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The Application of Parseval's Theorem to Integral Problems

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Abstract This paper uses the mathematical software Maple as an auxiliary tool to study six types of definite integrals. We can obtain the infinite series forms of these definite integrals by using Parseval's theorem. On the other hand, we provide some examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying the answers by using Maple.

Keywords: *definite integrals, infinite series forms, Parseval's theorem, Maple*

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1. Introduction

As the information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Beethoven, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to use the mathematical software Maple to conduct mathematical research. The main reasons of using Maple in this paper are its simple instructions and ease of use, which enable the beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximations calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research.

In calculus and engineering mathematics courses, we learnt many methods to solve the integral problems including the change of variables method, the integration by parts method, the partial fractions method, the trigonometric substitution method, and so on. In this study, we evaluate the following six types of definite integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int_0^{2\pi}\left[\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^2(r\cos x)+\cos^2(r\sin x)}\right]^2 dx \tag{1}$$

$$\int_0^{2\pi}\left[\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^2(r\cos x)+\cos^2(r\sin x)}\right]^2 dx \tag{2}$$

$$\int_0^{2\pi}\frac{\sinh^2(r\cos x)+\sin^2(r\sin x)}{\sinh^2(r\cos x)+\cos^2(r\sin x)}dx \tag{3}$$

$$\int_0^{2\pi}\left[\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)}\right]^2 dx \tag{4}$$

$$\int_0^{2\pi}\left[\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)}\right]^2 dx \tag{5}$$

$$\int_0^{2\pi}\frac{\sinh^2(r\cos x)+\cos^2(r\sin x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)}dx \tag{6}$$

where r is a real number. We can obtain the infinite series forms of these definite integrals by using Parseval's theorem; these are the major results of this paper (i.e., Theorems 1 and 2). The study of related integral problems can refer to [1-15]. Additionally, we propose some definite integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these answers by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce a notation, a definition and some formulas used in this article.

2.1. Notation

Let $z = a + ib$ be a complex number, where $i = \sqrt{-1}$, a, b are real numbers. We denote a the real part of z by Re(z), and b the imaginary part of z by Im(z).

2.2. Definition

Suppose $f(x)$ is a continuous function defined on [0, 2π], the Fourier series expansion of $f(x)$ is $\frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx)$, where $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$, $a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx dx$, $b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx dx$ for all positive integers k.

2.3. Formulas

2.3.1. Euler's Formula

$e^{ix} = \cos x + i\sin x$, where x is any real number.

2.3.2. DeMoivre's Formula

$(\cos x + i\sin x)^n = \cos nx + i\sin nx$, where n is any integer, and x is any real number.

2.3.3. ([16])

$\sinh(\alpha + i\beta) = \sinh\alpha\cos\beta + i\cosh\alpha\sin\beta$, where α, β are real numbers.

2.3.4. ([16])

$\cosh(\alpha + i\beta) = \cosh\alpha\cos\beta + i\sinh\alpha\sin\beta$, where α, β are real numbers.

2.3.5. Taylor Series Expansion of Hyperbolic Secant Function ([17])

$\operatorname{sech}(z) = \sum_{n=0}^{\infty}\frac{E_{2n}}{(2n)!}z^{2n}$, where z is a complex number, $|z| < \pi/2$, E_n are the Euler numbers for all non-negative integers n.

2.3.6. Taylor Series Expansion of Hyperbolic Cosecant Function ([17])

$\operatorname{csch}(z) = \frac{1}{z} + \sum_{n=1}^{\infty}\frac{2(1-2^{2n-1})B_{2n}}{(2n)!}z^{2n-1}$, where z is a complex number, $0 < |z| < \pi$, and B_n are the Bernoulli numbers for all positive integers n.

Next, we introduce an important theorem used in this study.

2.4. Parseval's Theorem ([18])

Suppose $f(x)$ is a continuous function defined on [0, 2π], $f(0) = f(2\pi)$. If the Fourier series expansion of $f(x)$ is $\frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then $\frac{1}{\pi}\int_0^{2\pi} f^2(x)dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$.

