

Thermodynamically Consistent Analysis of Magnetocaloric Effects

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Abstract We suggest a thermodynamically and mathematically consistent analysis of magnetocaloric effect for a plate immersed in a uniform static magnetic field. We ignore deformability of the plate, but make no assumption regarding the amplitude of the magnetic field - it can be arbitrarily large. Traditional presentations of magnetocaloric effect are rather simple and straightforward - they are based on the algebraic manipulations with thermodynamic identities, and no analysis of boundary value problems is required. But they suffer one conceptual drawback - they are dealing with the magnetic field inside the specimen. However, the interior field is, a priori, unknown and depends on the geometry of the specimen. In fact, the meaningful analysis should be based on usage of the experimentally controllable exterior field. The relevant analysis therefore, should be based on the consideration of the boundary value problem for the equations of magnetostatics. We establish the relevant relationships of the magnetocaloric effect for the sample in the shape of a plate.

Keywords: electromagnetism, thermodynamics, low temperature physics, insert, template

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1. Introduction

Per the classical definition (https://en.wikipedia.org/wiki/Magnetic_refrigeration), "The magnetocaloric effect (MCE, from magnet and calorie) is a magneto-thermodynamic phenomenon in which a temperature change of a suitable material is caused by exposing the material to a changing magnetic field..." The phenomenon was widely used in low temperature physics instead of traditional adiabatic cooling caused by mechanically induced expansion. Unfortunately, mechanically induced adiabatic cooling becomes unavailable at superlow temperatures because all substances become unacceptably brittle at those temperatures. Magnetocaloric adiabatic cooling appears to be the appropriate substitute. In the low temperature physics the magnetocaloric effect is known as adiabatic demagnetization.

Contemporary theoretical analysis of electrocaloric and magnetocaloric effects can be found in many books, including Landau and Lifshitz [1], Rosensweig [2].

Because of the remarkable progress with promising magnetocaloric materials, last years show a splash of experimental Giri and Cho (2014) and theoretical studies, including Tishin and Spichkin [3] and Kitanovski, Tušek, Tomc, Plaznik, Ozbolt, Poredoš [4]. The comprehensive review paper Franco, Blázquez [5] counts 1600 references for the topic of magnetocaloric effect.

2. Equations of Magnetostatics

Consider a nondeformable magnetizable plate immersed into unbounded domain with the originally uniform magnetic field, as shown in Figure 1. Let H and h be the magnetic fields outside and inside the body, respectively. The magnetic field should be distinguished from the magnetic induction B . They are different in the domains containing the distributed magnetization M ; namely, in the Gauss system of units they connected by the relationship

$$B = h + 4\pi M \quad (1)$$

In vacuum the fields H and B coincide.

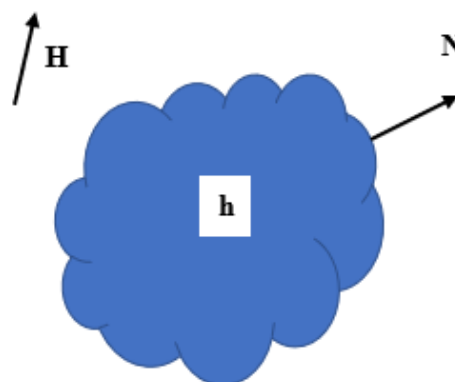


Figure 1. Magnetizable body immersed into magnetic field

The field \mathbf{H} should be distinguished from the external field \mathbf{H}° because the field \mathbf{H} also includes the field created by the distributed magnetization vectors \mathbf{M} .

In the following we assume that space is referred to the affine coordinates Z^i with the uniform covariant basis \mathbf{Z}_i and the uniform covariant metrics Z_{ij} , so that $\mathbf{H} = H^i \mathbf{Z}_i$, $\mathbf{h} = h^i \mathbf{Z}_i$, $\mathbf{H}^\circ = H^{\circ i} \mathbf{Z}_i$, $\mathbf{B} = B^i \mathbf{Z}_i$ and $\mathbf{M} = M^i \mathbf{Z}_i$.

Per currently dominating point of view, the magnetic field is created by the currents of charged substance. When there are no macroscopic moving charges, the magnetic field appears to be potential

$$H_i = -\nabla_i \phi. \quad (2)$$

Also, in magnetostatics the magnetics induction is divergence-free

$$\nabla_i B^i = 0. \quad (3)$$

In phenomenological thermodynamics of magnetism of nondeformable substances we introduce also free energy density per unit volume Ψ which is the function of the magnetization M^i per unit volume and the absolute temperature T - $\Psi = \Psi(M, T)$; here and in the following we omit indices in the arguments.

