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Application of Multivariable Predictive Control in a Hydropower Plant

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Abstract The hydroelectric energy is one of the most important renewable energy in the world. It does not encounter the problem of population displacement and is not as expensive as solar or wind energy. However, the hydro electrical generating units are usually isolated from the grid network; thus, they require control to maintain of constant the power for any working conditions. The simulation model of hydropower plant was constructed based on mathematical equations that summarize the behavior of the hydropower plant. The simulation model of power plant is useful in stability studies. This paper, presents the approach of Generalized Predictive Control (GPC) is applied to a multivariable model of the part turbine/generator of hydropower plant. In this study, the standard multivariable (GPC) algorithm is presented. It is then applied to achieve sets points tracking of the outputs of the plant. A Multi Input Multi Output (MIMO) model is used for control purposes. A comparative study is carried out using the named controller's multivariable Linear Quadratic Gaussian (LQG) and multivariable (GPC) Controls. The performance of the proposed controller is illustrated by a simulation example of hydropower plant. Encouraging results are obtained that motivate for further investigations.

Keywords: *Generalized Predictive Control, modeling, Linear Quadratic Gaussian Control, multivariable systems, hydropower plant*

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1. Introduction

The Hydro-electric energy is most important renewable energy in the world. It provides energy to various loads. User load requires a uniform and uninterrupted supply of input energy. The load demand varies continuously. It affects the terminal voltage and real power output at the generator terminals [1,2,3,4]. The objective of the control strategy is to generate and deliver power in an interconnected system as economically and reliably as possible while maintaining the voltage and frequency within permissible limits. Hydropower plant is equipped with hydraulic turbine governor and excitation control. The errors in the terminal voltage and in the output active power, with respect to their respective references, represent the controller inputs and the generator-exciter voltage and governor-valve position represent the controller outputs. The control of real power output and the terminal voltage keeps the system in the steady state [5-10].

This paper presents the application of multivariable GPC control to achieve sets points tracking of the outputs of the plant. The GPC control is one of the most favorite predictive control methods, popular in industry and also at universities. It was first published in 1987 [11,12]. The authors wanted to find one universal method to control different systems. Multivariable GPC Control has been successfully implemented in many industrial applications, showing good performance and a certain degree of robustness. It is applicable [13] to the systems with non-minimal phase, unstable systems in open loop, systems with unknown or varying dead time, systems with unknown order and nonlinear systems approximated by linear models.

The basic idea of GPC control [14,15] is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. The index to be optimized is the expectation of a quadratic function measuring the distance between the predicted system output and some reference sequence over the horizon plus a quadratic function measuring the control effort. The predictive model is carried out based on the solving Diophantine equations.

The paper is organized as follows. Section II presents the system modelling. Section III describes the designed multivariable LQG Controller. Section IV is devoted the description of multivariable GPC algorithm. In section V, the effectiveness and superiority of the proposed algorithm, is demonstrated by simulation example. Some concluding remarks end the paper.

2. System Modeling

List of Symbols

V_d, V_q Stator voltage in d-axis and q-axis circuit

V_t Terminal voltage

$E_q^{'}$ Transient EMF in the quadratic axis of the generator

x_{ad} Stator–rotor mutual reactance

E_{fd} Field voltage

r_{fd} Field resistance

X_{fd} Self reactance of field winding

U_e Exciter input

δ Rotor angle

P_m Mechanical power

P_w Water power

H Inertia constant

$\omega(t)$ Rotor speed of the generator

ω_0 Angular frequency of the infinite bus bar

K_d Mechanical damping torque coefficient

T_d Damping torque coefficient due to damper windings

P_t Real power output at the generator terminals

τ_e Exciter time constant

τ_g Governor valve time constant

τ_b Turbine time constant

U_g Governor input

G_v Governor valve position

K_v Valve constant

x_d Total d-axis synchronous reactance between the generator and the infinite busbar

x_q Total q-axis synchronous reactance between the generator and the infinite busbar

$x_d^{'}$ Total d-axis transient reactance including the generator and the infinite busbar

