

Numerical Simulation of the Magnetic Susceptibility of a Spin 1 System Interacting with an Oscillating Magnetic Field

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Abstract We made the numerical simulation of the theoretical results obtained by the resonant means, method in combination with Floquet's theorem of a set of spin particles interacting with an oscillating field. The paces that we obtained by the numerical simulation using Mathematica software of the magnetic susceptibility function theoretically obtained as a function of temperature or as a function of frequency represent a good agreement with the curves of the work published using experimental results. Our objective was to obtain a quality agreement between the experiments and the theoretical model studied.

Keywords: magnetic susceptibility, the resonant means method, floquet's theorem

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1. Introduction

The study of the magnetic susceptibility of materials has applications in several areas of physics such as electromagnetic, radar application, the optical propagation [1,3], Photonics [4,5] and semiconductors [6].

The properties of magnetic susceptibility have aroused the interest of many experimental and theoretical researchers [7]. Experimentally observed many properties of them were good compared to theoretical predictions based on spin theories of fluctuations. Dependence of temperature and frequency of magnetic susceptibility. The theory succeeded in deriving the Curie-Weiss law of the magnetic susceptibility of the nonlinear effect of thermal magnetic amplitudes greater than TC [14,19]. It is, however, known that the spontaneous magnetic moment does not vanish within the limit; $T \rightarrow TC$ [8,10] [20-25].

In this article, we will show the dependence of the magnetic susceptibility as a function of the frequency for several values of the temperature that we have chosen arbitrarily to compare our theoretical results [11] numerically with the results published in the literature, then we have although showed good agreement with the theoretical and experimental results of several authors published on the same subject [26,28].

2. Theoretical Study

2.1. Floquet Theorem

Consider a quantum system interacting with an oscillating field, Periodical période période $\tau = \frac{2\pi}{\omega_1}$, and amplitude ω_1 . The Schrödinger equation governing the evolution of this system is such that

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi(t)\rangle \quad (1)$$

$$H(t) = H_0 + \omega_1 H_1(t) \quad (2)$$

where H_0 is the Hamiltonian of the free system, and where H_1 is the Hamiltonian which reflects the interaction with aperiodic field. Let $V(t)$ the evolution operator on Eq. (1) $V(t)$ satisfies

$$i \frac{d}{dt} V(t) = H(t) V(t). \quad (3)$$

According Floquet's theorem, there is a decomposition $(R, T(t))$ such that :

$$V(t) = T(t) e^{-\frac{iRt}{\hbar}} \quad (4)$$

where $T(t)$ is a unitary operator, τ Periodical period (or a multiple de τ), and where R is a constant Hermitian operator.

$V(t)$ is the interaction representation

2.2. Resonating Main Values Method (MMR)

The method of resonant medium (MMR) is due to G. and M. Lochak Thiounn [12] and is a generalization of the method of Bogolyubov [13,28,29]. It allows the approximate solution of the Schrödinger equation governing the evolution of a quantum system interacting with a periodic field.

The solutions are obtained in the form of a series expansion in powers ω_1 . The Hamiltonian $H_I(t)$ can be written as :

$$H_I(t) = \sum (H_I)_k e^{i\Omega_k t} \quad (5)$$

where $(H_I)_k$ is a constant set of hermetic operators; Ω_k which is a sequence of frequencies, which can extract a suite including the frequency $\Omega_k = 0$. Suppose that this subsequence contains finite number k of frequencies that can direct $\Omega_0, \Omega_1, \Omega_2, \dots$. And the other Ω_k frequencies, Ω_{k+1} are neither harmonics. And the other Ω_k frequencies, Ω_{k+1} are neither harmonics or harmonic combinations of raw k . Then we can write the interaction Hamiltonian in the form:

$$H_I = \sum_{p=0}^{k-1} (H_I)_p e^{i\Omega_p t} + \sum_{p=k}^{\infty} (H_I)_p e^{i\Omega_p t} \quad (6)$$

We define two operators called "middle part" and oscillating part "of $(H_I)_k$ respectively denoted \bar{H} and asking

$$\bar{H}_I(t) = \sum_0^{k-1} (H_I)_p e^{i\Omega_p t} \quad (7)$$

$$\tilde{H}_I(t) = \sum_k^{\infty} (H_I)_p \frac{e^{i\Omega_p t}}{i\Omega_p}. \quad (8)$$

2.3. Application of the Method to a Set of Spin One Interacting with an Oscillating Field

Consider a statistical ensemble of N particles with spin 1 without mutual interactions; individual magnetic moment; subjected to a static field $\vec{h}_0 = h_0 \vec{k}$ and an oscillating field $\vec{h}_1(t) = 2h_1 \cos \omega t \vec{i}$.

The resulting Hamiltonian interaction with these fields is given by:

$$H(t) = \hbar \omega_0 I_Z + 2\hbar \omega_1 I_x \cos \omega t \quad (10)$$

2.3.1. Susceptibility to Improved First Order

From this result we note that the component of the magnetization depends only on the first order of the intensity of the oscillating field.