Before deriving the first major result in this study, we need a lemma.

2.5. Lemma 1

Suppose α, β are real numbers with $\sinh^2\alpha + \cos^2\beta \neq 0$. Then

$$\operatorname{sech}(\alpha+i\beta)=\frac{\cosh\alpha\cos\beta - i\sinh\alpha\sin\beta}{\sinh^2\alpha + \cos^2\beta} \tag{7}$$

$$\frac{\cosh^2\alpha\cos^2\beta + \sinh^2\alpha\sin^2\beta}{(\sinh^2\alpha + \cos^2\beta)^2} = \frac{1}{\sinh^2\alpha + \cos^2\beta} \tag{8}$$

2.5.1. Proof

$$\begin{aligned}&\operatorname{sech}(\alpha+i\beta)\\ &= \frac{1}{\cosh(\alpha+i\beta)}\\ &= \frac{1}{\cosh\alpha\cos\beta + i\sinh\alpha\sin\beta}\end{aligned}$$

(By Formula 2.3.4.)

$$\begin{aligned}&= \frac{\cosh\alpha\cos\beta - i\sinh\alpha\sin\beta}{\cosh^2\alpha\cos^2\beta + \sinh^2\alpha\sin^2\beta}\\ &= \frac{\cosh\alpha\cos\beta - i\sinh\alpha\sin\beta}{\sinh^2\alpha + \cos^2\beta}\end{aligned}$$

And

$$\begin{aligned}&\frac{\cosh^2\alpha\cos^2\beta + \sinh^2\alpha\sin^2\beta}{(\sinh^2\alpha + \cos^2\beta)^2}\\ &= \frac{\cosh^2\alpha\cos^2\beta + \sinh^2\alpha(1-\cos^2\beta)}{(\sinh^2\alpha + \cos^2\beta)^2}\\ &= \frac{1}{\sinh^2\alpha + \cos^2\beta}\end{aligned}$$

In the following, we determine the infinite series forms of the definite integrals (1), (2) and (3).

2.6. Theorem 1

If r is a real number with $|r| < \pi/2$. Then the definite integrals

$$\begin{aligned}&\int_0^{2\pi}\left[\frac{\cosh(r\cos x)\cos(r\sin x)}{\sinh^2(r\cos x) + \cos^2(r\sin x)}\right]^2 dx\\ &= 2\pi + \pi\cdot\sum_{n=1}^{\infty}\frac{(E_{2n})^2}{[(2n)!]^2}r^{4n}\end{aligned} \tag{9}$$

$$\int_0^{2\pi}\left[\frac{\sinh(r\cos x)\sin(r\sin x)}{\sinh^2(r\cos x)+\cos^2(r\sin x)}\right]^2 dx = \pi\cdot\sum_{n=1}^{\infty}\frac{(E_{2n})^2}{[(2n)!]^2}r^{4n} \quad (10)$$

$$\int_0^{2\pi}\frac{1}{\sinh^2(r\cos x)+\cos^2(r\sin x)}dx = 2\pi+2\pi\cdot\sum_{n=1}^{\infty}\frac{(E_{2n})^2}{[(2n)!]^2}r^{4n} \quad (11)$$

2.6.1. Proof Because

$$\frac{\cosh(r\cos x)\cos(r\sin x)}{\sinh^2(r\cos x)+\cos^2(r\sin x)} = \mathrm{Re}[\mathrm{sech}(re^{ix})]$$

(By (7))

$$= \mathrm{Re}\left[\sum_{n=0}^{\infty}\frac{E_{2n}}{(2n)!}(re^{ix})^{2n}\right]$$

(By Formula 2.3.5.)

$$= \mathrm{Re}\left[\sum_{n=0}^{\infty}\frac{E_{2n}}{(2n)!}r^{2n}e^{i2nx}\right]$$

