

Control Processes Optimization for Mechanical Systems with Active, Semi-Passive and Passive Actuators

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Abstract The influence of the geometric parameters of two-link manipulator on the energy expenditure necessary for the implementation of a given movement is investigated. It is proposed approximation-compensatory approach to replace the active actuators in the joints of the manipulator on semi-passive and entirely passive actuators. Advantages and disadvantages of such replacement are analyzed and the fields of use of such manipulators are proposed.

Keywords: two-link manipulator, control, optimization, passive actuator

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1. Introduction

Manipulators of various types and applications are used in many fields of research and industry. Problems of control and optimization of such systems were considered at various times by various authors [1,2,3]. One of the most used are two-link manipulators (TLM) with rapid and flexible links [4,5]. For TLM were solved the time-optimal [6] and energy-optimal control problems [7], vibration [8] and adaptive control problems [9], kinematic [10] and multi-objective [11] optimization problems etc. In these and many other articles research were focused on manipulators with active actuators. Less attention was paid to the manipulators with passive and semi-passive actuators [12,13,14]. To a large extent this is due to the more narrow field of use. However, there are devices in which the use of such manipulators is more than appropriate. These devices include autonomous space [15] and deep-sea [16] research vehicles containing system "one-time" deployment (solar panels, antennas, systems of video surveillance or sampling etc.). In these cases, simple structure, its minimal weight and power consumption is a significant advantage, which increases the reliability of the device and the time of its use. To realize this advantage often allows the use of semi-passive and passive actuators. In this paper, we show that for TLM with a given mass of manipulator and the load can be chosen such geometric parameters that significantly minimize the energy expenditure required to deploy it. But the main goal of this paper is to describe the easy-to-implement approximation-compensatory approach for sequential replacement of the active actuators semi-and fully passive and investigation the results of its use.

2. TLM with Active Actuators

Let's consider the motion of entirely actively controlled two-link manipulator. Assuming that it transfers the loads of m_c weight from point with Cartesian coordinates (X_0, Y_0) to the point with coordinates (X_k, Y_k) . Let us denote the manipulator mounting point as (X_3, Y_3) , and time required for transition from point (X_0, Y_0) to point (X_k, Y_k) as T . Let m_i , l_i , j_i , r_i denote weight, length, moment of inertia and centre of mass location for i -th link of manipulator, $i = 1, 2$, m_s – weight of grip. We assume that

$$l_1 + l_2 = l = \text{const}, \quad m_1 + m_2 = m = \text{const} \quad (1)$$

The components of system generalized coordinates vector $x = (x_1, x_2)$ present the angle between the axis OX and first link, and the angle between first and second manipulator links accordingly. Control force vector $u = u^a = (u_1^a, u_2^a)$ defines the torques acting in system joints for implementation of motion set.

Let us describe motion dynamics of given mechanical system using Lagrange's equation of second kind

$$a_{11} \dot{x}_1 + a_{12} \dot{x}_2 + b_{11} x_1 + b_{12} (x_2)^2 = u_1, \quad (2)$$

$$a_{21} \dot{x}_1 + a_{22} \dot{x}_2 + b_{21} (x_1)^2 = u_2, \quad (3)$$

where

$$a_{11} = j_1 + m_1 r_1^2 + (m_2 + m_s + m_c) l_1^2 + a_{12} + b_0 \cos x_2,$$

$$a_{12} = a_{22} + b_0 \cos x_2, \quad a_{21} = a_{12},$$

$$a_{22} = j_2 + m_2 r_2^2 + (m_s + m_c) l_2^2,$$

$$b_{11} = -2b_0 \sin x_2, \quad b_{12} = -b_0 \sin x_2,$$

$$b_{21} = -b_{12}, \quad b_0 = l_1(m_2 r_2 + m_s l_2 + m_c l_2).$$

Boundary conditions for given mechanical system generalized coordinates vector are easy to be received from given values (X_3, Y_3) , (X_0, Y_0) , and (X_k, Y_k) :

$$x(0) = x_0, \quad x(T) = x_T \tag{4}$$

We shall assume that

$$x'(0) = x'(T) = 0 \tag{5}$$

and F_i denotes values

$$F_i = \left\| u_i^a \right\|_{L_2[0,T]}^2 \tag{6}$$

which are proportional to local energy expenditures required for motion implementation of i -th system joint, $i = 1, 2$, and F denotes the value $F = F_1 + F_2$. The value F allows us to estimate the quality of control process in general, as long as it can be considered proportional to energy expenditures required for implementation of motion in given mechanical system within given time interval [12].

