

A Simple Scheme for Generation of N-Qubits Entangled Stated

Siamak Khademi*, Ghasem Naeimi, Ozra Heibati

Department of Physics, University of Zanjan, Zanjan, Iran

*Corresponding author: khademi@znu.ac.ir

Received November 06, 2013; Revised November 29, 2013; Accepted December 26, 2013

Abstract Generation and manipulation of multi-qubits or multi-partite entangled states are cornerstones of manufacturing quantum computers and developing quantum information. In this paper, we develop a new scheme for the generation of a multi-partite maximally entangled state generation. This method has less limitation and is simpler than the previous ones. It is based on the interactions of a chain of Rydberg Rubidium atoms with an array of five high quality cavities, including four classical and one quantum cavity in the middle.

Keywords: entangles state, N-qubit, quantum cavity, rydberg atoms

Cite This Article: Siamak Khademi, Ghasem Naeimi, and Ozra Heibati, "A Simple Scheme for Generation of N-Qubits Entangled Stated." *Applied Mathematics and Physics* 2, no. 1 (2014): 1-3. doi: 10.12691/amp-2-1-1.

1. Introduction

The concept of entanglement which was widely studied by Einstein, Podolski and Rozen [1] has a main importance in the fundamental quantum mechanics. Furthermore, entangled states find many applications in quantum information [2], quantum Teleportation [3,4,5,6], quantum cryptography [7,8], test of fundamental quantum concepts [9,10] etc. Maximally entangled state for three or more partite is given by Greenberger-Horne-Zeilinger state [11,12]. Many theoretical and experimental schemes have been proposed to prepare multipartite entangled states [13-18].

Recently, many experimental schemes have been also achieved to produce N-qubits entangled states [19,20,21,22]. The development of theoretical and experimental methods of manipulation and generation of entangled states into the N-partite entangled state [23,24] is very important for the development of quantum information, quantum teleportation, quantum computers and so on.

Cirac and Zoller [25] and Rauschenbeutel et al. [26] presented a scheme to prepare a GHZ (three-partite) entanglement state for three atoms. A GHZ state is an entangled state between three states of atoms (or photons' number, photon's polarization, ion trapped, spins and so on). In this paper, the generation of three-partite GHZ state is developed into an N-qubits entangled state

$$|\alpha\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle_1 |+\rangle_2 |+\rangle_3 \cdots |+\rangle_n + |-\rangle_1 |-\rangle_2 |-\rangle_3 \cdots |-\rangle_n \right), \quad (1)$$

where $|+\rangle$ and $|-\rangle$ are eigen states of a two-level system. In the Cirac and Zoller's method, they used a quantum cavity which was initially filled by a superposition

state $(|0\rangle - |3\rangle) / \sqrt{2}$. After the generation of a three-partite GHZ state, this quantum cavity was finally empty (or vacuum state). The three-qubits GHZ atomic state was produced where three atoms, which were initially in the $|g\rangle$ state, interacted with a linear array of cavities, consisted of one quantum cavity in the middle of two classical cavities. The atoms had a non-resonance (resonance) interaction with the classical (quantum) cavities (cavity). The output state of atoms was a three-qubits GHZ entangled state.

This method is straightforwardly developed for the generation of N-qubits entangled particle (called N-qubits GHZ entangled state or N-GHZ state) [27,28]. In this case, the quantum cavity in the proposed experimental setup is filled by a superposition state $(|0\rangle - |n\rangle) / \sqrt{2}$ and the previous procedure is followed to generate an N-qubits GHZ entangled state of atoms in the output of the experimental setup. In the Cirac and Zoller's method, the state of the quantum cavity which is initially in a superposition state $(|0\rangle - |n\rangle) / \sqrt{2}$, finally changed into the vacuum state. Therefore, the numbers of produced entangled particles depended on the initial state $|0\rangle$ of the cavity quantum electrodynamics. Here, another method is introduced for the generation of N-qubits GHZ state.

A non-demolition experimental setup for detecting the existence of a single photon in a cavity quantum electrodynamics by the measurement of Wigner distribution function of the electromagnetic field in the cavity in the origin of phase space is proposed by Davidovich and Lutterbach [29] and is experimentally achieved by Negues et al. [30]. In this paper, an experimental setup is proposed which is very similar to the Negues's experimental setup so as to generate an N-qubits GHZ entangled state. In the next section, the

method for the generation of N-qubits GHZ entangled state, including N-1 entangled atom all of which are entangled with one photonic state of quantum cavity, is introduced. In Section 3, the proposed method is developed to be used for the generation of N-qubits GHZ state of Rubidium atoms. The last section is devoted to the conclusions.