(By DeMoivre's formula)

$$= \sum_{n=0}^{\infty}\frac{E_{2n}}{(2n)!}r^{2n}\cos 2nx$$

(By Euler's formula)

$$= 1+\sum_{n=1}^{\infty}\frac{E_{2n}}{(2n)!}r^{2n}\cos 2nx \quad (12)$$

Thus, using Parseval's theorem, we obtain

$$\int_0^{2\pi}\left[\frac{\cosh(r\cos x)\cos(r\sin x)}{\sinh^2(r\cos x)+\cos^2(r\sin x)}\right]^2 dx = 2\pi+\pi\cdot\sum_{n=1}^{\infty}\frac{(E_{2n})^2}{[(2n)!]^2}r^{4n}$$

Similarly, because

$$\frac{\sinh(r\cos x)\sin(r\sin x)}{\sinh^2(r\cos x)+\cos^2(r\sin x)} = -\mathrm{Im}[\mathrm{sech}(re^{ix})] = -\mathrm{Im}\left[\sum_{n=0}^{\infty}\frac{E_{2n}}{(2n)!}(re^{ix})^{2n}\right] = -\sum_{n=1}^{\infty}\frac{E_{2n}}{(2n)!}r^{2n}\sin 2nx \quad (13)$$

By Parseval's theorem, we have

$$\int_0^{2\pi}\left[\frac{\sinh(r\cos x)\sin(r\sin x)}{\sinh^2(r\cos x)+\cos^2(r\sin x)}\right]^2 dx = \pi\cdot\sum_{n=1}^{\infty}\frac{(E_{2n})^2}{[(2n)!]^2}r^{4n}$$

On the other hand, from the summation of (9) and (10) and using (8),

$$\int_0^{2\pi}\frac{1}{\sinh^2(r\cos x)+\cos^2(r\sin x)}dx = 2\pi+2\pi\cdot\sum_{n=1}^{\infty}\frac{(E_{2n})^2}{[(2n)!]^2}r^{4n}$$

Before deriving the second major result of this paper, we also need a lemma.

2.7. Lemma 2

Suppose α, β are real numbers with $\sinh^2\alpha+\sin^2\beta\neq 0$. Then

$$\mathrm{csch}(\alpha+i\beta) = \frac{\sinh\alpha\cos\beta - i\cosh\alpha\sin\beta}{\sinh^2\alpha+\sin^2\beta} \quad (14)$$

$$\frac{\sinh^2\alpha\cos^2\beta+\cosh^2\alpha\sin^2\beta}{(\sinh^2\alpha+\sin^2\beta)^2} = \frac{1}{\sinh^2\alpha+\sin^2\beta} \quad (15)$$

2.7.1. Proof

$$\mathrm{csch}(\alpha+i\beta) = \frac{1}{\sinh(\alpha+i\beta)} = \frac{1}{\sinh\alpha\cos\beta+i\cosh\alpha\sin\beta}$$

(By Formula 2.3.3.)

$$= \frac{\sinh\alpha\cos\beta - i\cosh\alpha\sin\beta}{\sinh^2\alpha\cos^2\beta+\cosh^2\alpha\sin^2\beta} = \frac{\sinh\alpha\cos\beta - i\cosh\alpha\sin\beta}{\sinh^2\alpha+\sin^2\beta}$$

And

$$\frac{\sinh^2\alpha\cos^2\beta+\cosh^2\alpha\sin^2\beta}{(\sinh^2\alpha+\sin^2\beta)^2} = \frac{\sinh^2\alpha(1-\sin^2\beta)+\cosh^2\alpha\sin^2\beta}{(\sinh^2\alpha+\sin^2\beta)^2} = \frac{1}{\sinh^2\alpha+\sin^2\beta}$$

Finally, we find the infinite series forms of the definite integrals (4), (5) and (6).