$T_{do}^{'}$ d-axis transient open-circuit time constant

x_T Reactance of the transformer

x_L Reactance of the transmission line

x_s Reactance of the system

The block diagram of the sample controlled power system is shown in Figure 1 that comprises a hydraulic turbine driving a synchronous generator which is connected to an infinite bus via a step-up transformer and a transmission line. The output real power P_t and terminal voltage V_t at the generator terminals are measured and fed to the controller. The outputs of the controller (system control inputs) are fed into the generator-exciter and governor-valve. In the simulation studies described here, the nonlinear equations of the synchronous generator are represented by a third-order nonlinear model based on park's equations. The hydraulic turbine, governor valve and exciter are each represented by a first order model. The model equations are as follows [16-24]:

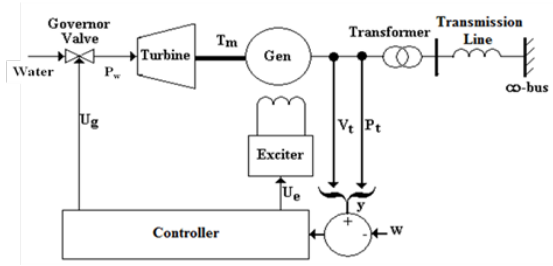

Figure 1. Controlled sample hydropower system

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The Mechanical equations

The rotor speed of the generator is given by:

$$\delta(t) = \omega(t) \tag{1}$$

The mechanical equation of the motion is as follows:

$$\frac{H}{\pi f_0}\frac{d\omega(t)}{dt} + D\omega = P_m - P_t$$

i.e

$$M\frac{d\omega(t)}{dt} + D\omega = P_m - P_t \tag{2}$$

Where,

$$M = \frac{H}{\pi f_0} \text{ and } f_0 = \frac{\omega_0}{2\pi}$$

The electrical generator dynamics equations

$$\frac{dE_q^{'}(t)}{dt} = \frac{1}{T_{do}^{'}}(E_{fd}(t) - E_q(t)) \tag{3}$$

The electrical equations (assumed $x_d^{'} = x_q$)

$$E_q(t) = E_q^{'}(t) + (x_d - x_q)I_d(t) \tag{4}$$

$$P_t(t) = E_q(t)I_q(t) \tag{5}$$

$$I_d(t) = \frac{E_q^{'}(t) - V_s\cos\delta(t)}{x_{ds}^{'}} \tag{6}$$

$$I_q(t) = \frac{V_s\sin\delta(t)}{x_{ds}^{'}} \tag{7}$$

$$E_q^{'}(t) = x_{ad}I_f(t) \tag{8}$$

$$V_t(t) = \left[(V_d(t))^2 + (V_q(t))^2\right]^{\frac{1}{2}} \tag{9}$$

$$V_d(t) = E_q^{'}(t) - x_d^{'}I_d(t) \tag{10}$$

$$V_q(t) = x_d^{'}I_q(t) \tag{11}$$

$$x_{ds} = x_d + x_T + x_L$$
$$x'_{ds} = x'_d + x_T + x_L$$
$$x_s = x_T + x_L$$

More details about power system modelling can be seen in [20,21,22,23].

Using the above equations, we can express $P_t(t)$ as

$$P_t(t) = \frac{V_s x_{ds}}{(x'_{ds})^2} E'_q(t)\sin\delta(t) - \frac{(x_d - x_q)(V_s)^2}{(x'_{ds})^2}\sin\delta(t)\cos\delta(t) \quad (12)$$

In terms of the state variables δ and $\omega(t) = \dot{\delta}(t)$, the equation (2) becomes

$$\frac{d\omega(t)}{dt} = (P_m - \frac{V_s x_{ds}}{(x'_{ds})^2} E'_q(t)\sin\delta(t) + \frac{(x_d - x_q)(V_s)^2}{(x'_{ds})^2}\sin\delta(t)\cos\delta(t) - D\omega(t))\frac{\omega_0}{2H} \quad (13)$$

In terms of the state variables $E'_q(t)$ and $E_f(t)$ the equation (3) becomes

$$\frac{dE'_q(t)}{dt} = \frac{\omega_0 r_{fd}}{x_{ad}} E_{fd}(t) + \frac{x_{ds}}{x'_{ds}T'_{do}} E'_q(t) + \frac{(x_d - x_q)V_s}{x'_{ds}T'_{do}} \quad (14)$$

