h^2}{(ab-gh)^2} \pm \sqrt{\left(\frac{b^2+g^2}{(ab-gh)^2} + \frac{a^2+h^2}{(ab-gh)^2}\right)^2 - 4\left(\frac{b^2+g^2}{(ab-gh)^2}\frac{a^2+h^2}{(ab-gh)^2} - \frac{(bh+ag)^2}{(ab-gh)^4}\right)}}{2}$$

$$\lambda_{1,2} = \frac{b^2+g^2+a^2+h^2 \pm \sqrt{(b^2+g^2+a^2+h^2)^2 - 4((b^2+g^2)(a^2+h^2) - (bh+ag)^2)}}{2(ab-gh)^2}.$$

The solution

$$\begin{cases} x = \lambda_1 \cos(t+\alpha) \\ y = \lambda_2 \sin(t+\alpha) \end{cases} \quad (4)$$

is obtained, but it is difficult to apply in the coefficients for the other calculations. Engineer must choose one right decisions of the two $\lambda_{1,2}$ additionally.

It is proposed to use another method [7,8,9]. The method is based on the permutation symmetry and other symmetries [8].

3. Direct Analytical Method of Arbitrary Linear Transformations

We calculate the angle β which is symmetrical own corner α for system (1)

$$\tan 2\beta = \frac{2(gb+ha)}{a^2-h^2-b^2+g^2} \quad (5)$$

in the first step. The angle is determined from the two equations

$$\tan \alpha_1 = \frac{a \tan \beta - h}{b - g \tan \beta} \quad (6)$$

and

$$\tan \alpha_2 = \frac{b \tan \beta + g}{a + h \tan \beta}. \quad (7)$$

Angles are equal if the calculation and transformation is correct. The coefficients $\lambda = \begin{pmatrix} c \\ d \end{pmatrix}$ are equal

$$c_1 = \frac{a \cos \beta + h \sin \beta}{\cos \alpha}, \quad (8)$$

$$c_2 = \frac{b \sin \beta + g \cos \beta}{\sin \alpha}, \quad (9)$$

$$d_1 = \frac{a \sin \beta - h \cos \beta}{\sin \alpha}, \quad (10)$$

$$d_2 = \frac{b \cos \beta - g \sin \beta}{\cos \alpha}. \quad (11)$$

The initial system will be:

$$\begin{cases} x = c_1 \cos(t+\alpha) \\ y = d_1 \sin(t+\alpha) \end{cases} \quad (12)$$

and

$$\begin{cases} x = c_2 \cos(t+\alpha) \\ y = d_2 \sin(t+\alpha) \end{cases}. \quad (13)$$

Both solutions are valid and the choice is not necessary.

General solution for any curve could not get unfortunately. If the result of the calculations obtained systems $\begin{cases} x = mf_x - nf_y \\ y = mf_x + nf_y \end{cases}$, $\begin{cases} x = kmf_x - nf_y \\ y = -knf_x + mf_y \end{cases}$, $\begin{cases} x = mf_x + knf_y \\ y = -nf_x + kmf_y \end{cases}$, where f_x and f_y arbitrary trigonometrically functions, the solution can be found similarly. For example, the system $\begin{cases} x = \cos t - 2\sin 2t \\ y = \cos t + 2\sin 2t \end{cases}$ can be reduced to the form $\begin{cases} x = c\cos(t+\alpha) \\ y = d\sin 2(t+\alpha) \end{cases}$.

4. Theoretical Tests

Submitted theorems are very simple and allow the reader to the proof without reference to sources in a foreign language. Let's consider the sufficiency of all the theorems only. Theorem 3 is published the first.

Theorem 1. Angles α_1 (6) and α_2 (7) is equivalence.

Proof.