2.8. Theorem 2

If r is a real number with $0<|r|<\pi$. Then the definite integrals

$$\int_0^{2\pi}\left[\frac{\sinh(r\cos x)\cos(r\sin x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)}\right]^2 dx = \pi\cdot\left[\left(\frac{1}{r}-\frac{r}{6}\right)^2+4\cdot\sum_{n=2}^{\infty}\frac{(1-2^{2n-1})^2(B_{2n})^2}{[(2n)!]^2}r^{4n-2}\right] \tag{16}$$

$$\int_0^{2\pi}\left[\frac{\cosh(r\cos x)\sin(r\sin x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)}\right]^2 dx = \pi\cdot\left[\left(\frac{1}{r}+\frac{r}{6}\right)^2+4\cdot\sum_{n=2}^{\infty}\frac{(1-2^{2n-1})^2(B_{2n})^2}{[(2n)!]^2}r^{4n-2}\right] \tag{17}$$

$$\int_0^{2\pi}\frac{1}{\sinh^2(r\cos x)+\sin^2(r\sin x)}dx = \pi\cdot\left[\frac{2}{r^2}+\frac{r^2}{18}+8\cdot\sum_{n=2}^{\infty}\frac{(1-2^{2n-1})^2(B_{2n})^2}{[(2n)!]^2}r^{4n-2}\right] \tag{18}$$

2.8.1. Proof Because

$$\frac{\sinh(r\cos x)\cos(r\sin x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)} = \mathrm{Re}[\mathrm{csch}(re^{ix})]$$

(By (14))

$$= \mathrm{Re}\left[\frac{1}{re^{ix}}+\sum_{n=1}^{\infty}\frac{2(1-2^{2n-1})B_{2n}}{(2n)!}(re^{ix})^{2n-1}\right]$$

(By Formula 2.3.6.)

$$=\left(\frac{1}{r}-\frac{r}{6}\right)\cos x+\sum_{n=2}^{\infty}\frac{2(1-2^{2n-1})B_{2n}}{(2n)!}r^{2n-1}\cos(2n-1)x \tag{19}$$

Using Parseval's theorem, we have

$$\int_0^{2\pi}\left[\frac{\sinh(r\cos x)\cos(r\sin x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)}\right]^2 dx = \pi\cdot\left[\left(\frac{1}{r}-\frac{r}{6}\right)^2+4\cdot\sum_{n=2}^{\infty}\frac{(1-2^{2n-1})^2(B_{2n})^2}{[(2n)!]^2}r^{4n-2}\right]$$

Similarly,

$$\frac{\cosh(r\cos x)\sin(r\sin x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)} = -\mathrm{Im}[\mathrm{csch}(re^{ix})]$$

(By (14))

$$= -\mathrm{Im}\left[\frac{1}{re^{ix}}+\sum_{n=1}^{\infty}\frac{2(1-2^{2n-1})B_{2n}}{(2n)!}(re^{ix})^{2n-1}\right]$$

(By Formula 2.3.6.)

$$=\left(\frac{1}{r}+\frac{r}{6}\right)\sin x-\sum_{n=2}^{\infty}\frac{2(1-2^{2n-1})B_{2n}}{(2n)!}r^{2n-1}\sin(2n-1)x \tag{20}$$

Therefore, using Parseval's theorem, we obtain

$$\int_0^{2\pi}\left[\frac{\cosh(r\cos x)\sin(r\sin x)}{\sinh^2(r\cos x)+\sin^2(r\sin x)}\right]^2 dx = \pi\cdot\left[\left(\frac{1}{r}+\frac{r}{6}\right)^2+4\cdot\sum_{n=2}^{\infty}\frac{(1-2^{2n-1})^2(B_{2n})^2}{[(2n)!]^2}r^{4n-2}\right]$$

In addition, from the summation of (16) and (17) and by (15), we have

$$\int_0^{2\pi}\frac{1}{\sinh^2(r\cos x)+\sin^2(r\sin x)}dx = \pi\cdot\left[\frac{1}{r^2}+\frac{r^2}{36}+8\cdot\sum_{n=2}^{\infty}\frac{(1-2^{2n-1})^2(B_{2n})^2}{[(2n)!]^2}r^{4n-2}\right]$$

3. Examples

In the following, for the six types of definite integrals in this study, we provide some definite integrals and employ Theorems 1 and 2 to determine their infinite series forms. In addition, we use Maple to calculate the approximations of these definite integrals and their solutions for verifying our answers.