The free energy density function $\Psi(M, T)$ cannot be determined with the methods of phenomenological thermodynamics. Thermodynamics, however, permits to determine various universal relationships, i.e., the relationships that should be valid for any particular models of substance. For instance, it allows to establish the universal identity

$$H_i = \frac{\partial \Psi(M, T)}{\partial M^i} \quad (4)$$

as it was done, even in more general context, in (Grinfeld and Grinfeld, [6]); other, more traditional form can be found in (Landau and Lifshitz [1], Rozenzweig [2], among others).

Contrary, to the widespread relationship $B^i = \mu^{ij} H_j$ (where μ^{ij} is the permittivity tensor), which is valid in the limited domain of the parameters, the thermodynamics identity is valid everywhere where the quite universal model $\Psi = \Psi(M, T)$ is applicable.

Basically, the thermodynamics-based analysis of magnetocaloric effect uses another thermodynamics identity

$$\frac{\partial T(S, h)}{\partial h} = -\frac{T}{C_h(T, h)} \frac{\partial M(T, h)}{\partial T} \quad (5)$$

which can be derived from thermodynamics and where $C_h(T, h)$ is the specific heat capacity per unit volume. In fact, all the derivatives in the Equation (5) can be expressed in terms of the free energy density $\Psi(M, T)$ and its derivatives.

The identity is applicable to the bodies of arbitrary geometry - no geometry whatsoever appears in this formula. The cost for this universality is rather high,

however. Namely, the formula uses the internal magnetic field h^i whereas in the experiments and applications we can control only the external field $H^{\circ i}$. Therefore, we need the relationship between the equilibrium temperature and the field $H^{\circ i} - T(H^{\circ i})$. Of course, this formula is not as universal as the Equation (5), it depends upon the geometry of the system and its determination is based on solving boundary value problems for different geometries of the magnetizable bodies. For that sake, we need not only the bulk Equations (2), (3) but also the boundary conditions and/or conditions at infinity.

The boundary conditions of the potential continuity reads

$$[\phi] = 0 \quad (6)$$

the other condition reflects continuity of the normal component of the magnetic induction

$$[B^i] N_i = 0. \quad (7)$$

Equation (6) implies the continuity of the tangential components of the magnetic field; that fact can be presented in the form

$$[H^k] (\delta_k^i - N^i N_k) = 0 \quad (8)$$

where $\delta_k^i - N^i N_k$ is the operator of projection onto the tangent planes.

Using the Equations (1) and (2), we can rewrite the Equation (3) as follows

$$\nabla_i (-\nabla^i \phi + 4\pi M^i) = 0. \quad (9)$$

Thus, we arrive at the 2 bulk equations - the nonlinear algebraic equation (4) and the partial differential equation (9), for 2 unknown functions $\phi(Z)$ and $M^i(Z)$.

The Equation (9) demands 2 boundary conditions at the interface, one of which is the potential continuity condition Equation (6). The second one is implied by the Equations (1), (2) and (7). It reads

$$[-\nabla^i \phi + 4\pi M^i] N_i = 0. \quad (10)$$

3. Magnetocaloric Effect for Magnetizable Plate

The simplest boundary value problem corresponds to the case of a uniform plate immersed into the uniform external field $H^{\circ i}$. Consider an unbounded plate made of a nondeformable magnetizable substance (see the Figure 2).

For this geometry, the equilibrium magnetization M^i will be uniform as well as the magnetic fields $H^i = H^{\circ i}$ and h^i outside and inside the plate. If so, then the bulk partial differential equation (9) will be satisfied automatically, and it remains to satisfy the boundary conditions (8) and (10) only. These boundary conditions can be presented in the form