Where,

$$T'_{do} = \frac{x_{ad}}{\omega_0 r_{fd}}$$

The governor valve equation is given by

$$\frac{P_w}{U_g} = \frac{K_v}{1 + \tau_g s} \quad (15)$$

The exciter equation defined by

$$\frac{E_{fd}}{U_e} = \frac{1}{1 + \tau_e s} \quad (16)$$

The turbine equation

$$\frac{P_m}{P_w} = \frac{1}{1 + \tau_b s} \quad (17)$$

In terms of the state variables E_{fd} P_w and P_m the equations (15,16,17) written as follow:

$$P_w, \frac{dP_w(t)}{dt} = \frac{-P_w}{\tau_g} + \frac{K_v}{\tau_g} U_g \quad (18)$$

$$\frac{dE_{fd}(t)}{dt} = \frac{-E_{fd}(t)}{\tau_e} + \frac{1}{\tau_e} U_e \quad (19)$$

$$\frac{dP_m}{dt} = \frac{-P_m}{\tau_b} + \frac{1}{\tau_b} P_w \quad (20)$$

Defining $x = [\delta \quad \dot{\delta} \quad E'_q \quad E_{fd} \quad P_w \quad P_m]^T$ the state variables vector then the equations above can be written in the key form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (x_6 - K_1 x_3 \sin x_1 - K_2 \sin x_1 \cos x_1 - Dx_2)\frac{\omega_0}{2H} \\ x_3 = \frac{\omega_0 r_{fd}}{x_{ad}} x_4 + K_3 x_3 - K_4 \cos x_1 \\ \dot{x}_4 = \frac{-x_4}{\tau_e} + \frac{1}{\tau_e} U_e \\ \dot{x}_5 = \frac{-x_5}{\tau_g} + \frac{1}{\tau_g} U_g \\ \dot{x}_6 = \frac{-x_6}{\tau_b} + \frac{x_5}{\tau_b} \end{cases} \quad (21)$$

The output y_1, y_2 may be expressed in terms of these state variable by

$$y_1 = P_t = K_1 x_3 \sin x_1 + K_2 \sin x_1 \cos x_1 \quad (22)$$

$$y_2 = V_t = \left(V_d^2 + V_q^2\right)^{1/2} \quad (23)$$

Where,

$$V_d = K_5 \sin x_1 \quad (24)$$

$$V_q = K_6 x_3 + K_7 \cos x_1 \quad (25)$$

2.1. Linear Model of Synchronous Generator

A linear Multi-Input Multi-output (MIMO) model of the generator system is required to design a controller for such system. It is derived from the system nonlinear model by linearizing the nonlinear equation (13) and the nonlinear equation (14) around a specific operating point. The linear state-space model is derived next where the variables shown represent small displacements around the selected operating point.

$$\begin{cases} \dot{x}(t) = F_X x(t) + F_U u(t) \\ y(t) = G_X x(t) + G_U u(t) \end{cases} \quad (26)$$

Where,

$$F_X = \left.\frac{\partial f}{\partial X^T}\right|_{\bar{X},\bar{U}} = \begin{bmatrix} \frac{\partial f_1}{\partial X_1} & \cdots & \frac{\partial f_1}{\partial X_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial X_1} & \cdots & \frac{\partial f_n}{\partial X_n} \end{bmatrix}_{\bar{X},\bar{U}}$$

$$F_U = \left.\frac{\partial f}{\partial U^T}\right|_{\bar{X},\bar{U}} = \begin{bmatrix} \frac{\partial f_1}{\partial U_1} & \cdots & \frac{\partial f_1}{\partial U_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial U_1} & \cdots & \frac{\partial f_n}{\partial U_n} \end{bmatrix}_{\bar{X},\bar{U}}$$

$$G_X = \left.\frac{\partial g}{\partial X^T}\right|_{\bar{X},\bar{U}} = \begin{bmatrix} \frac{\partial g_1}{\partial X_1} & \cdots & \frac{\partial g_1}{\partial X_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_p}{\partial X_1} & \cdots & \frac{\partial g_p}{\partial X_n} \end{bmatrix}_{\bar{X},\bar{U}}$$

$$G_U = \left.\frac{\partial g}{\partial U^T}\right|_{\bar{X},\bar{U}} = \begin{bmatrix} \frac{\partial g_1}{\partial U_1} & \cdots & \frac{\partial g_1}{\partial U_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_p}{\partial U_1} & \cdots & \frac{\partial g_p}{\partial U_m} \end{bmatrix}_{\bar{X},\bar{U}}$$

F_X, F_U, G_X et G_U are the Jacobian matrices of partial derivatives of f and g respectively to X and U evaluated at the point $(\bar{X},\bar{U})$.