2. Generation of N-qubits GHZ Entangled State

An experimental setup containing an array of five microwave cavities is introduced; each one of them is a Fabry-Perot resonator made up of two spherical superconductor mirrors [30]. The cavities are labeled by R1, R2, C, R3 and R4 in the order of sequence, as shown in Figure 1.

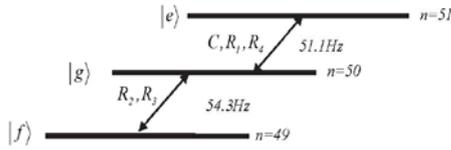


Figure 1. The Rydberg levels are at 49, 50 and 51th levels of Rubidium atom. The cavities R₂ and R₃ are tuned to interact resonantly at f and g levels and cavities C, R₁ and R₄ are tuned to interact (non-resonantly for C and resonantly for others) at g and e levels

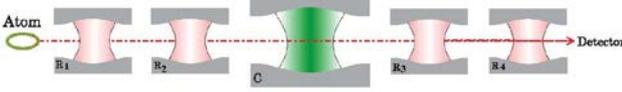


Figure 2. The experimental setup is made up of a quantum cavity C in the middle of four classical cavities labeled by R. Incoming atoms are interacted resonantly with the R's cavities and non-resonantly with the quantum cavity C

A beam of Rubidium atoms is excited into the Rydberg states passing through the cavities' array and interacts with their electromagnetic fields. There are three Rydberg levels for the Rubidium atom with the main quantum numbers of 49, 50 and 51 and their states are denoted by $|f\rangle$, $|g\rangle$ and $|e\rangle$, respectively (see Figure 2) [30]. The atom-field interaction in the C, R₁ and R₄ cavities are tuned with two-levels of e and g with phase ϕ . The atom-field interaction in the R₂ and R₃ cavities are tuned with the two-level of f and g with phase $\pi/2$. The non-resonant atom interaction with the quantized electromagnetic fields in the cavity C is investigated by the Jaynes-Cummings model [31]. Furthermore, the atom-field interaction (with the classical electromagnetic fields) in the R_i ($i=1, 2, 3$ and 4) cavities is investigated by the Rabi's model [27]. It makes a non-resonant interaction by a Stark effect, where a DC electric field is applied in the cavity C. It makes a detuning Δ between the electromagnetic and the transition frequencies for the corresponding two levels.

The initial state of the first incoming atom is set to be $|\psi\rangle_0 = |e\rangle_{a1}$, where a1 denotes the atom number 1. The atomic state of incoming atom changes to $|\psi\rangle_{R1} = (|e\rangle_{a1} + i|g\rangle_{a1})/\sqrt{2}$, where it passes through the

cavity R₁. In the next stage, it passes through the cavity R₂. After atom-field interaction in the cavity R₂, the first term $|e\rangle_{a1}$ remains the same because the cavity is tuned with g and f states; but, the second term is changed into $|f\rangle_{a1}$ with an additional phase factor (a minus sign). Then, the outgoing of cavity R₂ is given by $|\psi\rangle_{R2} = (|e\rangle_{a1} - |f\rangle_{a1})/\sqrt{2}$, when the state $|\psi\rangle_{R2}$ passes through the cavity C and interacts (non-resonantly) with its quantized electromagnetic fields, which is set to be initially in the superposition state $(|0\rangle + |1\rangle)/\sqrt{2}$, the eigenstate $|f\rangle_{a1}$ does not change; however, $|e\rangle_{a1}$ changes by a phase which depends on the number of photons in the cavity C. For the vacuum $|0\rangle$ and one photon $|1\rangle$ states, the phase changing are -1 and +1, respectively. Therefore, the Rubidium atom and photons in the cavity become an entangled state which is denoted by:

$$|\psi\rangle_C = \frac{1}{\sqrt{2}} \left(|e\rangle_{a1} (|0\rangle + |1\rangle) - |f\rangle_{a1} (|0\rangle + |1\rangle) \right) / \sqrt{2}. \quad (2)$$

In the next stage, the atom-field interaction within the cavity R₃, changes the eigenstate $|f\rangle_{a1}$ into $i|g\rangle_{a1}$; therefore, the outgoing state is

$$|\psi\rangle_{R3} = \frac{1}{\sqrt{2}} \left(|e\rangle_{a1} (|0\rangle + |1\rangle) - i|g\rangle_{a1} (|0\rangle + |1\rangle) \right) / \sqrt{2}.$$