Let's equating the formula (6) and (7) in value $\tan \alpha$: $\frac{b \tan \beta + g}{a + h\tan \beta} = \frac{a \tan \beta - h}{b - g \tan \beta}$, $(b \tan \beta + g)(b - g \tan \beta) = (a \tan \beta - h)(a + h\tan \beta)$ or $v_1 = v_2$. Let's show $v_1 = (b \tan \beta + g)(b - g \tan \beta)$: $v_1 = b^2 \tan \beta + gb - gb \tan^2 \beta - g^2 \tan \beta$, $v_1 = (b^2 - g^2)\tan \beta + gb(1 - \tan^2 \beta)$. The value $v_2 = (a \tan \beta - h)(a + h\tan \beta)$ is determinate: $v_2 = a^2 \tan \beta - ha + ah \tan^2 \beta - h^2 \tan \beta$, $v_2 = (a^2 - h^2)\tan \beta - ha(1 - \tan^2 \beta)$. Results equate to $(a^2 - h^2)\tan \beta - ha(1 - \tan^2 \beta) = (b^2 - g^2)\tan \beta + gb(1 - \tan^2 \beta)$, $(a^2 - h^2)\tan \beta - (b^2 - g^2)\tan \beta = gb(1 - \tan^2 \beta) + ha(1 - \tan^2 \beta)$,

$(a^2-h^2-b^2+g^2)\tan\beta=(gb+ha)(1-\tan^2\beta)$,

$\frac{2\tan\beta}{1-\tan^2\beta}=\frac{2(gb+ha)}{a^2-h^2-b^2+g^2}$,

$\frac{\tan\beta}{1-\tan^2\beta}=\frac{gb+ha}{a^2-h^2-b^2+g^2}$,

$\tan 2\beta=\frac{2(gb+ha)}{a^2-h^2-b^2+g^2}$. The resulting formula coincides with the formula (5) and hence the angle α is the same in both formulas.

Remark. Theorem is intended to linearly independent transformations only.

Theorem 2. Parameters $c_{1,2}$ when $b\sin\beta+g\cos\beta\neq 0$ and $d_{1,2}$ when $b\cos\beta-g\sin\beta\neq 0$ are equivalence.

Proof.

1. Let's assume the opposite for coefficient c. Let's $c_1\neq c_2$, $c_1,c_2\neq 0$ than $c_1-c_2=\Delta$ from (8) and (9): $\frac{b\sin\beta+g\cos\beta}{\sin\alpha}=\frac{a\cos\beta+h\sin\beta}{\cos\alpha}+\Delta$,

$\frac{1}{\sin\alpha}=\frac{a\cos\beta+h\sin\beta}{\cos\alpha(b\sin\beta+g\cos\beta)}+\frac{\Delta}{b\sin\beta+g\cos\beta}$,

$\frac{\cos\alpha}{\sin\alpha}=\frac{a\cos\beta+h\sin\beta}{b\sin\beta+g\cos\beta}+\frac{\Delta\cos\alpha}{b\sin\beta+g\cos\beta}$. We have from (7) and theorem 1 $\tan\alpha=\tan\alpha+\frac{\Delta\cos\alpha}{b\sin\beta+g\cos\beta}$.

Equality holds when $\cos\alpha=0$ or $\Delta=0$. Since $\cos\alpha$ in general not equal 0 then $\Delta=0$ and parameters are equivalence.

2. Let's assume the opposite for coefficient d. Let's $d_1\neq d_2$, $d_1,d_2\neq 0$ than $d_1-d_2=\Delta$ from (10) and (11): $\frac{b\cos\beta-g\sin\beta}{\cos\alpha}=\frac{a\sin\beta-h\cos\beta}{\sin\alpha}+\Delta$,

$\frac{1}{\cos\alpha}=\frac{a\sin\beta-h\cos\beta}{\sin\alpha(b\cos\beta-g\sin\beta)}+\frac{\Delta}{b\cos\beta-g\sin\beta}$,

$\frac{\sin\alpha}{\cos\alpha}=\frac{a\sin\beta-h\cos\beta}{b\cos\beta-g\sin\beta}+\frac{\Delta\sin\alpha}{b\cos\beta-g\sin\beta}$. We have from (6) and theorem 1 $\tan\alpha=\tan\alpha+\frac{\Delta\sin\alpha}{b\cos\beta-g\sin\beta}$.