The linear state-space model defined by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{27}$$

Where,

$$A = F_X,\ B = F_U,\ C = F_U,\ D = G_U$$

The matrices A, B, C and D have the form:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ K_8 & \frac{-D\omega_0}{2H} & K_9 & 0 & 0 & \frac{\omega_0}{2H} \\ K_{10} & 0 & K_3 & \frac{\omega_0 r_{fd}}{x_{ad}} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\tau_g} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\tau_b} & \frac{-1}{\tau_b} \end{bmatrix} B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\tau_e} & 0 \\ 0 & \frac{K_g}{\tau_g} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} K_{11} & 0 & K_{12} & 0 & 0 & 0 \\ K_{13} & 0 & K_{14} & 0 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where,

$x = [\delta \quad \dot{\delta} \quad E_q' \quad E_{fd} \quad P_w \quad P_m]^T$	State variables vector
$u = [U_e \quad U_g]^T$	Control input vector
$y = [P_t \quad V_t]^T$	Measurement vector
$P_t = K_{11}x_1 + K_{12}x_3$	Output Power
$V_t = K_{13}x_1 + K_{14}x_3$	Terminal voltage

Expressions for parameters K1, K2, K3, K4, K5, K6, K7, K8, K9, K10, K11, K12, K13 and K14 are given in Appendix.

2.2. State Space to Transform Function Conversion

Consider the state equation (27). We may take its Laplace transform and rearrange it as follows:

$$\begin{aligned} sX(s) &= AX(s) + BU(s) \\ \rightarrow (sI - A)X(s) &= BU(s) \end{aligned} \tag{28}$$

If we combine this with the transform of the output equation: $Y(s) = CX(s) + DU(s)$, we get $Y(s) = C(sI - A)^{-1}BU(s) + DU(s)$

Or, equivalently

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \tag{29}$$

In the Control Systems Toolbox, the command $[num, den] = ss2tf(A, B, C, D, i)$ converts the state equation to a transfer function for i^{th} input.

3. Multivariable Linear Quadratic Gaussian Control Algorithm

Let us, consider the following $m \times m$ process model [24]:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})\xi(t) \tag{30}$$

Where,

$$A(q^{-1}) = I_m + A_1 q^{-1} \ldots\ldots\ldots\ldots + A_{na} q^{-na},\ A_j \epsilon R^{m,m}$$

$$B(q^{-1}) = B_0 + B_1 q^{-1} \ldots\ldots\ldots\ldots + B_{nb} q^{-nb},\ B_j \epsilon R^{m,m}$$

$$C(q^{-1}) = C_0 + C_1 q^{-1} \ldots\ldots\ldots\ldots + C_{nc} q^{-nc},\ C \epsilon R^{m,m}$$

$y(t) \epsilon R^m$ is the output vector

$u(t) \epsilon R^m$ is the input vector

$\xi(t) \epsilon R^m$ is a sequence of independent random vectors with zero mean value and finite covariance matrix

q^{-1} is the backward shift operator such that $q^{-1}f(t) = f(t-1)$

To this model, we can associate the companion block state representation in the observable form by:

$$\begin{cases} x(t+1) = A'x(t) + B'u(t) + G'\xi(t) \\ y(t) = C'x(t) + \xi(t) \end{cases} \tag{31}$$

Where:

$$A' = \begin{bmatrix} -A_1 & I' & 0' & \cdots & 0' \\ -A_2 & 0' & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0' \\ \vdots & \vdots & 0 & \ddots & I \\ -A_n & 0' & \cdots & \cdots & 0' \end{bmatrix} \quad B' = \begin{bmatrix} B_0 \\ \vdots \\ B_n \end{bmatrix}$$

$$G' = \begin{bmatrix} C_1 + A_1 \\ \vdots \\ C_n + A_n \end{bmatrix} \quad C' = \begin{bmatrix} I_m & 0' & \dots & 0' \end{bmatrix}$$

$$A' \epsilon R^{mn,mn},\ B' \epsilon R^{mn,m},\ G' \epsilon R^{m,m},\ C' \epsilon R^{m,mn}$$