The atom, in its turn, interacts resonantly with the last cavity R₄; therefore, in the state $|\psi\rangle_{R3}$, the atomic states $|e\rangle_{a1}$ and $|g\rangle_{a1}$ change into $(|e\rangle_{a1} + i|g\rangle_{a1})/\sqrt{2}$ and $(|g\rangle_{a1} + i|e\rangle_{a1})/\sqrt{2}$ respectively. Therefore, the final state of atom-field is obtained as an entangled state between the first atom and photons' number in the cavity C

$$|\psi^{(2)}\rangle_{R4} = \frac{1}{\sqrt{2}} \left(|e\rangle_{a1} |1\rangle - i|g\rangle_{a1} |0\rangle \right). \quad (3)$$

In this stage, an atom-photon entangled state is prepared. The upper index $|\psi^{(N)}\rangle_{R4}$ denotes N-parts entangled states. Next, if the second atom follows the same procedure, the final state is an entangled state between three quantum states (two atomic and one photonic states), i.e., $|\psi^{(3)}\rangle_{R4} = (|e\rangle_{a2} |e\rangle_{a1} |1\rangle - i|g\rangle_{a2} |g\rangle_{a1} |0\rangle) / \sqrt{2}$. If a chain of N-1 atoms passes through the cavities by the same procedure, an entangled state between N-1 atomic state and one photonic state is obtained

$$|\psi^N\rangle = \frac{1}{\sqrt{2}} \left(\overbrace{|e\rangle_{a1} |e\rangle_{a2} |e\rangle_{a3} \cdots |e\rangle_{aN-1}}^{N-1} |1\rangle + (-i)^{N-1} \overbrace{|g\rangle_{a1} |g\rangle_{a2} |g\rangle_{a3} \cdots |g\rangle_{aN-1}}^{N-1} |0\rangle \right). \quad (4)$$

If the excited states ($|e\rangle$ or $|1\rangle$) and ground states ($|g\rangle$ or $|0\rangle$) are considered as the atomic or photonic qubits and are represented by $|+\rangle$ and $|-\rangle$, respectively, therefore, the quantum state (4) is just an entangled GHZ state between N qubits (N-1 atoms and one photon)

$$|\psi^{(N)}\rangle_{GHZ} = \frac{1}{\sqrt{2}} \left(\overbrace{|+\rangle|+\rangle|+\rangle\cdots|+\rangle}^N + (-i)^{N-1} \overbrace{|-\rangle|-\rangle|-\rangle\cdots|-\rangle}^N \right). \quad (5)$$

In this procedure, there is no limitation on the number of entangled atoms in the prepared quantum state.

3. N-qubits GHZ of the Rubidium Atoms

The next step is devoted to the separation of atomic and photonic states to obtain an N-qubits GHZ entangled state between N Rubidium atoms. To close the procedure in the previous section, after the generation of entanglement state between N-1 Rubidium atom and photon in the quantum cavity C, another Rubidium atom in the state $|e\rangle$ is entered into just the cavity C. Therefore, the last (Nth) Rubidium atom does not interact with the electromagnetic fields in the R_i 's cavities. Its atomic levels, e and g , now resonantly interact with the electromagnetic field within the cavity C, which is adjusted with the phase π . The photonic state in the quantum cavity C is finally separated from the N-1 entangled atomic states and the last Rubidium atom becomes entangle with the N-1 entangled atoms

$$|e\rangle_N |\psi^N\rangle_{GHZ} = \frac{1}{\sqrt{2}} \left(\overbrace{|e\rangle_{a_1} |e\rangle_{a_2} |e\rangle_{a_3} \cdots |e\rangle_N}^N + (-i)^n \overbrace{|g\rangle_{a_1} |g\rangle_{a_2} |g\rangle_{a_3} \cdots |g\rangle_N}^N \right) |1\rangle. \quad (6)$$