Equality holds when $\sin\alpha=0$ or $\Delta=0$. Since $\sin\alpha$ in general not equal 0 then $\Delta=0$ and parameters are equivalence.

Theorem 3. The own angle in classical method and direct analytical method is equivalence.

Proof.

Let's equating the formula (6) and (7) in value $\tan\beta$. Let's deduce the angle $\tan\beta$ of the formula (7): $\tan\alpha_1=\frac{a\tan\beta-h}{b-g\tan\beta}$, $\tan\alpha_1(b-g\tan\beta)=a\tan\beta-h$,

$\tan\alpha_1 b-g\tan\alpha_1\tan\beta=a\tan\beta-h$,

$\tan\alpha_1 b+h=a\tan\beta+g\tan\alpha_1\tan\beta$, $\frac{\tan\alpha_1 b+h}{a+g\tan\alpha_1}=\tan\beta$.

Let's deduce the angle $\tan\beta$ of the formula (6): $\tan\alpha_2(a+h\tan\beta)=b\tan\beta+g$,

$\tan\alpha_2 a+h\tan\alpha_2\tan\beta=b\tan\beta+g$

$\tan\alpha_2 a-g=b\tan\beta-h\tan\alpha_2\tan\beta$, $\frac{\tan\alpha_2 a-g}{b-h\tan\alpha_2}=\tan\beta$.

Results equate to $\frac{\tan\alpha_1 b+h}{a+g\tan\alpha_1}=\frac{\tan\alpha_2 a-g}{b-h\tan\alpha_2}$,

$(\tan\alpha_1 b+h)(b-h\tan\alpha_2)=(\tan\alpha_2 a-g)(a+g\tan\alpha_1)$. We have from theorem 1 $\alpha_1=\alpha_2$ that $(b\tan\alpha+h)(b-h\tan\alpha)=(a\tan\alpha-g)(a+g\tan\alpha)$ or $v_1=v_2$. Let's consider $v_1=(b\tan\alpha+h)(b-h\tan\alpha)$:

$v_1=b^2\tan\alpha+bh-bh\tan^2\alpha-h^2\tan\alpha$,

$v_1=(b^2-h^2)\tan\alpha+bh(1-\tan^2\alpha)$. Value $v_2=(a\tan\alpha-g)(a+g\tan\alpha)$ is

$v_2=a^2\tan\alpha-ga+ag\tan^2\alpha-g^2\tan\alpha$,

$v_2=(a^2-g^2)\tan\alpha-ag(1-\tan^2\alpha)$. Results equate to

$(a^2-g^2)\tan\alpha-ag(1-\tan^2\alpha)=(b^2-h^2)\tan\alpha+bh(1-\tan^2\alpha)$,

$(a^2-g^2)\tan\alpha-(b^2-h^2)\tan\alpha=ag(1-\tan^2\alpha)+bh(1-\tan^2\alpha)$,

$(a^2-g^2-b^2+h^2)\tan\alpha=(ag+bh)(1-\tan^2\alpha)$,

$\frac{\tan\alpha}{1-\tan^2\alpha}=\frac{ag+bh}{a^2+h^2-g^2-b^2}$, $\tan 2\alpha=\frac{2(ag+bh)}{a^2+h^2-g^2-b^2}$.

We find (2).

The proof of the transformation parameters can be obtained similarly to Theorem 2, but it is cumbersome for publication in the journal.