$x(t) \epsilon R^{mn}$ is the system state

$n = \max(na, nb, nc)$

The problem is to found a control vector by state feedback that minimizing the following criterion:

$$J = \lim_{T \to \infty} \sum_{t=0}^{T} \left\{ \begin{array}{l} \left[y(t) - y^*(t) \right]^T Q \left[y(t) - y^*(t) \right] \\ + u^T(t-1) \Lambda u(t-1) \end{array} \right\} \quad (32)$$

Where:

T is the control horizon

$y^*(t)$ is the reference vector sequence

Λ is a symmetric semi definite positive matrix

Q is a symmetric definite positive matrix

The solution is:

$$\Delta u(t) = -\Gamma(t) x(t) - v(t) + W(t) \quad (33)$$

Where,

$$\Gamma(t) = \left(B^T R B' + \Lambda \right)^{-1} B'^T R A' \quad (34)$$

$$W(t) = \left(B'^T R B' + \Lambda \right)^{-1} B'^T R G' \left[y(t) - C'x(t) \right] \quad (35)$$

$$v(t) = \left(B'^T R B' + \Lambda \right)^{-1} B'^T \begin{bmatrix} y^*(t+1) \\ + \left(A' - B'\Gamma \right)^T y^*(t+2) \\ + \ldots + \left(\left(A' - B'\Gamma \right)^T \right)^m y^*(t+l+1) \\ + \ldots \end{bmatrix} \quad (36)$$

$R(t)$ is the Riccati matrix

$$Y^{*T} = \begin{bmatrix} y^*(t) & 0' & \dots & 0' \end{bmatrix} \quad (37)$$

Remark

The solution is an explicit form of the state variables. But they are not available. Therefore a state observer is necessary.

The state observer is given by:

$$\hat{x}(t) = H \phi_e(t-1) \quad (38)$$

Where

$$H = \begin{bmatrix} -A_1 & \cdots & -A_n & -B_0 & \cdots & -B_n & -C_1 & \cdots & -C_n \\ \vdots & \iddots & \vdots & \vdots & \iddots & \vdots & \vdots & \iddots & \vdots \\ -A_n & \cdots & 0 & -B_n & \cdots & 0 & -C_n & \cdots & 0 \end{bmatrix}$$

$$\phi_e^T(t) = \begin{bmatrix} y(t) \ldots y(t-n+1) u(t) \\ \ldots u(t-n+1) \xi(t) \ldots \xi(t-n+1) \end{bmatrix}$$

$H \in R^{3mn}$

$\phi_e \in R^{3mn}$

$$\xi(t) = \hat{y}(t) - Cx(t) \quad (39)$$

4. Multivariable Generalized Predictive Control Algorithm

When considering regulation about a particular operating point, even a non-linear plant generally admits a locally-linearized model [11,12] given by the equation (30).

The objective of the GPC control is the output $y(t)$ to follow some reference signal $y^*(t)$ taking into account the control effort. This can be expressed in the following cost function:

$$J(h_i, h_p, h_c, t) = \left\{ \begin{array}{l} \sum_{h_i}^{h_p} \left[y(t+j) - y^*(t+j) \right]^T R [y(t+j) - y^*(t+j)] \\ + \sum_{h_i}^{h_c} \Delta u^T (t+j-1) Q \Delta u(t+j-1) \end{array} \right\} \quad (40)$$

Where:

h_p is the prediction horizon.

h_i is the initial horizon.

h_c is the control horizon.

$y^*(t)$ is the output reference.

R is the output weighting factor.

Q is the control weighting factor.