$$H^{\circ i} N_i = (h^i + 4\pi M^i) N_i \quad (11)$$

and

$$h^k (\delta_k^i - N^i N_k) = H^{\circ k} (\delta_k^i - N^i N_k). \quad (12)$$

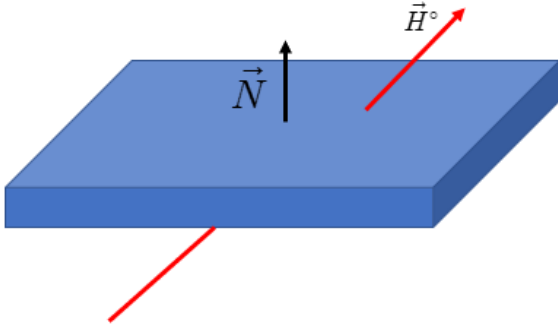


Figure 2. Polarizable plate in uniform magnetic field

The Equations (11), (12) imply the following relationship for the components of the magnetic field inside the plate

$$h^i = H^{\circ i} - 4\pi M^k N_k N^i \quad (13)$$

In order to establish Equation (13), we notice that any vector h^i can be presented in the form

$$h^i = h^k N_k N^i + h^k (\delta_k^i - N^i N_k) \quad (14)$$

which can be easily verified.

Also, we can rewrite Equation (11) as follows

$$h^i N_i = (H^{\circ i} - 4\pi M^i) N_i. \quad (15)$$

Using Equation (15), we can rewrite Equation (14) as

$$h^i = (H^{\circ k} - 4\pi M^k) N_k N^i + h^k (\delta_k^i - N^i N_k). \quad (16)$$

Now, using Equation (12), we can rewrite Equation (16) as

$$h^i = (H^{\circ k} - 4\pi M^k) N_k N^i + H^{\circ k} (\delta_k^i - N^i N_k). \quad (17)$$

At last, after cancellation of similar terms in Equation (16) we arrive at the required Equation (13).

To finish our calculations we have to use the thermodynamics relationship

$$h_i = \frac{\partial \Psi(M, T)}{\partial M^i} \equiv \Psi_{M^i}(M, T) \quad (18)$$

where $\Psi(M, T)$ is the free energy density per volume as function of the absolute temperature T and the magnetization density per unit volume M^i .

Eliminating the internal magnetic field h^i between the Equations. (13) and (18), we get

$$\Psi_{M^i}(M, T) = H_i^{\circ} - 4\pi M^k N_k N_i. \quad (19)$$

Let us fix the external field \vec{H}° and change the temperature T . Then, the magnetization M^i changes also, and we arrive at the function $M^i = M^i(T)$. We can find the derivative of this function, differentiating the identity

$$\Psi_{M^i}(M^k(T), T) \equiv H_i^{\circ} - 4\pi M^k(T) N_k N_i \quad (20)$$

with respect to T . We, then, get

$$\Psi_{M^i M^k} \frac{dM^k}{dT} + \Psi_{M^i T} \equiv -4\pi \frac{dM^k(T)}{dT} N_k N_i \quad (21)$$

or else

$$(\Psi_{M^i M^k} + 4\pi N_k N_i) \frac{dM^k}{dT} \equiv -\Psi_{M^i T}. \quad (22)$$

Let us now fix the absolute temperature T and begin changing the external field \vec{H}° . Then, the entropy of the plate will change, and we arrive at the function $S_{\text{therm}}(H_i^{\circ})$. Similarly, we arrive at the function $M_{\text{therm}}^i(H_m^{\circ})$. We, then get, by definition

$$\begin{aligned} \Psi_{M^i}(M, T) \Big|_{M^k = M_{\text{therm}}^k(H^{\circ P})} &= \\ H_i^{\circ} - 4\pi M^k \Big|_{M^k = M_{\text{therm}}^k(H^{\circ P})} N_k N_i & \end{aligned} \quad (23)$$

Differentiating the identity (2.6), we get

$$\Psi_{M^i M^k} \frac{\partial M_{\text{therm}}^k}{\partial H^{\circ m}} = Z_{im} - 4\pi \frac{\partial M_{\text{therm}}^k}{\partial H^{\circ m}} N_k N_i \quad (24)$$

or

$$(\Psi_{M^i M^k} + 4\pi N_i N_k) \frac{\partial M_{\text{therm}}^k(H^{\circ P})}{\partial H^{\circ m}} = Z_{im}. \quad (25)$$

We, then, use the standard definition of the entropy density per unit volume

$$S(M, T) \equiv -\frac{\partial \Psi(M, T)}{\partial T} \equiv -\Psi_T. \quad (26)$$