Let coefficients of matrix are $a=b=k$, $h=g=0$. Angle β is equivalence $\frac{2(gb+ha)}{a^2-h^2-b^2+g^2}=\tan 2\beta$,

$\frac{2(0\times k+0\times k)}{k^2-0^2-k^2+0^2}=\tan 2\beta$, $\frac{0}{0}=\tan 2\beta$. Since the angle is not define. Let coefficients h and g are equality value ε , where $\varepsilon\to 0$, than $\lim\limits_{\varepsilon\to 0}\frac{4\times\varepsilon\times k}{k^2-\varepsilon^2-k^2+\varepsilon^2}=\tan 2\beta$,

$\lim\limits_{\varepsilon\to 0}\frac{4\times\varepsilon\times k}{0}=\tan 2\beta$, $\infty=\tan 2\beta$ и $\beta=\pi/4$. Angle α is $\tan\alpha_1=\frac{a\tan\beta-h}{b-g\tan\beta}$, $\alpha_1=\frac{\pi}{4}$ or

$\tan\alpha_2=\frac{b\tan\beta+g}{a+h\tan\beta}$, $\alpha_2=\frac{\pi}{4}$. Angle α is turned out the same for both formulas. Coefficients c and d are

$c_1=\frac{a\cos\beta+h\sin\beta}{\cos\alpha}=\frac{k\cos(\pi/4)+0\sin(-\pi/4)}{\cos(\pi/4)}=k$,

$c_2=\frac{b\sin\beta+g\cos\beta}{\sin\alpha}=\frac{k\sin(\pi/4)+0\cos(\pi/4)}{\sin(\pi/4)}=k$,

$d_1=\frac{a\sin\beta-h\cos\beta}{\sin\alpha}=\frac{k\sin(\pi/4)-0\cos(\pi/4)}{\sin(\pi/4)}=k$,

$d_2=\frac{b\cos\beta-g\cos\beta}{\cos\alpha}=\frac{k\cos(\pi/4)-0\sin(\pi/4)}{\cos(\pi/4)}=k$.

Thus the formulas for converting performed in the vicinity of the transformation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, but do not exist for accurate conversion.

Let coefficients of matrix are $a = b = \cos\varphi$, $h = -\sin\varphi$, $g = \sin\varphi$. Angle β is equivalence $\frac{2(gb + ha)}{a^2 - h^2 - b^2 + g^2} = \tan 2\beta$, $\frac{2(\cos\varphi\sin\varphi - \cos\varphi\sin\varphi)}{\cos^2\varphi - \sin^2\varphi - \cos^2\varphi + \sin^2\varphi} = \frac{0}{0}$. Since the angle is not define also. Let coefficients of matrix are $a = b = \cos\varphi + \varepsilon$, $h = -\sin\varphi - \varepsilon$, $d = \sin\varphi + \varepsilon$, where $\varepsilon \to 0$. Angle β will seek, as $b^2 = a^2$:

$$\lim_{\varepsilon \to 0} \frac{2((\cos\varphi + \varepsilon)(\sin\varphi + \varepsilon) - (\cos\varphi + \varepsilon)(\sin\varphi + \varepsilon))}{-\sin^2\varphi + 2\varepsilon\sin\varphi - \varepsilon^2 + \sin^2\varphi + 2\varepsilon\sin\varphi + \varepsilon^2} = \tan 2\beta$$

, $\lim_{\varepsilon \to 0} \frac{0}{4\varepsilon\sin\varphi} = \tan 2\beta$, $0 = \beta$. Angle α is $\tan\alpha_1 = \frac{a\tan\beta - h}{b - g\tan\beta}$, $\tan\alpha_1 = \frac{\cos\varphi\tan 0 + \sin\varphi}{\cos\varphi - \sin\varphi\tan 0} = \tan\varphi$, $\alpha_1 = \varphi$ or $\tan\alpha_2 = \frac{b\tan\beta + g}{a + h\tan\beta}$, $\tan\alpha_2 = \frac{\cos\varphi\tan 0 + \sin\varphi}{\cos\varphi + \tan 0(-\sin\varphi)} = \tan\varphi$, $\alpha_2 = \varphi$. Angle α is turned out the same for both formulas. Coefficients c and d :