Let us consider the behaviours for TLM when the type of it's law of motion is set. We shall assume that components of system generalized coordinates vector are represented by cubic polynomials meeting conditions (4), (5).

Numerical experiments have confirmed that value of functional F decreases monotonously as the value of T increases, which is quite natural [1]. This is why the formulation of problems of control processes optimization for two-link mechanical systems requires clear determination of it's duration. From this point on, for the research of TLM control processes it is reasonable to question its optimal behaviour from energy expenditures point of view, while the mounting point (X_3, Y_3) and the length of links l_1, l_2 ($l_1 + l_2 = \text{const}$) for values of T , (X_0, Y_0) , and (X_k, Y_k) are being varied.

Natural limitations are applied to the values of (X_3, Y_3) as follows:

$$(X_3, Y_3) = \left\{ \begin{array}{l} (X, Y) : \max_{i=1,2} l_i^2 \leq (X - X_0)^2 + (Y - Y_0)^2 \leq l^2 \\ \max_{i=1,2} l_i^2 \leq (X - X_k)^2 + (Y - Y_k)^2 \leq l^2 \end{array} \right\}, \tag{7}$$

Let us consider that manipulator geometric and inertia specifications together with parameters defining it's law of motion are described with following input data:

$$l_1 + l_2 = 1.0 \text{ m}, \quad l_1 = 0.1k \text{ m}, \quad k = \overline{1,9},$$

$$m_1 + m_2 = 1.0 \text{ kg}, \quad m_1 = l_1(m_1 + m_2) / (l_1 + l_2) = 1.0 \text{ kg},$$

$$j_i = 0.229 l_i^2 m_i, \quad r_i = 0.4 l_i \tag{8}$$

$$m_s = 0.1 \text{ kg}, \quad m_c = 1.0 \text{ kg}, \quad T = 1 \text{ sec},$$

$$(X_0, Y_0) = (0.916, 0.379), \quad (X_k, Y_k) = (0.354, 0.854).$$

Values (X_0, Y_0) and (X_k, Y_k) are given in meters as well. Then, with $(X_3, Y_3) = (0.0, 0.0)$ we have $x_0 = (\pi / 12, \pi / 12)$ and $x_k = (\pi / 4, \pi / 4)$.

With above formulated values applied, let us consider the following sequence of problems for determining TLM control process:

A1. Determine the control efforts and the value of the functional F at fixed mounting point $(X_3, Y_3) = (0.0, 0.0)$ and $l_1 = l_2 = 0.5 \text{ m}$ ($k = 5$).

A2. Determine the lengths of links l_1 and l_2 that satisfy the condition (1), and a fixed position of mounting point $(X_3, Y_3) = (0.0, 0.0)$ minimizes the value of the functional F . This is a one-parameter constrained optimization problem, which is solved by the well-known method of variation of the value l_1 [17].

A3. Determine the position of mounting point (X_3, Y_3) , which, when fixing the values of the lengths of links $l_1 = l_2 = 0.5 \text{ m}$ ($k = 5$) minimizes the functional F . This is a two-parameter constrained optimization problem, which is solved by method of variation of the values (X_3, Y_3) .

A4. Determine the lengths of links l_1 and l_2 and the position of mounting point (X_3, Y_3) , which minimizes the functional F . This is a three-parameter constrained optimization problem, which is solved by method of variation of the values l_1 and (X_3, Y_3) .

The results of numerical experiments of solving above formulated problems are provided (Table 1).

Table 1. Energy Expenditures for Moving the Two-Link Manipulator with Active Actuators

	(X_3, Y_3)	(l_1, l_2)	F_1	F_2	F
A1	(0.00;0.00)	(0.5;0.5)	34.6	8.29	42.90
A2	(0.25;0.15)	(0.5;0.5)	8.55	12.0	20.53
A3	(0.00;0.00)	(0.7;0.3)	5.64	1.83	7.47
A4	(0.00;0.15)	(0.7;0.3)	2.06	0.89	2.96

Comparative analysis of data provided in Table 1 allows us to state that:

1) there exists optimal pair l_1, l_2 for every manipulator mounting point (X_3, Y_3) in terms of value of functional F , moreover, if this pair was chosen inappropriately, energy expenditures for transferring loads from point (X_0, Y_0) to point (X_k, Y_k) within the T period of time may increase several fold;

2) there exists optimal mounting point (X_3, Y_3) in terms of value of functional F , for manipulator links lengths set l_1, l_2 , moreover, if this point was chosen inappropriately, energy expenditures for transferring loads from point (X_0, Y_0) to point (X_k, Y_k) within the T period of time may increase several fold;

3) there exists optimal pair l_1, l_2 and mounting point (X_3, Y_3) , for every TLM in terms of value of functional F , moreover, if they were chosen inappropriately, energy expenditures for transferring loads from point (X_0, Y_0) to point (X_k, Y_k) within the T period of time may increase an order of magnitude more.

Numerical experiments have also confirmed that for every geometry of the manipulator it's optimal mounting

points are located on the borderline of permissible domain, defined by limitations (7).

Manipulator returns movement from point (X_k, Y_k) to point (X_0, Y_0) within T period of time with or without load is also optimal, with geometrical set in the course of problem A4 solving and defined position of mounting point.

Results provided above show that on the stage of design, it is possible to choose TLM geometry configuration and mounting point that would allow to decrease significantly energy expenditures required for implementation of different operational procedures.

3. TLM with Semi-Passive Actuators

Another way to optimize two-link mechanical systems control processes is to introduce passive control actuators (springs, dampers etc.) into system's joints. In general case, the control force vector will be as follows:

$$u = u^a + u^p = (u_1^a + u_1^p, u_2^a + u_2^p)$$

where $u_j^p = \phi_j(C_j, x_j, x_j', x_j'')$, $C_j = \{c_k^j\}_{k=1}^{q_j} \in \Omega_C^j \subset R^{q_j}$ is the vector with scalar parameters, which defines the mechanical properties of passive actuator applied in j -th joint of system, $j = 1, 2$.

Let us refer to systems of the kind as "semi-passively controlled" if control in one or two joints has both passive and active component and "partially passively controlled" if in one joint the control is entirely active, and in the other one – entirely passive.

Semi-passively controlled systems and partially passively controlled systems are more complicated objects of research than actively controlled ones. This results from existence of different types of actuators and more strict limitations for control forces. Naturally, the motion laws for such systems comprise often not defined subset of motion laws for actively controlled systems.

Let us consider the case when function u_j^p takes the following form:

$$u_j^p = C_j(x_j^0 - x_j) \tag{9}$$

e. g. the passive control is implemented by means of linear spring actuator. In this case, x_j^0 is the value of angle in j -th joint with idle spring and C_j is the stiffness ratio for weightless spring of passive actuator, $j = 1, 2$.

Both geometric and inertia parameters, as well as the conditions of motion of given system are similar to those described in previous section. Motion dynamics for such systems may be described in the form of problem (2)–(5). In order to perform control processes research it is reasonable to question their behaviour after introduction of semi-passive control into second TLM joint (problem B1), first TLM joint (problem B2), and both system joints (problem B3).

Let us presume that we have set the type of motion law and defined the control forces of actively controlled two-link manipulator of similar structure. Let us calculate passive component of control in j -th joint on condition that

$$\varepsilon_j = \left\| u_j - u_j^p \right\|_{L_2[0,T]} \rightarrow \min, \quad j = 1, 2. \tag{10}$$

Taking into account formulation of the former problem, let us call it **approximation**.

The values x_j^0 , $j = 1, 2$, are determined from the condition unloaded spring in the final position of the manipulator (X_k, Y_k) at a given position mounting point (X_3, Y_3) .

In the case (9) values C_j are defined by means of the formulas

$$C_j = \frac{\int_0^T (x_j^0 - x_j)^2 dt}{\int_0^T u_j(t)(x_j^0 - x_j) dt}$$

where $u_j(t)$, $t \in [0, T]$, is the solution of problem A1, $j = 1, 2$.

Numerical results of problems B1–B3 solving, achieved under similar (8) input data for $l_1=l_2=0.5$ m ($k = 5$), $(X_3, Y_3) = (0.0, 0.0)$ and values defined under condition (10) $C_1 = -35.2$ ($x_1^0 = -0.52$, $\varepsilon_1 = 0.13$), and $C_2 = -16.8$ ($x_2^0 = -0.53$, $\varepsilon_2 = 0.11$) are provided in Table 2. Comparative analysis on their basis allows us to state that:

1) solution of approximation problem (10) allows to decrease active control force an order of magnitude more due to passive actuator even of simple type being introduced into appropriate system joint;

2) energy expenditures required for implementation of motion set in appropriate joint are decreased almost an two order of magnitude more.