3.1. Example 1

Taking $r = 1/2$ into (9), we obtain the definite integral

$$\int_0^{2\pi}\left[\frac{\cosh(1/2\cdot\cos x)\cos(1/2\cdot\sin x)}{\sinh^2(1/2\cdot\cos x)+\cos^2(1/2\cdot\sin x)}\right]^2 dx = 2\pi+\pi\cdot\sum_{n=1}^{\infty}\frac{(E_{2n})^2}{[(2n)!]^2}\left(\frac{1}{2}\right)^{4n} \tag{21}$$

Next, we use Maple to verify the correctness of (21).

```
>evalf(int((cosh(1/2*cos(x))*cos(1/2*sin(x)))^2/((sinh(1/2*cos(x)))^2+(cos(1/2*sin(x)))^2)^2,x=0..2*Pi),18);
```

6.33281088710495438

```
>evalf(2*Pi+Pi*sum((euler(2*n))^2/((2*n)!)^2*(1/2)^(4*n),n=1..infinity),18);
```

6.33281088710495438

Similarly, if $r = 1/\sqrt{5}$ in (10), the definite integral

$$\int_0^{2\pi}\left[\frac{\sinh(1/\sqrt{5}\cdot\cos x)\sin(1/\sqrt{5}\cdot\sin x)}{\sinh^2(1/\sqrt{5}\cdot\cos x)+\cos^2(1/\sqrt{5}\cdot\sin x)}\right]^2 dx = \pi\cdot\sum_{n=1}^{\infty}\frac{(E_{2n})^2}{[(2n)!]^2}\left(\frac{1}{\sqrt{5}}\right)^{4n} \tag{22}$$

```
>evalf(int((sinh(1/sqrt(5)*cos(x))*sin(1/sqrt(5)*sin(x)))^2/((sinh(1/sqrt(5)*cos(x)))^2+(cos(1/sqrt(5)*sin(x)))^2)^2,x=0..2*Pi),18);
```

0.0316355454381394471

```
>evalf(Pi*sum((euler(2*n))^2/((2*n)!)^2*(1/sqrt(5))^(4*n),n=1..infinity),18);
```

0.0316355454381394471

Finally, taking $r = 4/5$ into (11), we have

$$\int_0^{2\pi} \frac{1}{\sinh^2(4/5 \cdot \cos x) + \cos^2(4/5 \cdot \sin x)} dx = 2\pi + 2\pi \cdot \sum_{n=1}^{\infty} \frac{(E_{2n})^2}{[(2n)!]^2} \left(\frac{4}{5}\right)^{4n} \quad (23)$$

>evalf(int(1/((sinh(4/5*cos(x)))^2+(cos(4/5*sin(x)))^2), x=0..2*Pi),18);

6.97565920844648007

>evalf(2*Pi+2*Pi*sum((euler(2*n))^2/((2*n)!)^2*(4/5)^(4*n),n=1..infinity),18);

6.97565920844648007

3.2. Example 2

If $r = 2$ in (16), then

$$\int_0^{2\pi} \left[\frac{\sinh(2\cos x)\cos(2\sin x)}{\sinh^2(2\cos x) + \sin^2(2\sin x)}\right]^2 dx = \pi \cdot \left[\frac{1}{36} + 4 \cdot \sum_{n=2}^{\infty} \frac{(1-2^{2n-1})^2 (B_{2n})^2}{[(2n)!]^2} 2^{4n-2}\right] \quad (24)$$

>evalf(int((sinh(2*cos(x))*cos(2*sin(x)))^2/((sinh(2*cos(x)))^2+(sin(2*sin(x)))^2)^2,x=0..2*Pi),18);