$$c_1 = \frac{a\cos\beta + h\sin\beta}{\cos\alpha} = \frac{\cos\varphi\cos 0 + \sin 0(-\sin\varphi)}{\cos\varphi} = 1 ,$$

$$c_2 = \frac{b\sin\beta + g\cos\beta}{\sin\alpha} = \frac{0\cos\varphi + 1\sin\varphi}{\sin\varphi} = 1 ,$$

$$d_1 = \frac{a\sin\beta - h\cos\beta}{\sin\alpha} = \frac{\cos\varphi\sin 0 + \sin\varphi\cos 0}{\sin\varphi} = 1 ,$$

$$d_2 = \frac{b\cos\beta - g\sin\beta}{\cos\alpha} = \frac{\cos\varphi\cos 0 - \sin\varphi\sin 0}{\cos\varphi} = 1 .$$

Findings are consistent with the conclusion of the previous conversion.

Method can not be used to rotate $\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$ and scaling (homothety) $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ and some other transformations (see theorem 2) [Lozhkin 2012].

5. Geometrical Modeling

Example of geometric modeling the method is presented to calculate the trajectory of the astroid in language AutoLisp. Astroid (hypocycloid modulo 4) is described by two identical systems of parametric equations $\begin{cases} x = \frac{3}{4}R\cos\frac{t}{4} + \frac{1}{4}R\cos\frac{3t}{4} \\ y = \frac{3}{4}R\sin\frac{t}{4} - \frac{1}{4}R\sin\frac{3t}{4} \end{cases}$ and $\begin{cases} x = R\cos^3\frac{t}{4} \\ y = R\sin^3\frac{t}{4} \end{cases}$ [Gibson 2001].

The program algorithm is quite simple:

1. The requested transformation parameters;
2. We output the desired astroid will transform every part of the line of it. Output is black;
3. Forward transformation parameters;
4. Output a new astroid with parameters obtained in yellow.

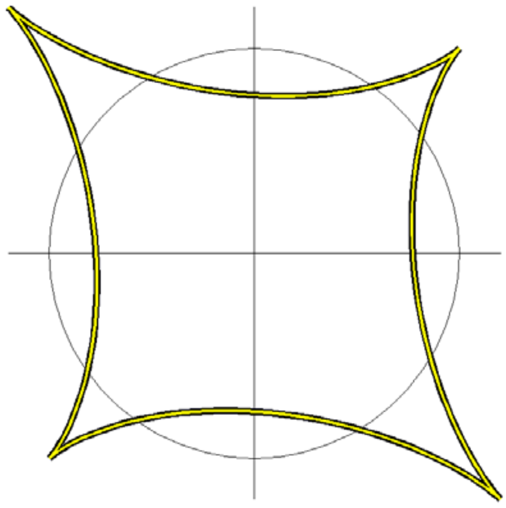

Figure 1. Normal transformation an astroid

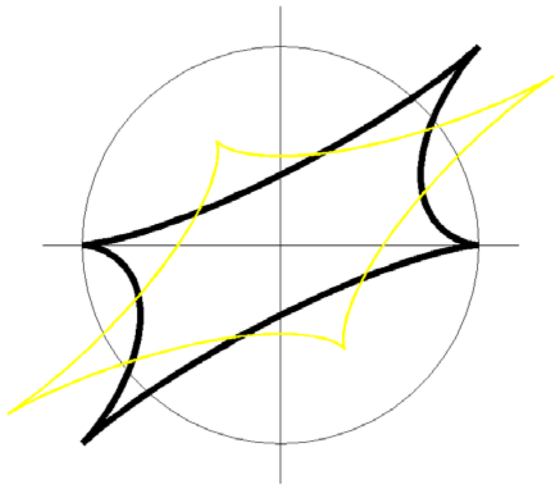

Figure 2. "Bad" transformation.