Differentiating Equation (26) along the equilibrium curve, we get

$$\begin{aligned} \frac{\partial S_{\text{therm}}(H^{\circ P})}{\partial H^{\circ m}} &= \\ = -\Psi_{TM^k}(M, T) \Big|_{M^k = M_{\text{therm}}^k(H^{\circ P})} \frac{\partial M_{\text{therm}}^k(H^{\circ P})}{\partial H^{\circ m}}. & \end{aligned} \quad (27)$$

Let G^{im} be the matrix such that

$$(\Psi_{M^i M^k} + 4\pi N_i N_k) G^{in}(M_k, T, N_l) \equiv \delta_k^n. \quad (28)$$

Contracting Equation (25) with G^{in} , we get

$$\frac{\partial M_{\text{therm}}^n(H^{\circ P})}{\partial H^{\circ m}} \equiv G^{in} Z_{im}. \quad (29)$$

Then, the Equation (28) implies

$$\frac{\partial S_{\text{therm}}(H^{\circ P})}{\partial H^{\circ m}} = -\Psi_{TM^k} G^{ik} Z_{im}. \quad (30)$$

Assume that the plate-shaped specimen is maintained in the adiabatically isolated environment. Let us begin

changing the external magnetic field \vec{H}° . Then, the magnetization vector M^k and the absolute temperature T of the specimen will change also. In the special, plate-like, geometry these functions will be identical at each point of the specimen. Thus, we arrive at the functions $M_S^k(H^{\circ P})$ and $T_S(H^{\circ P})$. By definition, we arrive at the identities

$$S(M^k, T) \Big|_{\substack{M^k = M_S^k(H^{\circ P}), \\ T = T_S(H^{\circ P})}} = S^\circ = \text{const} \quad (31)$$

and

$$\begin{aligned} \Psi_{M^i} (M, T) \Big|_{\substack{M^k = M_S^k(H^{\circ P}), \\ T = T_S(H^{\circ P})}} \\ = \left(H^{\circ k} - 4\pi M^k \Big|_{M^k = M_S^k(H^{\circ P})} \right) N_k N_i. \end{aligned} \quad (32)$$

Differentiating the identities (31), (32) with respect to $H^{\circ m}$, we get the following relationships for derivatives along equilibrium curves

$$S_{M^k} \frac{\partial M_S^k(H^{\circ P})}{\partial H^{\circ m}} + S_T \frac{\partial T_S(H^{\circ P})}{\partial H^{\circ m}} = 0 \quad (33)$$

and

$$\begin{aligned} \Psi_{M^i M^k} \frac{\partial M_S^k(H^{\circ P})}{\partial H^{\circ m}} + \Psi_{M^i T} \frac{\partial T_S(H^{\circ P})}{\partial H^{\circ m}} \\ = \left(\delta_m^k - 4\pi \frac{\partial M_S^k(H^{\circ P})}{\partial H^{\circ m}} \right) N_k N_i. \end{aligned} \quad (34)$$

Equation (34) can be rewritten as

$$\begin{aligned} \left(\Psi_{M^i M^k} + 4\pi N_k N_i \right) \frac{\partial M_S^k(H^{\circ P})}{\partial H^{\circ m}} + \\ \Psi_{M^i T} \frac{\partial T_S(H^{\circ P})}{\partial H^{\circ m}} = N_m N_i \end{aligned} \quad (35)$$

Using (33), we can rewrite (35) as follows

$$\begin{aligned} \left(\Psi_{M^i M^k} - \Psi_{M^i T} \frac{S_{M^k}}{S_T} + 4\pi N_k N_i \right) \times \\ \frac{\partial M_S^k(H^{\circ P})}{\partial H^{\circ m}} = N_m N_i \end{aligned} \quad (36)$$

or else

$$R_{ik} \frac{\partial M_S^k(H^{\circ P})}{\partial H^{\circ m}} = N_m N_i \quad (37)$$

where R_{ik} is the following symmetric matrix

$$R_{ik} \equiv \frac{(\Psi_{M^i M^k} + 4\pi N_k N_i) \Psi_{TT} - \Psi_{M^i T} \Psi_{M^k T}}{\Psi_{TT}} \quad (38)$$

Let Q^{ij} be the inverse of R_{ik} . Then, the relationship (37) can be rewritten as

$$\frac{\partial M_S^j(H^{\circ P})}{\partial H^{\circ m}} = Q^{ij} N_m N_i. \quad (39)$$

Now, inserting (39) in (34), we get the final relationship

$$\frac{\partial T_S(H^{\circ P})}{\partial H^{\circ m}} = -\frac{\Psi_{TM^k}}{\Psi_{TT}} Q^{ik} N_m N_i, \quad (40)$$

connecting the change of the equilibrium temperature with changing exterior magnetic field.

