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A New Collocation Method for Systems of Nonlinear Fredholm Integral Equations

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Abstract In this paper we present a new method for solving nonlinear Fredholm integral equations system in terms of continuous Legendre multi-wavelets on the interval [0, 1). To begin with we describe the characteristic of Legendre multi-wavelets and will go on to indicate that through this method a system of Fredholm integral equations can be reduced to an algebraic equation. Convergence analysis of this method is also presented. Finally, numerical results are given which support the theoretical results.

Keywords: *nonlinear Fredholm integral equation, system of integral equations, Legendre multi-wavelets, collocation method, Multiresolution of analysis (MRA), algebraic equations*

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1. Introduction

The subject of integral equations is one of the most important mathematical tools in both pure and applied mathematics. Integral equations play a very important role in modern science such as numerous problems in engineering and mechanics, for more details see [1,2,3]. For such equations as well as a system of such equations, various techniques such as iterative, extrapolation, Galerkin, collocation, quadrature, projection, spline, orthogonal polynomial, and multiple grid methods have been presented to determine desired solutions (see e.g. [1,2,3] and the references quoted there).

Let us consider the system of non linear Fredholm integral equations of the form:

$$F(x) = G(x) + \int_{\Gamma} K(x,t)h[F(t)]dt, \quad x \in \Gamma = [0,1], \quad (1)$$

where,

$$F(x) = [f_1(x), f_2(x), ..., f_n(x)]^T,$$
$$G(x) = [g_1(x), g_2(x), ..., g_n(x)]^T,$$
$$K(x,t) = [k_{i,j}(x,t)]^T, \quad i, j = 1, 2, ..., n.$$

In system (1) the known kernel $K(x, t)$ is continuous, the function $G(x)$ is given, and $F(x)$ is the solution to be determined [4].

There have been considerable interests in solving integral equation (1). In addition to the well-known techniques, there are several new techniques for solving integral equation systems [5-17]. As we know, it is important to select a suitable basis function in numerical methods for system of integral equations. One of the most attractive proposals made in the recent years was an idea connected to the application of wavelets as basis functions in the method of moments [18]. The wavelet technique allows the creation of very fast algorithms when compared to the algorithms ordinarily used and the main advantage of the wavelet technique is its ability to transform complex problems into a system of algebraic equations. Various wavelet basis are applied. In addition to the conventional Duabechies wavelets, Haar wavelets [19], linear B-splines [18], Walsh functions [20] have been used.

In this paper, we propose the application of the linear Legendre multi-wavelets as basis functions in collocation’s method for numerical solution of the system of nonlinear Fredholm integral equations (1). The method is tested with the numerical examples.

2. Legendre Multi-Wavelets

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, we have the following family of continuous wavelets as [21].

$$\varphi_{a,b}(t) = |a|^{-1/2} \phi(\frac{t-b}{a}), \quad a, b \in \Re, a \neq 0.$$

If we restrict the parameters a and b to discrete values as $a = 2^{-k}, b = n2^{-k}$, then

$$\varphi_{k,n}(t) = 2^{-k/2} \phi(2^k t - n),$$

form an orthogonal basis [21].

The linear Legendre multi-wavelets are described in [22] and applied in [23,24,25,26]. For constructing the linear

Legendre multi-wavelets, at first we describe the following scaling functions;

$$\varphi_0(t)=1,\quad \varphi_1(t)=\sqrt{3}(2t-1),\quad 0\le t\le 1.$$

Now let $\psi^0(t)$ and $\psi^1(t)$ be the corresponding mother wavelets, then by Multiresolution of analysis (MRA) and applying suitable conditions [22] on $\psi^0(t)$ and $\psi^1(t)$ the explicit formula for linear Legendre mother wavelets will obtain as;

$$\psi^0(t)=\begin{cases}-\sqrt{3}(4t-1), & 0\le t\le \frac{1}{2},\\ \sqrt{3}(4t-3), & \frac{1}{2}\le t\le 1,\end{cases}\tag{2}$$

$$\psi^1(t)=\begin{cases}6t-1, & 0\le t\le \frac{1}{2},\\ 6t-5, & \frac{1}{2}\le t\le 1,\end{cases}\tag{3}$$

and the family $\{\psi^j_{k,n}\}=\{2^{k/2}\psi^j(2^k t-n)\}$, k is any nonnegative integer, $n=0,1,\dots,2^k-1$ and j = 0, 1, forms an orthonormal basis for $L^2(\Re)$.