The result of the programs for conversion $\begin{pmatrix} 1 & 1.2 \\ 1 & -1.2 \end{pmatrix}$ shown in Figure 1. The method works. Parameters are calculated correctly. The result of the programs for conversion $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is shown in Figure 2. Method does not work. The experiment was carried out in AutoCAD 2009. Two rules to observe when moving algorithm to other languages:

1. Calculate the arc tangent function of two arguments;
2. Remember in which quadrant is a calculation.

It should be noted that the theoretical and modeling researches of Jordan curves is not full.

6. Experiment

Experiments were carried out on the basis of the developed stand with dimensions 1300x1400x1500 mm to check the adequacy of the developed mathematical model. Photo booth is shown in Figure 3.

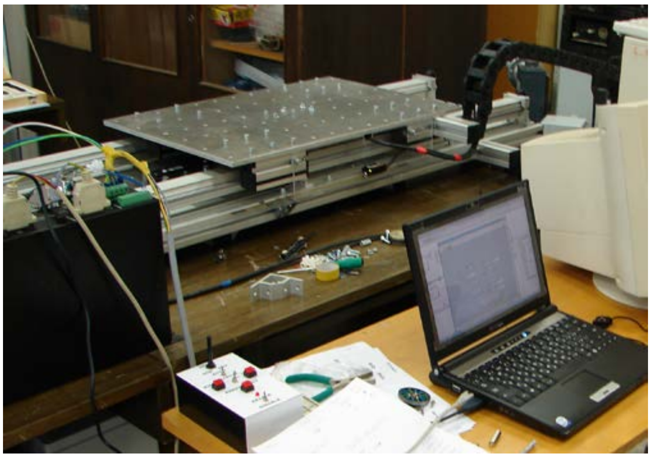

Figure 3. Photo experimental stand

The design of the stand was assembled from aluminum profiles, linear modules and fasteners RK Rose + Krieger. Aluminium profiles RK Rose + Krieger Blocan shown in Figure 4.

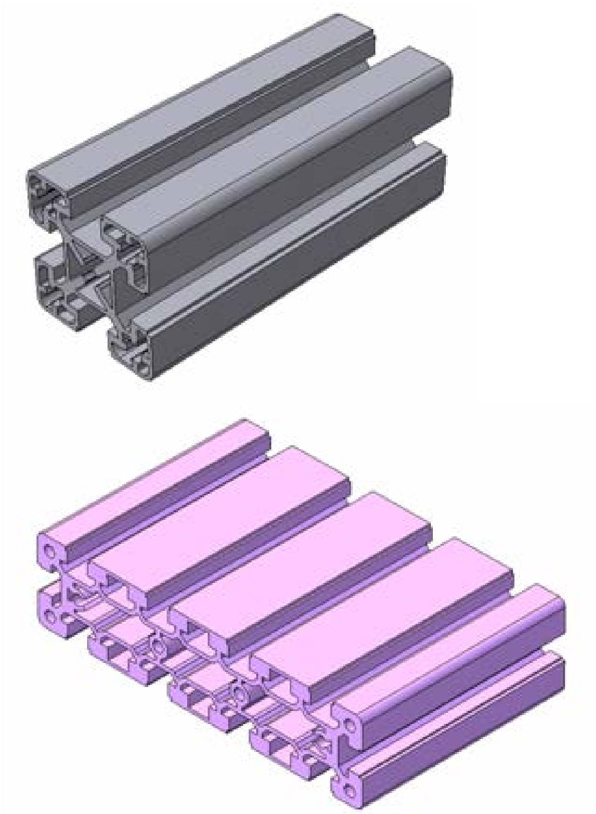

Figure 4. Aluminum profiles RK Rose+Krieger Blocan (3D model)

Aluminum profiles are profiled rods with longitudinal T-slots. The surface profile is finishing. Linear Module RK Rose + Krieger Blocan have aluminum construction, possess high hardness, large carrying capacity. Assembly is protected from dust and dirt. Linear guides have been installed on a wide aluminum profile and secured the appropriate bracket, Figure 5.