Thus, we can replace the active actuator to a pair of passive actuator and active actuator less power. Note that the use of semi-passive actuators in both joints of TLM at non-optimal geometric pattern (see Table 2, problem B3) gives a better result than the optimum geometry of the manipulator with active actuators (see Table 1, problem A4). Obviously, TLM with semi-passive actuators is not useful for performing cyclic work operations, as for the return of such manipulator in the initial position a power of active component has to be considerably greater than in the absence of passive component.

4. TLM with Passive Actuator

Motion dynamics for partially passively controlled two-link mechanical system may also be described in the form of problem (2)–(5). To perform research of this system's control processes, let us question it's behaviour after entirely passive type of control is introduced into second joint (introduction of passive control into first or both joints does not seem reasonable from the point of view of such systems technical application).

Supposing motion law is set and control forces are defined for entirely actively controlled TLM of similar structure. Let us distinguish passive component of control in second joint u_2^p on condition (10), where function u_2^p takes the form (9). Absence of active component in this joint is compensated by active control in first joint of system as follows.

Supposing u_2 in equation (3) takes the following form $u_2 = u_2^a + u_2^p$, and the function x_2 behaves similarly to the solution of appropriate problem for actively controlled analogue of given mechanical system. Let us solve boundary problem for equation (3) in relation to x_1 supposing that $u_2^a = 0$, with boundary conditions

$$x_1(0) = x_1^0, \quad x_1(T) = x_1^k. \quad (11)$$

For numerical solution of problem (3), (11) we use well-known multiple shooting technique [18].

Let us calculate compensative force \tilde{u}_1^a in first joint, with active component of control being absent in second joint, from formula (2), with x_2 known and x_1 value defined from the solution of problem (3), (11).

Solution for the latest problem (let us call it **compensatory**) allows relocating forces between actively and passively controlled joints of the system with respect to limitations applied to the latest.

The need to solve compensatory problem is defined by fact (and this is confirmed by numerical experiments) that simple neglect of active component of control in the second joint of system, even if it's value is quite small, doesn't allow to transfer it from defined initial position to defined final position.

Numerical results of solving the problem of replacement in the second joint of the active actuator by completely passive (problem C) acquired under similar (8) input data for $l_1=l_2=0.5$ m ($k=5$), $(X_3, Y_3) = (0.0, 0.0)$ and $C_2 = -16.8$ ($x_2^0 = -0.53$, $\varepsilon_2 = 0.11$) defined with respect to statement (10) are presented in Table 2.

Table 2. Energy Expenditures for Moving the Two-Link Manipulator with Semi-Passive and Passive Actuators

	F_1	F_2	F
A1	34.61	8.29	42.90
B1	34.61	0.15	34.76
B2	0.86	8.29	9.15
B3	0.86	0.15	1.15
C	39.68	–	39.68

Analysis obtained results show that after passive control was introduced into second system joint, energy expenditures in first joint is increased, but total energy expenditures required for implementation of motion set is decreased.

In the case of an entirely passive actuator to return the manipulator to the initial position corresponding joint must have an additional active actuator significantly more power than in the case of conventional active control. However, to perform "one-off" operations, the use of semi-passive or passive actuators greatly simplifies the design and reduces its weight and power consumption.

5. Conclusions

Results achieved also allow stating that:

1) the values of geometric parameters of two-link mechanical systems have significant impact on energy expenditures of their control processes;

2) the use of semi-passive actuators allows to use engines that implement active component control, almost 10 times less power than the fully active case;

3) the use one of the joints of the manipulator entirely passive actuator can be limited to only one engine in the second joint, and its capacity will be less than the total engine power in case of fully active control;

4) introduction of partially or entirely passively controlled actuators does not provide decrease in general energy expenditures required for implementation of manipulator cyclic motion.

At present the perspectives of the compound of the proposed approaches to significantly expand the field of use of manipulators with partially and fully passive actuators are analyzed. Also methods for replacing the active actuators on passive, based on the approximation-compensatory approach, in the case of multi-link mechanical systems are developed.

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