3. Function Approximation

A function $f(t)$ defined over [0, 1) can be expanded as;

$$f(t)=f_0\varphi_0(t)+f_1\varphi_1(t)+\sum_{k=0}^{\infty}\sum_{j=0}^{1}\sum_{n=0}^{\infty}f^j_{k,n}\psi^j_{k,n}(t),\tag{4}$$

where,

$$\begin{aligned}f_0&=<f(t),\phi_0(t)>,\\ f_1&=<f(t),\phi_1(t)>,\\ f^j_{k,n}&=<f(t),\psi^j_{k,n}(t)>.\end{aligned}\tag{5}$$

In Eq.(5), $<.,.>$ denoting the inner product. If the infinite series of Eq.(4) is truncated, then it can be written as;

$$\begin{aligned}&f(t)\approx f_0\phi_0(t)+f_1\phi_1(t)+\\ &\sum_{k=0}^{M}\sum_{j=0}^{1}\sum_{n=0}^{2^k-1}f^j_{k,n}\psi^j_{k,n}(t)=C^T\varphi=\sum_{i=1}^{2M+2}c_i\varphi_i(t),\end{aligned}\tag{6}$$

where,

$$\begin{aligned}&C=[f_0,f_1,f^0_{0,0},f^1_{0,0},\dots,f^0_{M,0},f^0_{M,1},\dots\\ &,f^0_{M,(2^M-1)},\dots,f^1_{M,0},f^1_{M,1},\dots,f^1_{M,(2^M-1)}]^T,\\ &\varphi=[\phi_0(t),\phi_1(t),\psi^0_{0,0}(t),\psi^1_{0,0}(t),\\ &\dots,\psi^0_{M,0}(t),\psi^0_{M,1}(t),\dots,\psi^0_{M,(2^M-1)}(t),\\ &\dots,\psi^1_{M,0}(t),\psi^1_{M,1}(t),\dots,\psi^1_{M,(2^M-1)}(t)]^T,\end{aligned}$$

and M is a nonnegative integer.

4. System of Nonlinear Fredholm Integral Equations

In this section we apply collocation method to convert Eq.(1) to algebraic system of nonlinear equations and then solve this system by well known solver. We assume that Eq. (1) has a unique solution. However, the necessary and sufficient conditions for existence and uniqueness of the solution of system Eq.(1) could be found in [4]. We approximate $f_i(x)$'s, such that;

$$f_i(x)\approx\sum_{k=1}^{2M+2}c_{i,k}\varphi_k(x),\quad i=1,\dots,n.\tag{7}$$

where $\phi_k(x)$ defined in Eq. (6) and $c_{i,k}$'s are unknown coefficients which are determined by solving algebraic system.

By substituting Eq. (7) in Eq. (1) we have;

$$\begin{cases}\sum_{k=1}^{2M+2}c_{1,k}\phi_k(x)=g_1(x)+\sum_{i=1}^{n}\int_\Gamma k_{1,i}(x,t)h[\sum_{k=1}^{2M+2}c_{i,k}\phi_k(t)]dt,\\ \sum_{k=1}^{2M+2}c_{2,k}\phi_k(x)=g_2(x)+\sum_{i=1}^{n}\int_\Gamma k_{2,i}(x,t)h[\sum_{k=1}^{2M+2}c_{i,k}\phi_k(t)]dt,\\ \vdots\\ \sum_{k=1}^{2M+2}c_{n,k}\phi_k(x)=g_n(x)+\sum_{i=1}^{n}\int_\Gamma k_{n,i}(x,t)h[\sum_{k=1}^{2M+2}c_{i,k}\phi_k(t)]dt.\end{cases}$$

Now, we choose some collocation points such as;

$$x_i=\frac{i}{2M+2},\quad i=1,\dots,2M+2,$$

which are equidistant, also define system of residual equations by:

$$\begin{cases}E_1(x)=\sum_{k=1}^{2M+2}c_{1,k}\varphi_k(x)-g_1(x)-\\ \sum_{i=1}^{n}\int_\Gamma k_{1,i}(x,t)h[\sum_{k=1}^{2M+2}c_{i,k}\varphi_k(t)]dt,\\ E_2(x)=\sum_{k=1}^{2M+2}c_{2,k}\varphi_k(x)-g_2(x)-\\ \sum_{i=1}^{n}\int_\Gamma k_{2,i}(x,t)h[\sum_{k=1}^{2M+2}c_{i,k}\varphi_k(t)]dt,\\ \vdots\\ E_n(x)=\sum_{k=1}^{2M+2}c_{n,k}\varphi_k(x)-g_n(x)-\\ \sum_{i=1}^{n}\int_\Gamma k_{n,i}(x,t)h[\sum_{k=1}^{2M+2}c_{i,k}\varphi_k(t)]dt.\end{cases}$$

Then, by imposing the conditions;

$$E_i(x_j)=0,\quad i=1,\dots,n\quad and\quad j=1,\dots,2M+2;$$

we can conclude algebraic system of nonlinear equations.

Next, we present the convergence analysis of our method.

Let us consider the following norms;

$$\|u(t)\|_{L_2[0,1]} = \sqrt{\int_0^1 u^2(t)dt},$$

$$\|u(t)\|_{H^k(0,1)} = \sqrt{\sum_{i=0}^{k}\int_0^1 \left|u^{(i)}(t)\right|^2 dt}.$$

Lemma 4.1. Let $u(t) \in H^k(0,1)$ (*Sobolev space*) and $P_\nu(u(t)) = \sum_{k=1}^{\nu} c_k \varphi_k(t)$ be the best approximation polynomial of $u(t)$ in L_2-norm. Then we have,

$$\|u(t) - P_\nu(u(t))\|_{L_2[0,1]} \le \omega_0 \nu^{-k} \|u(t)\|_{H^k(0,1)},$$

where ω_0 is a positive constant, which depends on the selected norm and is independent of $u(t)$ and ν.