0.179526459374120175

>evalf(Pi*(1/36+4*sum((1-2^(2*n-1))^2*(bernoulli(2*n))^2/((2*n)!)^2*2^(4*n-2),n=2..infinity)),18);

0.179526459374120175

In addition, if $r = \sqrt{7}$ in (17), then

$$\int_0^{2\pi} \left[\frac{\cosh(\sqrt{7}\cos x)\sin(\sqrt{7}\sin x)}{\sinh^2(\sqrt{7}\cos x) + \sin^2(\sqrt{7}\sin x)}\right]^2 dx = \pi \cdot \left[\frac{1183}{1764} + 4 \cdot \sum_{n=2}^{\infty} \frac{(1-2^{2n-1})^2 (B_{2n})^2}{[(2n)!]^2} (\sqrt{7})^{4n-2}\right] \quad (25)$$

>evalf(int((cosh(sqrt(7)*cos(x))*sin(sqrt(7)*sin(x)))^2/((sinh(sqrt(7)*cos(x)))^2+(sin(sqrt(7)*sin(x)))^2)^2,x=0..2*Pi),18);

2.96653319615568147

>evalf(Pi*(1183/1764+4*sum((1-2^(2*n-1))^2*(bernoulli(2*n))^2/((2*n)!)^2*(sqrt(7))^(4*n-2),n=2..infinity)),18);

2.96653319615568147

On the other hand, if $r = 9/5$ in (18), we have

$$\int_0^{2\pi} \frac{1}{\sinh^2(9/5 \cdot \cos x) + \sin^2(9/5 \cdot \sin x)} dx = \pi \cdot \left[\frac{3229}{4050} + 8 \cdot \sum_{n=2}^{\infty} \frac{(1-2^{2n-1})^2 (B_{2n})^2}{[(2n)!]^2} \left(\frac{9}{5}\right)^{4n-2}\right] \quad (26)$$

>evalf(int(1/((sinh(9/5*cos(x)))^2+(sin(9/5*sin(x)))^2), x=0..2*Pi),18);

2.59613527226230114

>evalf(Pi*(3229/4050+8*sum((1-2^(2*n-1))^2*(bernoulli(2*n))^2/((2*n)!)^2*(9/5)^(4*n-2),n=2..infinity)),18);

2.59613527226230114

4. Conclusion

In this article, we provide a new technique to determine some definite integrals. We hope this technique can be applied to solve another definite integral problems. On the other hand, the Parseval's theorem plays a significant role in the theoretical inferences of this study. In fact, the applications of this theorem are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

5. Appendix

In the following, we introduce some Maple's commands.

5.1. evalf( ); calculating the approximation.

Example. Evaluating the approximation of $\sqrt{2}$. > evalf(sqrt(2));

1.414213562

5.2. sum( ); Determining the summation.

Example. Evaluating the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. > sum(1/n^2,n=1..infinity);

$$\frac{1}{6}\pi^2$$

5.3. int( ); Determining the integral.

Example. Finding the integral $\int \cos x dx$. > int(cos(x),x);

$\sin(x)$

5.4. bernoulli(n); Evaluating the n-th Bernoulli number.

Example. Determining the 12-th Bernoulli number. > bernoulli(12);

$$-\frac{691}{2730}$$

5.5. euler(n); Calculating the n-th Euler number.

Example. Evaluating the 8-th Euler number. > euler(8);

1385

5.6. laplace(f(t),t,s); Evaluating the Laplace transform of $f(t)$.

Example. Finding the Laplace transform of $\cos at$. > laplace(cos(a*t),t,s);

$$\frac{s}{s^2+a^2}$$

5.7. invlaplace(F(s),s,t); Evaluating the inverse Laplace transform of $F(s)$.

Example. Determining the inverse Laplace transform of $\frac{a}{s^2-a^2}$. > invlaplace(a/(s^2-a^2),s,t);

$$\sinh(at)$$

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