4. Isotropic Polarizable Substance

In the case of isotropic substance, the free energy density can be presented as

$$\Psi(M^k, T) \equiv \psi(\mathcal{M}, T) \quad (41)$$

where the invariant \mathcal{M} is defined as follows

$$\mathcal{M} \equiv M^i M_i. \quad (42)$$

Using Equations (41), (42), we get for the derivatives of the free energy

$$\begin{aligned} \Psi_{M^i} &\equiv 2\psi_{\mathcal{M}} M_i, \\ \Psi_{M^i M^k} &\equiv 2\psi_{\mathcal{M}} Z_{ik} + 4\psi_{\mathcal{M}\mathcal{M}} M_i M_k. \end{aligned} \quad (43)$$

Inserting Equation (43) in Equation (19), we get

$$(2\psi_{\mathcal{M}} Z_{ik} + 4\pi N_k N_i) M^k = -H^{\circ k} N_k N_i. \quad (44)$$

Contracting Equation (44) with N^i , we get

$$(2\psi_{\mathcal{M}} + 4\pi) N_k M^k = -H^{\circ k} N_k \quad (45)$$

or

$$N_k M^k = -\frac{1}{2(\psi_{\mathcal{M}} + 2\pi)} H^{\circ k} N_k. \quad (46)$$

Using Equation (46), we can rewrite (44) as follows

$$M_i = -\frac{1}{2\psi_{\mathcal{M}}} (H^{\circ k} N_k N_i + 4\pi N_i N_k M^k). \quad (47)$$

Eliminating $N_k M^k$ between Equations (46), (47), we get

$$M_i = -\frac{1}{2(\psi_{\mathcal{M}} + 2\pi)} H^{\circ k} N_k N_i. \quad (48)$$

The Equation (48) implies that the polarization M_i is oriented along the normal N_i . Therefore, we get

$$M_i = M_k N^k N_k. \quad (49)$$

The Equations (48), (49) imply the relationship

$$\mathcal{M} = \frac{1}{4(\psi_{\mathcal{M}} + 2\pi)^2} (H^{\circ k} N_k)^2 \quad (50)$$

which permits to calculate \mathcal{M} .

When we change the absolute temperature of the plate and keep the external polarization $H^{\circ i}$ fixed, the

equilibrium polarization changes, and it is characterized by the function $M_i(T)$, which we call the equilibrium curve. Differentiating Equation (50), we arrive at the following relationship for the derivative of the function $M_i(T)$

$$\frac{d\mathcal{M}}{dT} \left[\frac{2(\psi_{\mathcal{M}} + 2\pi)^3}{(H^{\circ k} N_k)^2} + \psi_{\mathcal{M}\mathcal{M}} \right] = -\psi_{\mathcal{M}T} \quad (51)$$

or after some algebraic transformations we the following equation of the equilibrium curve.

$$\frac{d\mathcal{M}}{dT} = -\frac{\psi_{\mathcal{M}T}(H^{\circ k} N_k)^2}{2(\psi_{\mathcal{M}} + 2\pi)^3 + \psi_{\mathcal{M}\mathcal{M}}(H^{\circ k} N_k)^2}. \quad (52)$$

5. Magnetocaloric Effect in Isotropic Substance

The magnetocaloric effect in isotropic plates is described by the following analogies of Equations (39) and (40):

$$2M_k S_{\mathcal{M}} \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} + S_T \frac{DT_S(H^{\circ p})}{DH^{\circ m}} = 0 \quad (53)$$

and

$$\begin{aligned} & (\psi_{\mathcal{M}} Z_{ik} + 2\psi_{\mathcal{M}\mathcal{M}} M_i M_k + 2\pi N_k N_i) \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} + \\ & \psi_{\mathcal{M}T} M_i \frac{DT_S(H^{\circ p})}{DH^{\circ m}} = -\frac{1}{2} N_m N_i \end{aligned} \quad (54)$$

where $D/DH^{\circ m}$ is the symbol of differentiation along the equilibrium curve.