The control objective is to compute at each time t, control inputs that minimize the quadratic criterion $J(h_i, h_p, h_c, t)$ for this there are two cases:

Case 1. $h_c = h_p,\ h_i = 1$

Let us first build j-step ahead predictors with following Diophantine equation:

$$1_m = E^j(q^{-1}) A(q^{-1}) \Delta(q^{-1}) + q^{-j} F^j(q^{-1}) \quad (41)$$

$$j = 1 \ldots hp$$

Where:

$$E^j(q^{-1}) = 1_m + E_1 q^{-1} \ldots\ldots\ldots\ldots + E_{j-1} q^{-(j-1)},\ E_j \epsilon R^{m,m}$$

$$F^j(q^{-1}) = F_0^j + F_1^j q^{-1} \ldots\ldots\ldots\ldots + F_{na}^j q^{-na},\ F_j^j \epsilon R^{m,m}$$

The polynomial matrices $F^j\left(q^{-1}\right)$ and $E^j\left(q^{-1}\right)$ are uniquely defined by: $A\left(q^{-1}\right)$, $\Delta\left(q^{-1}\right)$ and j.

Using equation (30) and (40) we obtain:

$$\begin{aligned} &y(t+j) \\ &= E^j(q^{-1})B(q^{-1})\Delta u(t+j-1) \\ &+F^j(q^{-1})y(t)+E^j(q^{-1})\xi(t+j) \end{aligned} \tag{42}$$

The optimal predictor $y(t+j)$ at time t is given by:

$$\hat{y}(t+j/t) = G^j(q^{-1})\Delta u(t+j-1)+F^j(q^{-1})y(t) \tag{43}$$

Where:

$$G^j(q^{-1}) = E^j(q^{-1})B(q^{-1})$$

Defining $G^j\left(q^{-1}\right) = g_0^j + g_1^j q^{-1} \ldots\ldots\ldots\ldots + g_{j-1}^j q^{-(j-1)}$ then the equation above can be written in the key vector form:

$$\hat{Y} = G\Delta U_t + Y_0 \tag{44}$$

$$\begin{aligned} \hat{Y}^T &= [\hat{y}^T(t+1)\ldots\ldots\ldots\ldots\hat{y}^T(t+h_p)] \\ \Delta U_t^T &= [\Delta U_t^T(t)\ldots\ldots\ldots\ldots\Delta U_t^T(t+h_p-1)] \\ Y_0^T &= [y_0^T(t+1)\ldots\ldots\ldots\ldots y_0^T(t+h_p)] \end{aligned}$$

Note that: $G^j\left(q^{-1}\right) = \dfrac{B\left(q^{-1}\right)\left[1-q^{-j}F^j\left(q^{-1}\right)\right]}{A\left(q^{-1}\right)}$ so that one way to computing G^j is simply to consider the Z-transform plant's step-response and to take the first j terms and therefore $g_j^i = g_i$ for j=0, 1, 2 …< i independent of the particular G polynomial [11].

The matrix G is then lower-triangular of dimension $mh_p \times mh_p$

$$G = \begin{bmatrix} g_0 & 0 & \cdots\cdots & 0 \\ g_1 & g_0 & \cdots\cdots & 0 \\ \cdot & \cdot\cdot & \cdots\cdots & \cdot \\ \vdots & & & \vdots \\ g_{h_p-1} & g_{h_p-2} & \cdots\cdots & g_0 \end{bmatrix}$$

From the definitions above of the vectors and with:

$$Y^{*T} = \left[y^{*T}(t+1)\ldots\ldots\ldots\ldots y^{*T}(t+h_p)\right] \tag{45}$$

The expectation of the cost-function of (4) can be written as follow:

$$\begin{aligned} &J\left(h_i, h_p, h_c, t\right) \\ &= \left(G\Delta U_t + Y_0 - Y^*\right)^T \bar{R}\left(G\Delta U_t + Y_0 - Y^*\right) \\ &+\Delta U_t \bar{Q} \Delta U_t^T \end{aligned} \tag{46}$$

The solution, ΔU_t minimizing the criterion can be explicitly found, using:

$$\frac{\partial J}{\partial \Delta U_t} = 0 \tag{47}$$

it follows that:

$$\Delta U_t^* = \left(G^T G + Q\right)^{-1} G^T R\left(Y_0 - Y^*\right) \tag{48}$$

Note that the first element ΔU_t^* of is $\Delta u(t)$ so that the current control u(t) is given by:

$$u(t) = u(t-1) + (G^T G + Q)^{-1} G^T R\left(Y_0 - Y^*\right) \tag{49}$$

Case 2. $h_c < h_p,\ h_i = 1$

It is possible to reduce computational burden by imposing a constant control input vector after a fixed horizon h_c.