Proof. [26].

Theorem 4.1. Le $K(x,t) \in L_2$ and $F_\nu(x)$ be the numerical solution of the Eq. (1). Then we have;

$$\sup_{x\in[0,1]} |F(x) - F_\nu(x)| \le \omega \nu^{-k} \|h[F(t)]\|_{H^k(0,1)},$$

where, ω is a positive constant.

Proof. By Eq.(1), we get;

$$\sup_{x\in[0,1]} |F(x) - F_\nu(x)|$$

$$\le \left|\int_0^1 K(x,t)h[F(t)]dt - \int_0^1 K(x,t)P_\nu(h[F(t)])dt\right|$$

$$= \left|\int_0^1 K(x,t)(h[F(t)] - P_\nu(h[F(t)]))dt\right|$$

$$\le \sqrt{\int_0^1 K^2(x,t)dt}.\|h[F(t)] - P_\nu(h[F(t)])\|_{L_2[0,1]}.$$

Since $K(x,t) \in L_2$,

$$\max_{x\in[0,1]} \sqrt{\int_0^1 K^2(x,t)dt} \le l$$

Furthermore, by Lemma 4.1, we obtain;

$$\|F(t) - P_\nu(F(t))\|_{L_2[0,1]} \le \omega_0 \nu^{-k} \|h[F(t)]\|_{H^k(0,1)},$$

and therefore,

$$\sup_{x\in[0,1]} |F(x) - F_\nu(x)| \le l\omega_0 \nu^{-k} \|h[F(t)]\|_{H^k(0,1)},$$

and by choosing $\omega = l\omega_0$ the proof is completed.

5. Numerical Experiments

In this section, we give some numerical experiments to illustrate the results obtained in previous sections. All the numerical experiments presented in this section were computed by a Maple 16 on a PC with a 1.86GHz 32-bit processor and 1GB memory. Moreover, these examples are solved by $M = 1$.

Example 5.1. Consider the following nonlinear system of Fredholm integral equations:

$$\begin{cases} u(x) = x - \dfrac{5}{18} + \displaystyle\int_0^1 \dfrac{1}{3}(u(t) + v(t))dt, \\ v(x) = x^2 - \dfrac{2}{9} + \displaystyle\int_0^1 \dfrac{1}{3}(u^2(t) + v(t))dt. \end{cases}$$

With the exact solutions $u(x) = x$ and $v(x) = x^2$.

Table 1 is the numerical results for Example 5.1.

Table 1. Shows the results of example 5.1

x	Error for $u(x)$	Error for $v(x)$
0.0	0	7. 511244674302861e-004
0.1	0	3. 953643559701100e-004
0.2	0	9.164898412450000e-004
0.3	0	9.999171234270000e-004
0.4	0	1.315241555600000e-004
0.5	0	8.020750977720000e-003
0.6	0	5.943176245000000e-004
0.7	0	2.143840797500000e-003
0.8	0	1.967102310300000e-003
0.9	0	3.328815824250000e-004

Example 5.2. Consider the following nonlinear system of Fredholm integral equations:

$$\begin{cases} u(x) = g_1(x) + \displaystyle\int_0^1 (\dfrac{1}{2}t^2)u^2(t)dt + \int_0^1 xv(t)dt, \\ v(x) = g_2(x) + \displaystyle\int_0^1 u(t)dt + \int_0^1 \dfrac{1}{4}v^3(t)dt. \end{cases}$$

Where;

$g_1(x) = \cos(2)/8+\sin(2)/16+\sin(x)-(x(2e-1))/2-1/12,$

$g_2(x) = x+\cos(1)-(3e)/4-(3e^2)/16-e^3/12+e^x+1/3.$

With the exact solutions $u(x) = \sin(x)$ and $v(x) = x+ex$.

Table 2 is the numerical results for Example 5.2.

Table 2. Shows the results of example 5.2

x	Error for $u(x)$	Error for $v(x)$
0.0	1.237199648446416e-004	1.928834164976521e-003
0.1	1.285952655211734e-004	6.457289107191100e-003
0.2	3.016884831493089e-004	2.662123924387300e-004
0.3	2.102334944864110e-004	3.814554230790300e-004
0.4	2.350729473931064e-004	4.225617057763000e-004
0.5	2.124969267227670e-004	4.135473077509300e-004
0.6	1.877840052618000e-004	4.078534215453400e-004
0.7	3.012506013852670e-004	4.625691262619500e-003
0.8	4.121226878424960e-004	5.225841488011300e-003
0.9	4.129353976856850e-004	7.532537490083000e-003

6. Conclusions

In this paper, the systems of nonlinear Fredholm integral equations are investigated and a practical projection method known as collocation method based on

Legendre multi-wavelets is established. Finally, illustrative examples are included to demonstrate the validity and applicability of the technique.

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