The Equation (53) implies

$$\frac{DT_S(H^{\circ p})}{DH^{\circ m}} = -2 \frac{S_{\mathcal{M}}}{S_T} M_k \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} \quad (55)$$

Using Equation (54), we can rewrite Equation (53) as follows

$$\begin{aligned} & (\psi_{\mathcal{M}} Z_{ik} + 2\psi_{\mathcal{M}\mathcal{M}} M_i M_k + 2\pi N_k N_i) \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} - \\ & \psi_{\mathcal{M}T} M_i \frac{2S_{\mathcal{M}}}{S_T} M_k \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} = -\frac{1}{2} N_m N_i \end{aligned} \quad (56)$$

and then

$$\begin{aligned} & \left[\frac{\psi_{\mathcal{M}} Z_{ik} + 2\pi N_i N_k +}{2 \frac{\psi_{\mathcal{M}\mathcal{M}} S_T - \psi_{\mathcal{M}T} S_{\mathcal{M}}}{S_T} M_i M_k} \right] \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} \\ & = -\frac{1}{2} N_m N_i. \end{aligned} \quad (57)$$

Equation (48) implies

$$M_i M_k = \frac{1}{4(\psi_{\mathcal{M}} + 2\pi)^2} (H^{\circ l} N_l)^2 N_i N_k \quad (58)$$

Inserting Equation (58) in Equation (57), we get

$$(\psi_{\mathcal{M}} Z_{ik} + \Gamma N_i N_k) \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} = -\frac{1}{2} N_m N_i \quad (59)$$

where the function Γ is defined as

$$\Gamma = \frac{B}{A} \quad (60)$$

where

$$A \equiv 2\psi_{TT}(\psi_{\mathcal{M}} + 2\pi)^2$$

$$B = (\psi_{\mathcal{M}\mathcal{M}}\psi_{TT} - \psi_{\mathcal{M}T}^2)(H^{\circ l} N_l)^2 + 4\pi\psi_{TT}(\psi_{\mathcal{M}} + 2\pi)^2.$$

Contracting both sides of Equation (59) with the components of unit normal N_i , we get

$$N_k \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} = -\frac{1}{2(\psi_{\mathcal{M}} + \Gamma)} N_m. \quad (61)$$

Let us rewrite Equation (59) as

$$\begin{aligned} & \psi_{\mathcal{M}} Z_{ik} \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} \\ & = -\frac{1}{2} N_m N_i - \Gamma N_i N_k \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}}. \end{aligned} \quad (62)$$

Using Equation (61), we can rewrite Equation (62) as follows

$$\frac{DM_S^i(H^{\circ p})}{DH^{\circ m}} = \frac{1}{2} \frac{1}{\psi_{\mathcal{M}} + \Gamma} N^i N_m. \quad (63)$$

Inserting Equation (63) in Equation (56), we get

$$\frac{DT_S(H^{\circ p})}{DH^{\circ m}} = \frac{\Psi_{\mathcal{T}\mathcal{M}} H^{\circ k} N_k N_m}{2\Psi_{TT}(\psi_{\mathcal{M}} + \Gamma)(\psi_{\mathcal{M}} + 2\pi)} \quad (64)$$

as implied by the following chain:

$$\begin{aligned} & \frac{DT_S(H^{\circ p})}{DH^{\circ m}} = -2 \frac{S_{\mathcal{M}}}{S_T} M_k \frac{DM_S^k(H^{\circ p})}{DH^{\circ m}} \\ & = -\frac{\Psi_{\mathcal{T}\mathcal{M}}}{\Psi_{TT}} \frac{1}{\psi_{\mathcal{M}} + \Gamma} M_k N^k N_m \\ & = \frac{\Psi_{\mathcal{T}\mathcal{M}}}{2\Psi_{TT}(\psi_{\mathcal{M}} + \Gamma)(\psi_{\mathcal{M}} + 2\pi)} H^{\circ k} N_k N_m. \end{aligned}$$

Equation (64) permits to analyze the magnetocaloric effect for the working magnetocaloric solids in the shape of a plate. It allows to make different quantitative and qualitative conclusions. In particular, it shows that when the angle between the external field and the normal direction to the plate approaches zero, then, the magnetocaloric effects approaches zero.

6. Conclusion

Based on the principles of thermodynamics, we established the Equations (30), (31), (39), (40), (52), (63), (64), allowing to analyze magnetocaloric effects in the

magnetocaloric plates, exposed to uniform magnetic field. The relationships (30), (31), (39), (40), concern plates made of the substances of arbitrary symmetry. The relationships (54), (63), (64) assume that the plates are made of isotropic substance.

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