$$\Delta u(t+j-1) = 0,\quad j > h_c$$

In this case the vector ΔU_t and the matrix G become:

$$\Delta U_t^T = \left[\Delta U_t^T(t)\ldots\ldots\ldots\ldots\Delta U_t^T(t+h_c-1)\right]$$

$$G = \begin{bmatrix} g_0 & 0 & \cdots\cdots & 0 \\ g_1 & g_0 & \cdots\cdots & 0 \\ \cdot & \cdot\cdot & \cdots\cdots & g_0 \\ \vdots & & & \vdots \\ g_{h_p-1} & g_{h_p-2} & \cdots\cdots & g_{h_p} - g_{h_c} \end{bmatrix}$$

Where,

$$G \in R^{mhp \times mhc}$$

5. Simulation and Discussion

To demonstrate the effectiveness of the above presented multivariable GPC Control algorithm, the result are presented and compared with those of the multivariable LQG control. The simulation results are obtained by using Matlab Toolbox.

Initial condition (operating point) for the non linear system:

$$x = [0.775 \quad 0 \quad 1.434 \quad -0.0016 \quad 0.8 \quad 0.8]^T$$

The hydropower plant model is as follow:

$$A\left(q^{-1}\right)y(t) = B\left(q^{-1}\right)u(t-1)$$

Where:

$$A_1 = \begin{bmatrix} -1.826 & 0 \\ 0 & -1.54 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1.21 & 0 \\ 0 & 0.7693 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.3479 & 0 \\ 0 & -0.1275 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0.03653 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 1.367 & -0.03534 \\ -0.07828 & 1.218 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -2.107 & -0.06115 \\ 0.1818 & -0.8947 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1.043 & 0.06876 \\ -0.1276 & 0.1182 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} -0.1629 & -0.01589 \\ 0.02571 & 0.02204 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 0.003059 & 0 \\ 0 & 0 \end{bmatrix}$$

The simulation has been done with respect to the following considerations:

Parameters of the GPC controller

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$h_p = 10, h_C = 5$$

Parameters of the LQG controller

$$\Lambda = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The reference is chosen as a square wave.

Simulations were carried out to verify the advantages of using multivariable GPC control in this application.

In the figures above, it can be observed the comparative results between multivariable GPC Control and multivariable LQG control.

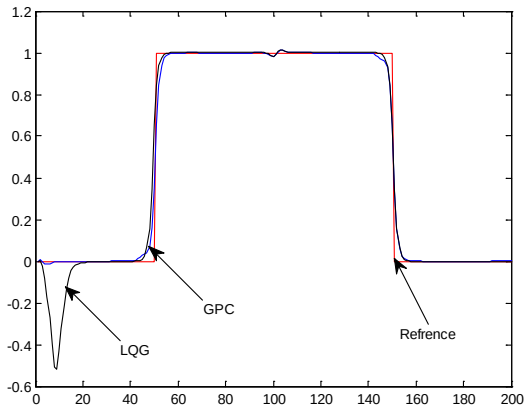

Figure 2. Power output P_t

The output real power P_t, the exciter input voltage U_e, the terminal voltage V_t and governor valve position U_g, under GPC and LQG controls are shown, respectively, in Figures 2, 3, 4 and 5. Best performance is characterized by best tracking, robustness, lower or no over/undershoots less or no oscillations. Based on this, for Pt response, GPC shows the best response whereas LQG shows the worse one with a bigger non minimum phase undershoot, which is eliminated by using GPC control. For V_t response GPC control produces the best response in terms of tracking, and overshoot, cancellation of oscillation. The LQG response has a very high overshoot, which is eliminated by using GPC control.