Knowing that the magnetic susceptibility can be written:

$$\chi(\omega) = \chi'(\omega) - i\chi''(\omega) \quad (11)$$

we can deduce the value of the magnetic susceptibility at first order improved depending on the density of the oscillating field, we obtained the following results

The term dispersion is given by the equation:

$$\chi'(\omega) = \frac{2N\gamma\hbar}{(\omega^2 - \omega_0^2)} \left(\frac{\hbar\omega_0}{3KT} - \frac{\hbar^2}{3} \right) \omega_0 \quad (12)$$

The term absorption is written

$$\chi''(\omega) = \frac{2N\gamma\hbar}{(\omega^2 - \omega_0^2)} \left(\frac{\hbar\omega_0}{3KT} - \frac{\hbar^2}{3} \right) \omega \quad (13)$$

Susceptibility to order a is expressed by the equation :

$${}^1\chi(\omega) = {}^1\chi'_1(\omega) - i {}^1\chi''_1(\omega). \quad (14)$$

Hence we have deduced the following equation:

$${}^1\chi(\omega) = \frac{2N\gamma\hbar}{(\omega^2 - \omega_0^2)} \left(\frac{\hbar\omega_0}{3KT} - \frac{\hbar^2}{3} \right) \{\omega_0 - i\omega\} \quad (15)$$

We note that the magnetic susceptibility to order one is linear, and takes its minimum value if ω tends to zero.

2.3.2. Magnetic Susceptibility to Third Order

In order three, only the Magnetic susceptibility to this order of approximation is given by

$$\chi_3(\omega) = \chi_1(\omega) + \omega_1^2 {}^1\chi_3(\omega). \quad (16)$$

We obtained the coefficient of dispersion to order three, it is as

$$\chi'_3(\omega) = \chi'_1(\omega) + \omega_1^2 {}^1\chi'_3(\omega) \quad (17)$$

with

$${}^1\chi\chi'_3(\omega\omega) = \frac{\hbar^5 \omega_0 e^{-i\omega_0 t}}{12KT(\omega + \omega_0)(\omega - \omega_0)^2} - \frac{\hbar^4 \omega_0}{6\omega(\omega^2 - \omega_0^2)(\omega - \omega_0)} \quad (18)$$

We deduced the absorption coefficient at this order of approximation as

$$\chi\chi''_3(\omega\omega) = \chi\chi''_1(\omega\omega) + \omega\omega_1^2 {}^1\chi\chi''_3(\omega\omega) \quad (19)$$

or

$${}^1\chi\chi''_3(\omega\omega) = \frac{\hbar^5 \omega e^{-i\omega_0 t}}{12KT(\omega + \omega_0)(\omega - \omega_0)^2} - \frac{\hbar^4 \omega}{6\omega(\omega^2 - \omega_0^2)(\omega - \omega_0)} \quad (20)$$

This result highlights the nonlinearity which appeared in the formula of magnetic susceptibility in this order approximation, this translates that has a population of difference in the transitions between the stationary states.

3. Simulation

3.1. Variation of Magnetic Susceptibility as a Function of Frequency

The magnetic susceptibility varies linearly as a function of the frequency, and presents a slope which differs for precise values of the temperature. The simulation of the magnetic susceptibility in absorption therm of the

following equation $\chi'(\omega) = \frac{2N\gamma\hbar}{(\omega^2 - \omega_0^2)} \left(\frac{\hbar\omega_0}{3KT} - \frac{\hbar^2}{3} \right) \omega_0$ as

a function of the temperature shows an interesting decrease as the temperature increases. We compared this curve with the one published in the article [11] we will find a good agreement.

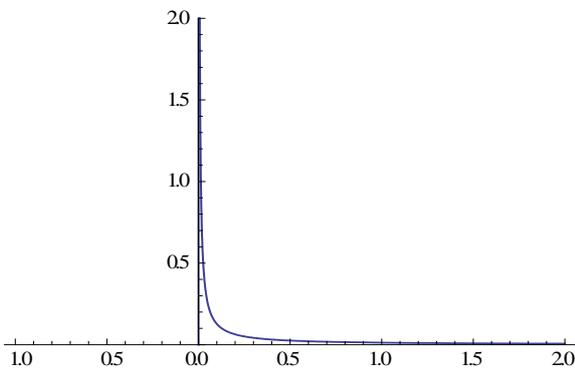


Figure 1. Variation of magnetic susceptibility as a function of ω and T

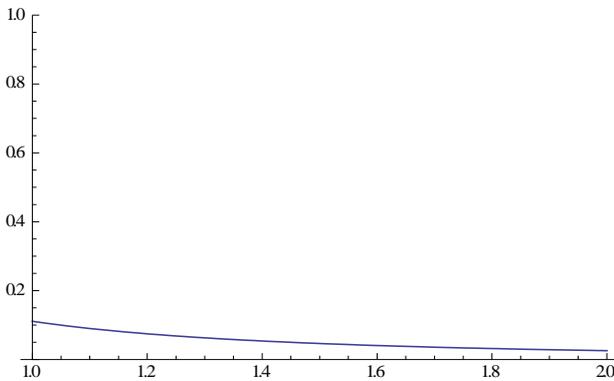


Figure 2. Temperature dependence of magnetic susceptibility

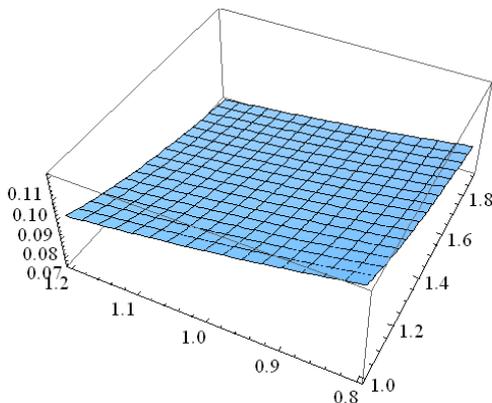


Figure 3. A variation of the dispersion term of order 3 depending on the temperature

4. Conclusion

In conclusion, the linear effect is due to the transition level of the photon. In addition, the increase in relaxation time, may improve this effect ; we have also shown that the nonlinearity depends on the temperature , and other parameters such as the structure of the material. We approached the study of a simple example where it is possible, without too many complications, pushing the calculations in higher orders. We have set thus identified "non-linear" interesting effects: saturation effects and nonlinear susceptibility

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