4. Conclusion

The methods for the manipulations or generation of atomic or photonic qubits are the milestones for the manufacturing quantum computers and the developing quantum information processing. In the present article, a new and simpler method is introduced for the generation of N-qubits GHZ-like states. In this case, an array of five cavities (four classical cavities and one quantum cavity in their middle), interacting with a chain of Rydberg Rubidium atoms, is used. The cavity in the middle, is a quantum cavity initially filled with a superposition of photonic states, $(|0\rangle+|1\rangle)/\sqrt{2}$. In the present method a chain of Rubidium atoms, which are excited in a Rydberg state (49th level), are passed through the array of cavities and interacted with their electromagnetic fields. Finally, the Rubidium atoms are entangled with each other and

with the electromagnetic field in the quantum cavity. The quantum cavity state is a stationary state during interaction therefore in spite of the previous methods has no technical limitation on the number of entangled parties. The outgoing state is an N-qubits GHZ-like entangled state. In this case, the whole N number of Rubidium atoms are entangled as an N-qubits GHZ state and the photonic state of the quantum cavity is separable, finally. In the generation of N-qubits GHZ-like entangled state, there is no limitation on the number of entangled atoms.

References

- [1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev., 48 (10), 24 (1995).
- [2] C. H. Bennett, Phys. Today 48 (10), 24 (1995).
- [3] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
- [4] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) 390, 575 (1997).
- [5] W. Mei-Yu and Y. Feng-Li, Chinese Phys. B, 20, 120309 (2011).
- [6] T. Jing-Wu, Z. Guan-Xiang and H. Xiong-Hui, Chinese Phys. B, 20, 050312 (2011).
- [7] Alexander Treiber et. al, New J. Phys., 11, 045013 (2009).
- [8] X. B. Wang, C. Z. Peng, J. Zhang, L. Yang, and J.W. Pan., Phys. Rev. A, 77, 042311 (2008).
- [9] A. Zeilinger, Reviews of Modern Physics, 71, 288 (1999).
- [10] M. Zukowski, A. Zeilinger and H. Weinfurter, Proceedings of the Fundamental Problems in Quantum Theory, (1995).
- [11] D. M. Greenberger, M. Horne and A. Zeilinger, Am. J. Phys., 58), 1131 (1990).
- [12] D. M. Greenberger, M. Horne and A. Zeilinger, Am. J. Phys., 69, (1989).
- [13] X. S. Hua, S. Bin and S. Ke-Hui, Commun. Theor. Phys., 52, 835 (2009).
- [14] P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko and Y. Shih, Phys. Rev. Lett , 75, 4337 (1995).
- [15] L. S. Bishop, L. Tornberg, D. Price, E. Ginosar, A. Nunnenkamp, A. A. Houck, J. M. Gambetta, Jens Koch, G. Johansson, S. M. Girvin and R. J. Schoelkopf, New J. Phys., 11, 073040 (2009).
- [16] G. P. Guo, C. F. Li, Jian Li and G. C. Guo, Phys. Rev. A, 65, 042102 (2002).
- [17] N. Bin-Bin, G. Yong-Jian, C. Xiao-Dong, L. Hong-Hui, L. Xiu and L. Xiu-Min, Chinese Phys. B, 19, 090316 (2010).
- [18] X. Shao-Hua, S. Bin and S. Ke-Hui, Commun. Theor. Phys., 52, 835 (2009).
- [19] D. Gonta, S. Fritzsche and T. Radtke, Phys. Rev. A, 77, 062312 (2008).
- [20] S. S. Sharma, Physics Letters A , 311, 111 (2003).
- [21] X. Zou, K. Pahlke and W. Mathis, Phys. Rev. A, 68, 024302 (2003).
- [22] R. J. Nelson, D. G. Cory and S. Lloyd, Phys. Rev. A, 61, 022106 (2000).
- [23] N. Bin-Bin, G. Yong-Jian, C. Xiao-Dong, L. Hong-Hui, L. Xiu and L. Xiu-Min, Chinese Phys. B, 19, 090316 (2010).
- [24] A. Zheng and J. Liu, J. Phys. B, 44, 165501 (2011).
- [25] J. I. Cirac and P. Zoller, Phys. Rev. A, 50, 2799 (1994).
- [26] A. Rauschenbeutel and et. al, Science, 288, 2024 (2004).
- [27] Y. Xia, J. Song, H. S. Song, Optics Communication, 277, 219 (2007).
- [28] L. Xiu, L. Hong-Cai, L. Xiuin, L. Xing-Hua and Y. Rong-Can, (vit Chinese Phys., 16, 1209 (2007).
- [29] L. G. Lutterbach and L. Davidovich, Phys. Rev. Lett., 78, 547 (1997).
- [30] G. Noguees, Phys. Rev. A, 62, 54101 (2000).
- [31] E. T. Jaynes and F. W. Cummings, Proc. IEEE, 51, 89 (1963).