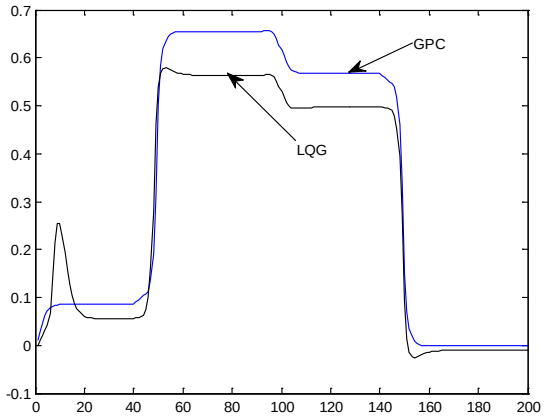

Figure 3. Exciter input U_e

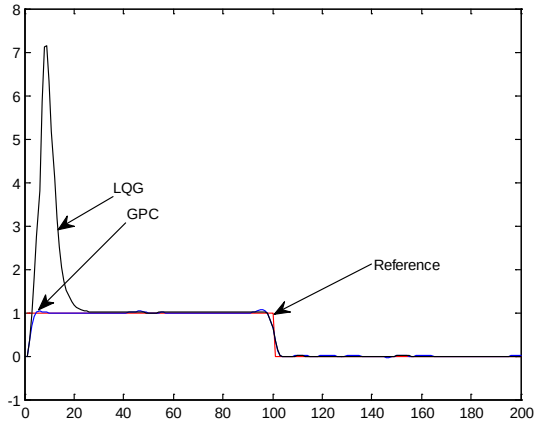

Figure 4. Terminal voltage V_t

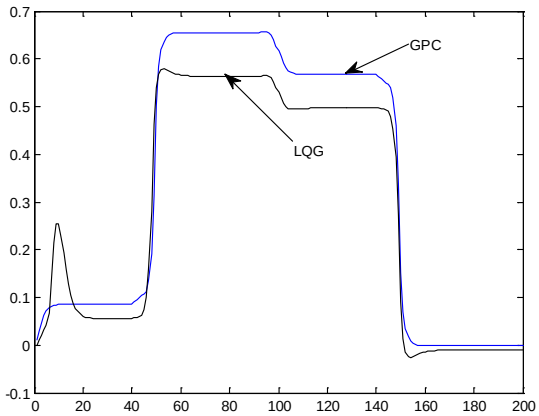

Figure 5. Governor Input Ug

6. Conclusion

In this paper, a Multivariable Generalized Predictive Controller and Multivariable Linear Quadratic Gaussian controller were designed for a sample power system comprising a water turbine driving a synchronous generator. From the simulation results, it is clear that, the GPC control exhibits better performance for P_t and V_t responses than the LQG control.

Appendix

Expressions for parameters K1, K2, K3, K4, K5, K6, K7, K8, K9, K10, K11, K12, K13 and K14 in the system model are:

$$K_1 = \frac{V_s x_{ds}}{\left(x'_{ds}\right)^2}$$

$$K_2 = -\frac{(x_d - x_q)V_s^2}{\left(x'_{ds}\right)^2}$$

$$K_3 = -\frac{x_{ds}}{x'_{ds}T'_{do}}$$

$$K_4 = -\frac{(x_d - x_q)V_s}{x'_{ds}T'_{do}}$$

$$K_5 = \frac{x_q V_s}{x'_{ds}}$$

$$K_6 = \frac{x_{t+}x_l}{x'_{ds}}$$

$$K_7 = \frac{x'_d V_s}{x'_{ds}}$$

$$K_8 = -K_1 x_{30}\cos(x_{10}) - K_2\cos(2x_{10})$$

$$K_9 = -K_1\sin(x_{10})$$

$$K_{10} = -K_4\sin(x_{10})$$

$$K_{11} = -K_1 x_{30}\cos(x_{10}) + K_2\cos(2x_{10})$$

$$K_{12} = K_1\sin(x_{10})$$

$$\begin{aligned} K_{13} = {} & ((K_5 - K_7^2)\sin(x_{10})\cos(x_{10}) \\ & - K_6 K_7 x_{30}\sin(x_{10}))((K_5\sin(x_{10}))^2 \\ & + (K_6 x_{30} + K_7\cos(x_{10}))^2)^{-1/2} \end{aligned}$$

$$\begin{aligned} K_{14} = {} & 2K_6(K_6 x_{30} + K_7\cos(x_{10}))((K_5\sin(x_{10}))^2 \\ & + (K_6 x_{30} + K_7\cos(x_{10}))^2)^{-1/2} \end{aligned}$$

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