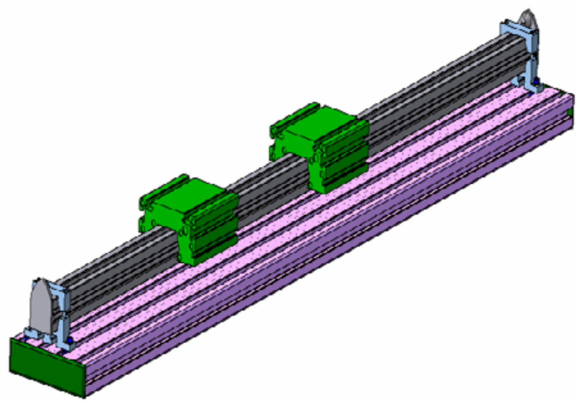

Figure 5. Model roller linear guides

Two profiles with guides were bonded together using an aluminum sheet on the top side. Four movable slider on the guide are connected to the same sheet of aluminum, 3D model shown in Figure 6.

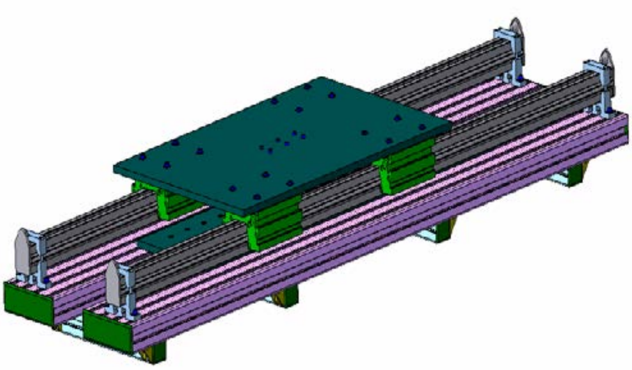

Figure 6. Model design guide stand

Experiments have shown a difference between the theoretical and actual trajectory less than 5%.

Simple and cheap robot to move along the path of the executive body of the ellipse and the circle may develop on the basis of the proposed mathematical model. Photo pneumatic robot is shown in Figure 7.

Figure 7. Photograph of the pneumatic robot

This robot can be carved with milling cutter details in the form of an ellipse and a circle made of different materials, such as glass, metal, ceramics and wood. The robot can with laser burn in different materials is not necessarily circular holes.

7. Plane Differentiable Curves

The authors studied the expression of symmetry Euler not only for conic sections but also more complex forms. Four groups of linear $\mathbf{H}_1 = \begin{pmatrix} m & -n \\ m & n \end{pmatrix}$, $\mathbf{H}_2 = \begin{pmatrix} -m & n \\ m & n \end{pmatrix}$, $\mathbf{H}_3 = \begin{pmatrix} km & n \\ -kn & m \end{pmatrix}$, $\mathbf{H}_4 = \begin{pmatrix} m & kn \\ -n & km \end{pmatrix}$, where $k, m, n \in \mathbf{R}$, can be used DAM for any Jordan curve, but the method is not applicable for rotation and dilation for any curve, because they not change the discrete structure of Euclidean plane.

The proposed theory allows the calculation of complex mechatronic systems. Let there be a simple kinematical mechanism built on the basis of three gears: central, moving inside the central wheel and moving outside of the central wheel (Figure 8). Let a point on the first wheel moves according to the law $\begin{cases} x = r\cos t \\ y = r\sin t \end{cases}$. Let the motion of the second and third wheels are defined by the systems of parametric equations $\begin{cases} x = r\cos^3 \dfrac{t}{4} \\ y = r\sin^3 \dfrac{t}{4} \end{cases}$ (asteroid), $\begin{cases} x = 3r\cos t - r\cos 3t \\ y = 3r\sin t - r\sin 3t \end{cases}$ (epicycloids).

It is necessary to design an elliptical gear mechanism to the trajectory of the first wheel to a system of parametric equations $\begin{cases} x = -1.2r\cos t + r\sin t \\ y = 1.2r\cos t + r\sin t \end{cases}$ from second group, where $m = 1.2$ and $n = 1$. The method of calculation can be applied to all wheels. Geometric modeling results are shown in Figure 9.

Solution of the characteristic equation $\mathbf{T}\vec{v} = \lambda\vec{v}'$ can be different. Vector $\vec{v}$ does not change after the transformation of the ellipse. Vector belongs to the set $\vec{v}' \in \left\{ \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} y \\ x \end{pmatrix}, \begin{pmatrix} -y \\ -x \end{pmatrix} \right\}$ after the transformation of the planar differentiable curve. Parameters symmetric transformations find possible and to simplify the system of parametric equations for the complex movements as a result. Real coefficients can be used in systems of parametric equations. Thus the analytical solution for calculating opened for the complex trajectories of the mechatronic system. Precision of designing is defined within a computer data storage and precision of machining equipment.

Symmetries has an important influence on the trajectory of this is obvious. Let anticipated changes in trajectory when applying an arbitrary linearly independent transformation. The system of parametric equations is divided into two or four portions of movement: $\begin{cases} x = c_1 f_x(t) \\ y = d_1 f_y(t) \end{cases}$, $\begin{cases} x = c_1 f_y(t) \\ y = d_1 f_x(t) \end{cases}$, $\begin{cases} x = -c_1 f_x(t) \\ y = -d_1 f_y(t) \end{cases}$, $\begin{cases} x = -c_1 f_y(t) \\ y = -d_1 f_x(t) \end{cases}$. This hypothesis is tested in research now.

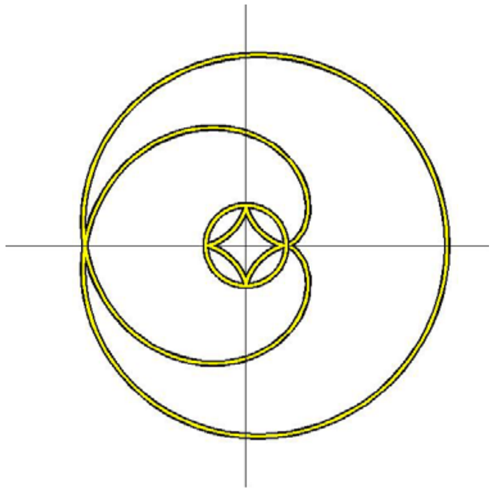

Figure 8. Simple kinematical mechanism

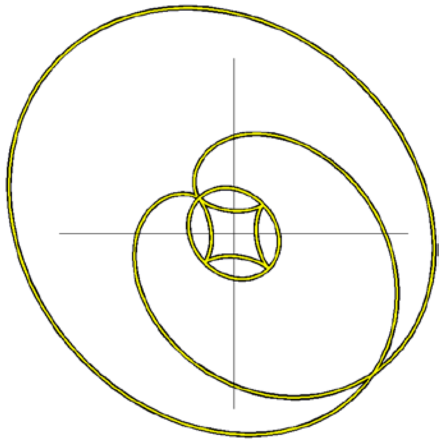

Figure 9. Complex kinematical mechanism

8. Summary

If the trajectory of the point of the kinematic mechanism is represented by the parametric equations that can be used a new effective method. Parametric system is used to convert the equations of motion in a circle. The method allows to find the exact characteristics of the mechanism are based on the kinematic trajectory of its motion. The method described by simple algorithms more uncomplicated compared with the classical method.

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